BIPM Capacity Building & Knowledge Transfer Programme

2024 BIPM - TÜBİTAK UME Project Placement

REPORT

Project Name	Analysis of the influence of factors on the result of mass measurements using Kibble Balance
Description	Assessment of the influence of factors of external magnetic fields, free fall acceleration on the result of mass measurements using Kibble Balance
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Date	01.09.2024 – 29.11.2024

Motivation

This project was necessary for me to gain practical experience of working and measuring using the Kibble Balance, as well as assessing the influence of factors. As part of the project, I planned to carry out measurements, but due to circumstances, I was able to thoroughly study the process of setting up the balance and gain experience of working with a gravimeter and interferometers. This experience will help improve the prototype Kibble Balance at VNIIM. During my internship at UME, the following works were carried out:

- research of the influence of external mag1netic fields on the Kibble Balance measurements
- research of the influence of non-vertical motions of the magnet assembly (alignment procedure) magnetic fields on the Kibble Balance measurements
- measurements of free fall acceleration.

In this report I will focus on the novel alignment procedure which reflects the latest improvements of UME KB.

Introduction

In UME KB-3 the mass artefact with a nominal value of 1 kg is kept under ambient air conditions. The load cell is used for the comparative measurements of the vertical Lorentz force on the coil pair and the gravitational force acting on the mass artefact. The Lorentz force on the coil pair is generated with the help of programmable Keithley 6220 current source supplying an electrical current of 38.4 mA to the coil pair. The coil pair experiences a downward Lorentz force for the positive values of the electrical current while for the negative ones the direction of the force is reversed. The mass loading device places the mass artefact on the handler when the Lorentz force is directed upward and lifts it from the handler for the downward one so that the range of the load cell is not exceeded. One cycle of successive mass-on and mass-off configuration lasts 180 s and the difference between the load cell readings for the downward (the mass artefact is off the handler) and upward (the mass artefact is on the handler) Lorentz forces is obtained for each cycle. The mean value of these differences throughout the experiment is denoted by ΔF. Similarly, the mean values of the differences between the downward and upward Lorentz forces and between the positive and negative values of the electrical current are represented by Fz and I, respectively. Load cell reading data ΔF represents the small difference between the Lorentz force Fz and the effective weight of the artifact mass [1]. The mass m of a mass artefact is assigned via

$$m = \frac{F_z - \Delta F}{g(1 - \frac{\rho_{air}}{\rho})} \tag{1}$$

where g is the gravitational acceleration in the center of mass of the artefact, ρ_{air} is the air density and ρ is the density of the mass artefact [1].

$$F_z = \frac{I\Phi}{Z}\ (1 + Q_{al} + Q_{ext} + Q_{nu}) \tag{2}$$

where I is the current on the coil pair; Φ is the magnetic flux passing through the coil pair; Z is the vertical displacement; Q_{al} is the correction for the effects due to the nonvertical motion of the magnet assembly; Q_{ext} is the correction for the external magnetic field; Q_{nu} is the non-uniformity permanent magnet magnetic field [1]. In the KB experiment along with motion of the magnet assembly parallel to the direction of earth free fall acceleration there appear non vertical motions two of which are displacements in horizontal plane $\vec{r} = (x, y)$ and three ones are rotations $\vec{\Theta} = (\theta_X, \theta_Y, \theta_Z)$ around the corresponding axis. The parasitic magnetic flux induced by such motions is of the form

$$\Phi_{par} = \ \vec{G} \cdot \vec{r} + \vec{K} \cdot \vec{\Theta} \tag{3}$$

where $\vec{G}$ and $\vec{K}$ are geometric factors which are proportional to the horizontal force and torque in the coil when the electrical current flows through it. The alignment factor in (2) then reads as $Q_{al} = \Phi_{par}/\Phi$.

Alingment Factor estimation technique

As we see below another representation for the parasitic magnetic flux (3) appears to be more suitable:

$$\Phi_{par} = \ \vec{H} \cdot \vec{O} \tag{4}$$

Here $\vec{H}$ and $\vec{O}$ are five dimensional vectors constructed from geometric factors and displacements:

$$\vec{H} = (\ |\vec{r}|\vec{G}, |\vec{\Theta}|\vec{K}) \tag{5}$$

$$\vec{O} = \left(\frac{\vec{r}}{|\vec{r}|}, \frac{\vec{\theta}}{|\vec{\theta}|}\right) \tag{6}$$

Note that such a representations allows us to treat linear and rotational displacements in a unified way by collecting these degrees of freedom in the single five vector $\vec{O}$. Note that our strategy in the study of the parasitic effects are based on the estimation of the whole effect given by the scalar product in the right hand side of (4) rather than on the estimation of each components of the horizontal force and torque.

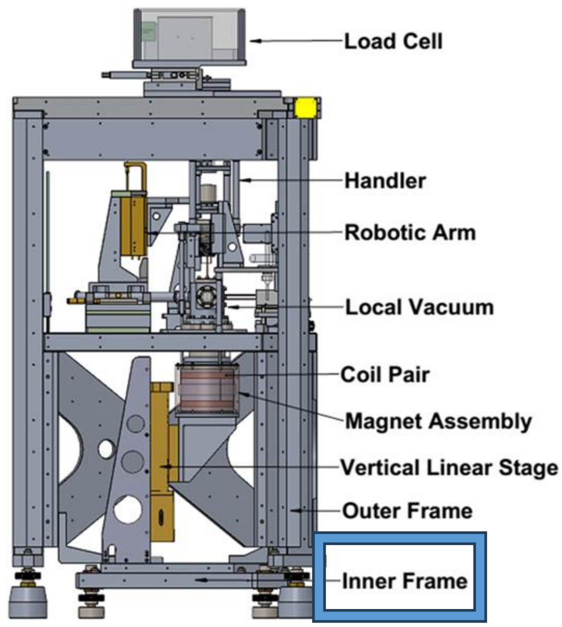

Figure 1. The UME KB-3 apparatus [1].

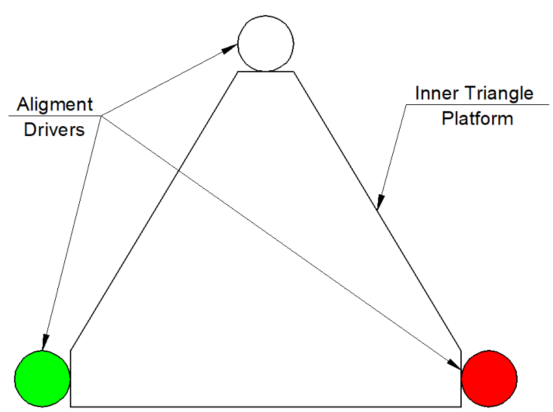

Figure 2. Alignment Drivers and Inner Triangle Platform.

To reach this purpose we use three lines motors which moves the legs of the inner platform of the UME Kibble Balance which support the magnet assembly stage. At the first stage we generate five independent motions $\overrightarrow{O_i}$, $i = 1...5$, of the magnet assembly by operating these three motors in appropriate way. Such motions induce magnetic flux Φ_i through the coil. Note that legs motor along horizontal and rotational displacements gives rise to vertical displacement Z_i of the magnet assembly also. Since geometric factors in the alingment experiment are the same as in the main KB experiment we have

$$\Phi_i - G_z Z_i = \vec{H} \cdot \overrightarrow{O_i} \tag{7}$$

Here G_z is the geometric factor which is proportional to the vertical Lorentz force in the coil: $G_z = \frac{\mathrm{F_z}}{\mathrm{I}}$. Using Gauss MatLab code we decompose the displacement five vector $\vec{O}$ of the main KB experiment into the displacements $\overrightarrow{O_1}, \overrightarrow{O_2}, \overrightarrow{O_3}, \overrightarrow{O_4}, \overrightarrow{O_5}$ measured in the alignment experiment:

$$\vec{O} = c_1\overrightarrow{O_1} + c_2\overrightarrow{O_2} + c_3\overrightarrow{O_3} + c_4\overrightarrow{O_4} + c_5\overrightarrow{O_5} \tag{8}$$

Where c_1 ... c_5 are the decomposition coefficients. Inserting the above decomposition into (4) we finally get the estimation for the parasitic magnetic flux during the KB experiment

$$\Phi_{par} = \sum_{j=1}^{5} c_k \ (\Phi_j - G_z Z_j) \tag{9}$$

Using the definition defined in the İntroduction section one can see that the uncertainty of the alignment factor Q_{al} is

$$\sigma_{al} = |c| \cdot \sqrt{\frac{\sigma_\Phi^2}{\Phi^2} + \frac{\sigma_z^2}{Z^2}} \tag{10}$$

We see that the uncertainty of the alignment factor is proportional to the norm $|c|$ of the decomposition vector. To reduce the uncertainty of the alignment factor experimental arrangments should be done in a such way that the value of the decomposition vector norm will be a small as possible. Name the following conditions should be fulfilled:

1. $|\overrightarrow{O_i}| >> |\vec{O}|$;
2. $\overrightarrow{O_i}$ vectors are independent in the well-defined way.

Results and analysis

The magnet assembly positions are measured by 6 channel Michelson interferometers using cubic mirror attached rigidly to the top surface of the magnet assembly. Before starting determination of alignment factor preliminary works are carried out to reduce unwilled horizontal and rotational displacements. This is achieved by using legs motors and piezo stage which support the cubic mirror. The legs motors provide periodic motions which fundamental period is arranged to be one second. The duration of each alignment experiment is chosen to be five minutes. Magnet assembly one period position curve which is the averages of all one period curves is shown in the Figure 3. In this figure two linear components of the position vector in *X* and *Y* directions and three rotational components are depicted.

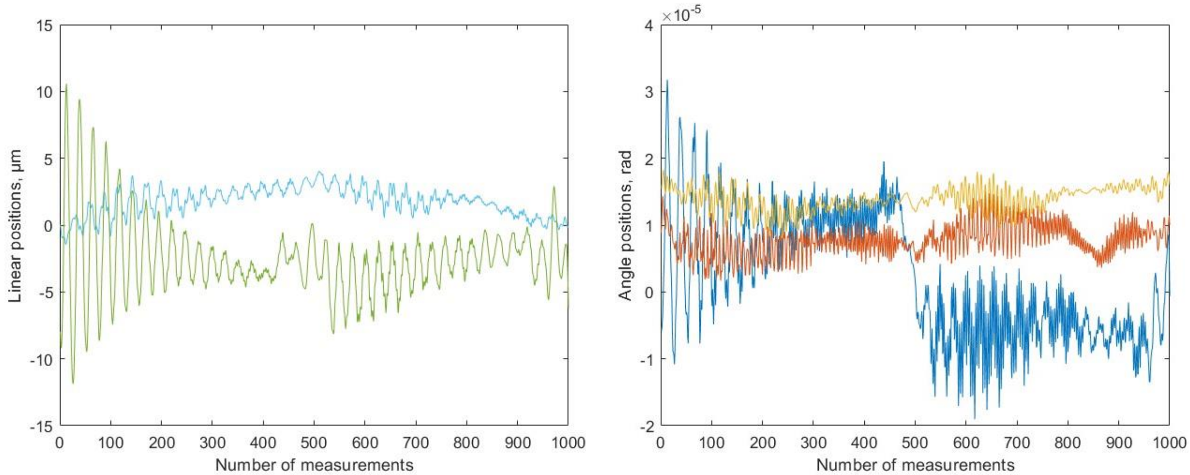

Figure 3. The mean curves magnet positions (X_O-axis – light-blue curve; Y_O-axis – light-green curve; θ_{Xo} – dark-blue curve; θ_{Yo} – orange curve; θ_{Zo} – yellow curve).

In the applications we need magnet assembly displacements rather than positions. During the experiment magnetic flux in the coil is measured by integration multimeter. To meet Faraday Induction law requirements, we need to define displacements of the magnet assembly by taking difference of its final positions and initial positions. Initial and positions are corresponded to time when integration multimeter starts and ends its measurements. On the Figure 4 linear and angular displacements are depicted. The integration time is chosen to be 360 ms.

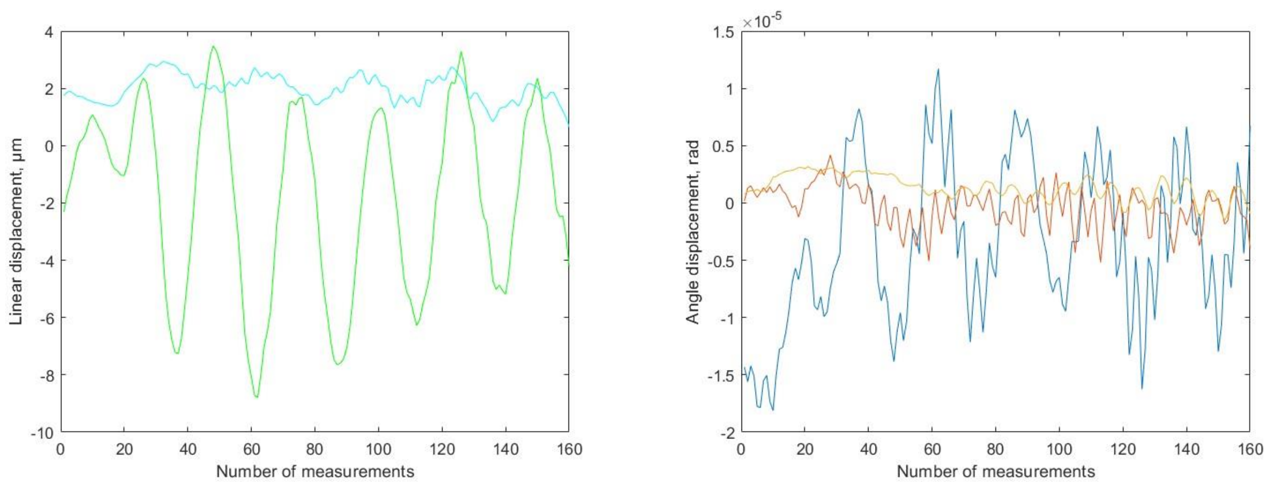

Figure 4. The mean curves magnet displacement (X_O-axis – light-blue curve; Y_O-axis – light-green curve; θ_{Xo} – dark-blue curve; θ_{Yo} – orange curve; θ_{Zo} – yellow curve).

İn ordinal axis In Figure 4 the curves show the dependence of linear and angular displacements on the chose of the initial time or initial position where magnetic flux measurements start. From these graphics we see that the displacements acquire minimal values in the region between 60 ms and 70 ms. In our further analysis we choose the value of linear and angular displacements at 67 ms. The norms of linear vector and rotational three vectors are $|\vec{r}| = 2.5229\ \mu m$ and , $|\vec{\Theta}| = 2.5192\ \mu Rad$ and the normalized five vector is of the form

$$\vec{O} = (0.9467, -0.3220, -0.6376, 0.5384, 0.5510) \qquad (11)$$

To emulate the non-vertical motions of the magnet assembly in the main KB experiment below mentioned motions are initiated according to the motor colors from Figure 2:

1. Movement of the Red alignment driver;
2. Movement of the Wight alignment driver;
3. Movement of the Green alignment driver;
4. Movement of the Green and Red alignment drivers;
5. Movement of the Green and White alignment drivers;
6. Movement of the Red and White alignment drivers;
7. Movement of the Green and Red alignment drivers with 90° phase delay;
8. Movement of the Green and White alignment drivers with 90° phase delay;
9. Movement of the Red and White alignment drivers with 90° phase delay.

Linear and angular displacements are normalized with respect to the norms $|\vec{r}|$ and $|\vec{\Theta}|$ to reach dimensionless five vectors for each of the above motion

$$\vec{Q}_k = \left(\frac{\vec{r_k}}{|\vec{r}|}, \frac{\vec{\Theta}_k}{|\vec{\Theta}|}\right)\ , k = 1 \dots 9 \qquad (12)$$

which components are given in Table 1.

Table 1. Components of $\vec{Q}_k$vectors.

k	$\frac{\vec{r_k}}{\lvert\vec{r}\rvert}$		$\frac{\vec{\Theta}_k}{\lvert\vec{\Theta}\rvert}$		
1	-13.0970	8.3160	-7.5394	-9.5547	-0.3243
2	-0.0389	-16.5129	9.7129	-1.2748	0.6733
3	13.3242	8.7488	-9.2986	8.1481	-0.6766
4	-0.0593	17.1336	-16.3324	2.4991	-0.0785
5	13.4369	- 7.2795	1.5296	8.8149	0.6360
6	-13.0970	8.3160	-7.5394	-9.5547	-0.3243
7	15.3515	7.6141	-7.3825	15.8637	2.3245
8	12.1941	10.4063	-12.4063	9.1941	0.0359

9	-13.5127	10.6875	-11.7243	-12.3075	-1.4447

To minimize the value of the decomposition vector C norm from the above set of motions one has to choose 5 linear independent motions in a suitable way. To reach this goal we follow the following procedure. Firstly, we chose the vector $\vec{Q}_{k_a}$ which will the best approximation for the five vectors: the projection of the vector 0 on the plane orthogonal to the vector $\vec{Q}_a$ takes minimal value among all other vectors in the set of motion. We denote the orthogonal projection of the vector 0 by 0' which is

$$\overrightarrow{O'} = \vec{O} - \frac{\vec{O} \times \overrightarrow{Q_a}}{\overrightarrow{Q_a} \times \overrightarrow{Q_a}} \times \overrightarrow{Q_a} \tag{13}$$

The above condition is satisfied by $a = 7$ which corresponds to the motion of Green and Red motors started with 90° phase delay. After the above procedure we have $\overrightarrow{O'}$ and $\overrightarrow{Q'}_k$ vector instead of $\vec{O}$ and $\vec{Q}_k$, where $\overrightarrow{Q'}_k$ is the projection of $\vec{Q}_k$ vectors on the plane orthogonal to $\overrightarrow{Q_a}$.

$$\overrightarrow{Q'}_k = \overrightarrow{Q_k} - \frac{\overrightarrow{O_k} \times \overrightarrow{Q_7}}{\overrightarrow{Q_7} \times \overrightarrow{Q_7}} \times \overrightarrow{Q_7} \quad , k = 1 \dots 9 \tag{14}$$

Since by definition $\overrightarrow{Q'}_7 = 0$ only 8 vectors $\overrightarrow{Q'}_k$ are nonzero. At the second stage we apply the same procedure to the vector $\overrightarrow{O'}$ by choosing suitable vector $\overrightarrow{Q'}_b$ which approximate $\overrightarrow{O'}$ in a mauch stutable way. As result we arrive at the vector $\overrightarrow{O''}$ and $\overrightarrow{Q''}_k$. Repeating these steps we finally get the set of indexes $a = 7, 5, 3, 4, 9$ and $\vec{O}, \overrightarrow{O'}, \overrightarrow{O''}, \overrightarrow{O'''}, \overrightarrow{O''''}$. There figures are given in Figure 5.

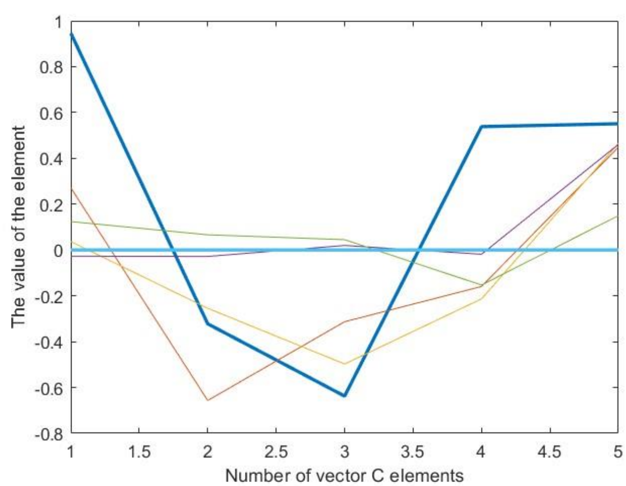

Figure 5. 5 iterations of the $\vec{O}, \overrightarrow{O'}, \overrightarrow{O''}, \overrightarrow{O'''}, \overrightarrow{O''''}$ vector ($\vec{O}$ – dark-blue curve; $\overrightarrow{O'}$ – red curve; $\overrightarrow{O''}$ – yellow curve; $\overrightarrow{O'''}$ – purple curve; $\overrightarrow{O''''}$ – light-blue curve).

Thus, we selected five linearly independent vector $\overrightarrow{Q_7}, \overrightarrow{Q_5}, \overrightarrow{Q_3}, \overrightarrow{Q_4}, \overrightarrow{Q_9}$ from the set of nine motions. To agree with notations in (8) we redefine these vectors by $\overrightarrow{O_1}, \overrightarrow{O_2}, \overrightarrow{O_3}, \overrightarrow{O_4}, \overrightarrow{O_5}$ respectively. Using a MATLAB code, we get the decomposition vector $\vec{c}$ and its norm $|\vec{c}|$.

$$\vec{c} = (0.4687,\ 0.0602,\ 0.1308,\ -0.5132,\ 0.3978) \tag{15}$$

$$|\vec{c}| = 0.8137 \tag{16}$$

Conclusions and Future Work

I have gained experience working on the Kibble Balance. I have carried out the setup and assembly procedures, and my knowledge of the process for measuring mass using the Kibble Balance has been deepened. I also acquired skills in working with relative gravimeters and interferometers. All this will help improve the prototype Kibble Balance at VNIIM.

Acknowledgements

I would like to express my sincere gratitude to Doç. Dr. Haji AHMADOV for his help in mastering the materials. Additionally, I would like to extend my thanks to Ms. Müge ATAM, Mr. Chingis KUANBAYEV, and other organizers of the internship for their efforts.

References

1. Hacı Ahmedov, Recep Orhan, Beste Korutlu “UME Kibble balance operating in air”, Metrologia (60), 2023, 015003.