A few considerations on measurement uncertainties expressed in dB

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- Case of sound pressure level (SPL)

By definition, at a given frequency for a sound source

$$x_{\mathrm{SPL}} = 20\log\left(\frac{P}{P_{\mathrm{ref}}}\right)$$

to which a measurement uncertainty is associated, so you may say

$$x_{SPL} = x_{\mathrm{measured}} \pm u_x \ ; \quad k=1$$

$$x_{\mathrm{SPL}} = x_{\mathrm{measured}} \pm 2u_x \ ; \quad k=2$$

or equivalent $x_{\mathrm{SPL}} = x_{\mathrm{measured}} \pm U$; k=2

However,

- what is the measurand?
- What is the expression for u_x
- Since u_x may be expressed as $u_x = 20\log\left(\frac{P^*}{P^*_{\mathrm{ref}}}\right)$; what is P^*_{ref}

If acoustic pressure is the measurant, then

$$x_{\mathrm{SPL}} = 20\log\left(\frac{P \pm u_p}{P_{\mathrm{ref}}}\right)$$

then, there are upper and lower bounds given by

$$x^+_{\mathrm{SPL}} = 20\log\left(\frac{P + u_p}{P_{\mathrm{ref}}}\right)$$

$$x^-_{\mathrm{SPL}} = 20\log\left(\frac{P - u_p}{P_{\mathrm{ref}}}\right)$$

take the upper bound,

$$x_{\text{SPL}}^{+} = 20\log\left(\frac{P + u_p}{P_{\text{ref}}}\right)$$

$$x_{\text{SPL}}^{+} = 20\log\left(\frac{P}{P_{\text{ref}}} + \frac{u_p}{P_{\text{ref}}}\right)$$

$$x_{\text{SPL}}^{+} = 20\log\left(\frac{P}{P_{\text{ref}}}\left(1 + \frac{u_p}{P}\right)\right)$$

$$x_{\text{SPL}}^{+} = 20\log\left(\frac{P}{P_{\text{ref}}}\right) + 20\log\left(1 + \frac{u_p}{P}\right)$$

then,

$x_{\text{SPL}} = x_{\text{measured}} + u_x$; k=1

- what about for a coverage factor $k = 2$?
- Is it $x_{\text{SPL}}^{+} = x_{\text{measured}} + 20\log\left(1 + \frac{u_p}{P}\right)^2$? Is this a deviation from the GUM?

or

* Is it $x_{\text{SPL}}^{+} = x_{\text{measured}} + 20\log\left(1 + \frac{2u_p}{P}\right)$?

- What if a larger coverage factor k is needed?
- Central limit theorem applies to measurements of acoustics pressure P but does not for measurements of SPL (¿?)
- Values of $u_r = u_p/p > 0.1$ would lead to sever overestimations of u_x.

Remark

Make your uncertainty estimations using SI units or use relative uncertainties. After that, transform your uncertainty estimation to dBs.