**Instructions for Linking New Comparisons to CCT Key Comparison 3**

The following instructions describe how to link new comparisons of ITS-90 temperature realizations to, or through, CCT Key Comparison 3 (KC3). While the accompanying example does not explicitly cover the range of situations that may come up in the use of KC3, it covers the most common case that is likely to be of interest. Following the general ideas behind the example should allow it to be easily extended to other situations by analogy.

1. Collect the necessary information from the new comparisons needed to compute the relevant estimates of $\boldsymbol{\Delta T}$ and their expanded uncertainties. The necessary information includes:
    a. the mean resistance ratios, $\overline{\boldsymbol{W}}_{\boldsymbol{i,j}}$, for the $\boldsymbol{i}$ laboratories participating in the new comparison and the $\boldsymbol{j}$ SPRT's used by each lab,
    b. the number of measurements averaged to obtain each value of $\overline{\boldsymbol{W}}_{\boldsymbol{i,j}}$,
    c. the freeze-to-freeze repeatabilities of the resistance ratios for each of the $\boldsymbol{i}$ laboratories and their associated degrees of freedom (referred to as $\boldsymbol{S_A}$ and $\boldsymbol{DF_A}$ in the KC3 report),
    d. the combined uncertainties from other all other sources of uncertainty within each laboratory that are fixed for a given laboratory and fixed point cell and their associated degrees of freedom (referred to as $\boldsymbol{S_B}$ and $\boldsymbol{DF_B}$ in the KC3 report), and
    e. the combined uncertainties associated with the $\boldsymbol{j}$ SPRT's used by the laboratories (referred to as $\boldsymbol{u_{SPRT}}$ in the KC3 report) and their degrees of freedom ($\boldsymbol{DF_{SPRT}}$).
2. Collect the relevant information (as in a-e above) from KC3 needed for the comparisons of interest. All of this information is available in the full report on KC3 or in the summary of the report published in Metrologia.
3. Set up the equation for the new estimate of $\boldsymbol{\Delta T}$ based on the intermediate comparisons with each individual SPRT, averaging multiple comparisons between particular pairs of laboratories made with multiple SPRT's.
4. Compute the combined standard uncertainty of $\boldsymbol{\Delta T}$ based on the equation constructed in step 3 using $\boldsymbol{S_A}$, $\boldsymbol{S_B}$, and $\boldsymbol{u_{SPRT}}$ for the uncertainties of the end laboratories and only $\boldsymbol{S_A}$ and $\boldsymbol{u_{SPRT}}$ for the intermediate laboratories (this accounts for the covariances in the data arising from the normalization of transfer instruments through the intermediate laboratories).
5. Compute the effective degrees of freedom for the combined standard uncertainty of $\boldsymbol{\Delta T}$ using the Welch-Satterthwaite formula and the equation used in step 4, after collecting the terms from each laboratory so that each unique uncertainty estimate appears only once in the equation.
6. Compute the expanded uncertainty for $\boldsymbol{\Delta T}$ using the results from steps 4 and 5 and the appropriate coverage factor obtained from the Student's *t* distribution.

The following example will help clarify these instructions and show that process is not as complicated as it might seem at first.

## Example

Suppose that an NMI that did not participate in KC3 does a bilateral comparison with PTB at the Sn point and also wishes to link to IMGC, another KC3 participant. In the example all temperature differences and uncertainties are reported in mK.

1. Collect the relevant information from the new comparison. (Note: Hypothetical numbers created for the example are given in blue.)

| Lab | SPRT | $\overline{W}$ | $n$ | $S_A$ | $DF_A$ | $S_B$ | $DF_B$ | $u_{SPRT}$ | $DF_{SPRT}$ |
|---|---|---|---|---|---|---|---|---|---|
| NMI | 1000 | 1.89270407 | 5 | 0.23 | 53 | 0.28 | ∞ | 0.047 | 3.727 |
| PTB | 1000 | 1.89270654 | 5 | 0.15 | 149 | 0.43 | ∞ | | |

2. Collect the relevant information from KC3.

| Lab | SPRT | $\overline{W}$ | $n$ | $S_A$ | $DF_A$ | $S_B$ | $DF_B$ | $u_{SPRT}$ | $DF_{SPRT}$ |
|---|---|---|---|---|---|---|---|---|---|
| PTB | 1030A | 1.89271583 | 3 | 0.15 | 149 | 0.43 | ∞ | 0.094 | 4.421 |
| NIST | 1030A | 1.89271603 | 3 | 0.12 | 301 | 0.02 | ∞ | | |
| PTB | 1030B | 1.89271880 | 1 | 0.15 | 149 | 0.43 | ∞ | 0.094 | 4.421 |
| NIST | 1030B | 1.89271723 | 2 | 0.12 | 301 | 0.02 | ∞ | | |
| PTB | 4385 | 1.89268664 | 5 | 0.15 | 149 | 0.43 | ∞ | 0.000 | NA |
| NIST | 4385 | 1.89268574 | 4 | 0.12 | 301 | 0.02 | ∞ | | |
| NIST | 1098B | 1.89271355 | 2 | 0.12 | 301 | 0.02 | ∞ | 0.000 | NA |
| IMGC | 1098B | 1.89271307 | 1 | 0.10 | 7 | 0.18 | ∞ | | |

3. Compute the estimate of $\Delta T$ based on the intermediate comparisons with each individual SPRT, averaging multiple comparisons between particular pairs of laboratories made with multiple SPRT's.

$$\begin{aligned}\Delta T = \{ & (\overline{W}_{NMI,1000} - \overline{W}_{PTB,1000}) \\ & +[(\overline{W}_{PTB,1030A} - \overline{W}_{NIST,1030A}) \\ & +(\overline{W}_{PTB,1030B} - \overline{W}_{NIST,1030B}) \\ & +(\overline{W}_{PTB,4385} - \overline{W}_{NIST,4385})]/3 \\ & +(\overline{W}_{NIST,1098B} - \overline{W}_{IMGC,1098B})\} / (dW_r / dT)\end{aligned}$$

$$\begin{aligned}\Delta T = \{ & (1.89270407-1.89270654) \\ & +[(1.89271583-1.89271603) \\ & +(1.89271880-1.89271723) \\ & +(1.89268664-1.89268574)]/3 \\ & +(1.89271355-1.89271307)\} / 0.000003713 \\ = & -0.3321663\end{aligned}$$

Note: Many digits are carried through all the calculations to avoid round-off errors. Rounding is only done when reporting final results that will not be used in further calculations.

4. Compute the combined standard uncertainty of $\Delta T$ based on the equation constructed in step 3 using $S_A$, $S_B$, and $u_{SPRT}$ for the uncertainties of the end laboratories (NMI and IMGC) and only $S_A$ and $u_{SPRT}$ for the intermediate laboratories.

$$\begin{aligned}u_c(\Delta T) = [ & (\frac{S^2_{A,NMI}}{n_{NMI,1000}} + S^2_{B,NMI} + \frac{S^2_{A,PTB}}{n_{PTB,1000}} + u^2_{1000}) \\ & +[(\frac{S^2_{A,PTB}}{n_{PTB,1030A}} + \frac{S^2_{A,NIST}}{n_{NIST,1030A}} + u^2_{1030A}) \\ & +(\frac{S^2_{A,PTB}}{n_{PTB,1030B}} + \frac{S^2_{A,NIST}}{n_{NIST,1030B}} + u^2_{1030B}) \\ & +(\frac{S^2_{A,PTB}}{n_{PTB,4385}} + \frac{S^2_{A,NIST}}{n_{NIST,4385}})]/3^2 \\ & +(\frac{S^2_{A,NIST}}{n_{NIST,1098B}} + \frac{S^2_{A,IMGC}}{n_{IMGC,1098B}} + S^2_{B,IMGC})]^{1/2}\end{aligned}$$

$$\begin{aligned}u_c(\Delta T) = [ & (\frac{0.23^2}{5} + 0.28^2 + \frac{0.15^2}{5} + 0.047^2) \\ & +[(\frac{0.15^2}{3} + \frac{0.12^2}{3} + 0.094^2) \\ & +(\frac{0.15^2}{1} + \frac{0.12^2}{2} + 0.094^2) \\ & +(\frac{0.15^2}{5} + \frac{0.12^2}{4})]/3^2 \\ & +(\frac{0.12^2}{2} + \frac{0.10^2}{1} + 0.18^2)]^{1/2} \\ = & \ 0.390921\end{aligned}$$

5. Compute the effective degrees of freedom for the combined standard uncertainty of $\Delta T$ using the Welch-Satterthwaite formula and the equation used in step 4, after collecting the terms from each laboratory so that each unique uncertainty estimate appears only once in the equation. Note: the same estimate of uncertainty was used for SPRT's ***1030A*** and ***1030B***. Therefore, when terms are collected to reduce the equation for $DF_{\Delta T}$ to independent inputs, the terms associated with those SPRT's are denoted by $u_{1030}$ and also collected. Typically, however, each SPRT used will have an uncertainty estimate that is independent of all other SPRT's used.

$$
\begin{aligned}
u_c = \{ & [\frac{1}{n_{NMI,1000}}] S^2_{A,NMI} + S^2_{B,NMI} \\
& + [(\frac{1}{n_{PTB,1030A}} + \frac{1}{n_{PTB,1030B}} + \frac{1}{n_{PTB,4385}}) \frac{1}{3^2}] S^2_{A,PTB} \\
& + [(\frac{1}{n_{NIST,1030A}} + \frac{1}{n_{NIST,1030B}} + \frac{1}{n_{NIST,4385}}) \frac{1}{3^2} + \frac{1}{n_{NIST,1098B}}] S^2_{A,NIST} \\
& + [\frac{1}{n_{IMGC,1098B}}] S^2_{A,IMGC} + S^2_{B,IMGC} \\
& + u^2_{1000} + [\frac{2}{3^2}] u^2_{1030} \}^{1/2}
\end{aligned}
$$

$$
\begin{aligned}
u_c = \{ & [\frac{1}{5}] 0.23^2 + 0.28^2 \\
& + [(\frac{1}{3} + \frac{1}{1} + \frac{1}{5}) \frac{1}{3^2}] 0.15^2 \\
& + [(\frac{1}{3} + \frac{1}{2} + \frac{1}{4}) \frac{1}{3^2} + \frac{1}{2}] 0.12^2 \\
& + 0.10^2 + 0.18^2 \\
& + 0.047^2 + [\frac{2}{3^2}] 0.094^2 \}^{1/2}
\end{aligned}
$$

$$
\begin{aligned}
DF_{\Delta T} = u_c^4 \{ & [\frac{1}{n_{NMI,1000}}]^2 \frac{S^4_{A,NMI}}{DF_{A,NMI}} + \frac{S^4_{B,NMI}}{DF_{B,NMI}} \\
& + [(\frac{1}{n_{PTB,1030A}} + \frac{1}{n_{PTB,1030B}} + \frac{1}{n_{PTB,4385}}) \frac{1}{3^2}]^2 \frac{S^4_{A,PTB}}{DF_{A,PTB}} \\
& + [(\frac{1}{n_{NIST,1030A}} + \frac{1}{n_{NIST,1030B}} + \frac{1}{n_{NIST,4385}}) \frac{1}{3^2} + \frac{1}{n_{NIST,1098B}}]^2 \frac{S^4_{A,NIST}}{DF_{A,NIST}} \\
& + [\frac{1}{n_{IMGC,1098B}}]^2 \frac{S^4_{A,IMGC}}{DF_{A,IMGC}} + \frac{S^4_{B,IMGC}}{DF_{B,IMGC}} \\
& + \frac{u^4_{1000}}{DF_{1000}} + [\frac{2}{3^2}]^2 \frac{u^4_{1030}}{DF_{1030}} \}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
DF_{\Delta T} = 0.390321^4 \{ & [\frac{1}{5}]^2 \frac{0.23^4}{53} + \frac{0.28^4}{\infty} \\
& + [(\frac{1}{3} + \frac{1}{1} + \frac{1}{5}) \frac{1}{3^2}]^2 \frac{0.15^4}{149} \\
& + [(\frac{1}{3} + \frac{1}{2} + \frac{1}{4}) \frac{1}{3^2} + \frac{1}{2}]^2 \frac{0.12^4}{301} \\
& + \frac{0.10^4}{7} + \frac{0.18^4}{\infty} \\
& + \frac{0.047^4}{3.727} + [\frac{2}{3^2}]^2 \frac{0.094^4}{4.421} \}^{-1} \\
= {} & 653.6026
\end{aligned}
$$

6. Compute the expanded uncertainty for $\Delta T$ using the results from steps 4 and 5 and the appropriate coverage factor obtained from the Student's ***t*** distribution.

$$U = t_{0.975, DF_{\Delta T}} u_c$$

$$
\begin{aligned}
U &= 1.9636(0.390321) \\
&= 0.766434
\end{aligned}
$$

After completing all of the calculations the answer can be rounded to a reasonable number of significant digits and reported as $\Delta T \pm U$. It is also important to report the confidence level used, the degrees of freedom associated with the combined standard uncertainty, and the coverage factor used to obtain the expanded uncertainty. For this example the answer would be reported as ***0.33 mK ± 0.77 mK*** at the ***95%*** confidence level, based on a combined standard uncertainty with ***653*** degrees of freedom and with a coverage factor obtained from Student's ***t*** distribution of ***1.9636***.