ON THE UNIT OF REACTIVE POWER

Draft Proposal to be discussed as working document at the next CCEM meeting in 2017

1. Introduction

According to the draft 9th SI Brochure [1]:

(1) The SI is a consistent system of units for use in international trade, high-technology manufacturing, human health and safety, protection of the environment, global climate studies and in the basic science that underpins all of these.
(2) Individual countries have established rules concerning the use of units by national legislation, either for general use or for specific areas such as commerce, health, public safety, and education. In almost all countries this legislation is based on the SI. The International Organization of Legal Metrology (OIML) is charged with the international harmonization of the technical specifications of this legislation.
(3) It is recognized that some non-SI units are widely used and are expected to continue to be used for many years. Therefore, the CIPM has accepted some non-SI units for use with the SI; these are listed in Table 8.

In fact, national legislations regarding the use of units are in general based on the SI. The SI Brochure has been increasingly adopted as a reference for that purpose. This is good for promoting the SI in the world. Nevertheless, problems occur when a non-SI unit which is relevant for national commerce and economics is not included in Table 8 of that document. This table lists the non-SI units accepted for use with the SI units. National agencies are then required to look instead for other standards to base their legislation on the use of units. In fact, they may prefer to do so at the outset as the SI Brochure does not provide some of the information they need for legal and commerce purposes. This may limit the utilization of the SI Brochure.

The non-SI unit we are concerned here is: var. The quantity *reactive power* whose unit name is var with unit symbol var is relevant for national commerce and economics; it is part of legal legislation in developed countries; and it is expected to continue to be used in the future. Our objective here is to justify the inclusion of such unit in Table 8 of the SI Brochure.

The quantity *active power*, whose unit name is watt with unit symbol W, should not be included in the SI Brochure as the quantity *power*, whose unit name is watt with unit symbol W, is already included in Table 4 which lists the 22 SI units with special names and symbols.

It is not necessary to include the quantity *apparent power*, whose unit name is volt-ampere with unit symbol V A, in Table 6 of the SI Brochure which lists examples of coherent derived units whose names and symbols also include derived units. The reason is clearly stated in page 12 of that document: it is not possible to provide a complete list of derived quantities and derived units.

Though all these three quantities (active power, reactive power and apparent power) have the same unit expressed in terms of base units as kg m^2 s^{-3}, electrical engineers need to compute separately each of them when designing electrical facilities. To avoid confusion, electrical engineers have historically assigned different units for each of those quantities. The active power is the average energy flow from the utility to the costumer. In fact, it is the actual average power supplied which is consumed in realizing work and dissipating heat. The reactive power is related with the energy that is swapped back and forth between the utility and the consumer facility. It is proportional to the average energy stored in the electric and magnetic fields. The active power does not account for such component since there is no average energy flow related to such component, or in other words, there is no average increase in the energy stored in the costumer facility. The apparent power is the square root of the squared sum of the previous two power components and provides relevant information concerning the facility size and its short-circuit requirements.

Utilities are increasingly interested in measuring separately the active power, reactive power and apparent power. There is increasing concern in taxing separately each of these power components even for the residential costumer. The electrical instruments to be used should give clear indication on each of these power components so that the costumer knows what is being paid for. As the instrument readings are designed to help both the utility and the costumer, the instruments provide different units for each of these power components that are widely accepted.

As stated above, our objective here is to justify the inclusion of the unit var in Table 8 of the SI Brochure. In section 2, the average power flow in steady-state ac problems is computed. In section 3, a similar calculation is performed using circuit theory approximations. In section 4, the complex Poynting vector theorem is deduced. The Poynting power density is related to the circuit power input in section 5. The active power, reactive power and apparent power are finally defined in section 6. The role of reactive power is discussed in section 7. A few applications of the reactive power in electrical engineering work are described in section 8. The impact on Appendices B and C of the MRA is discussed in section 9. An alternative polarity convention is commented in section 10. Conclusions are presented in section 11. Acknowledgments are in section 12. The references and the bibliography used to compile this text are added at the end.

2. Time-averages of Time-harmonic Electric and Magnetic Fields

Sinusoidal time variation at a fixed frequency ω is of practical interest, as many of our sources generate sinusoidal outputs.

For sinusoidal time variation it is convenient to use the complex notation (phasor method) to represent the instantaneous values of the electric field **E**(t) and the magnetic field **H**(t) as the real parts of complex exponentials $\hat{\mathbf{E}}e^{j\omega t}$ and $\hat{\mathbf{H}}e^{j\omega t}$, respectively, where $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ are complex vector functions of position, and j is the imaginary number.

We can write then

$$\begin{aligned}\mathbf{E}(t) &= \mathrm{Re}\left(\hat{\mathbf{E}}e^{j\omega t}\right) = \mathrm{Re}\left[\left(\mathbf{E}_r + j\mathbf{E}_i\right)\left(\cos\omega t + j\sin\omega t\right)\right] \\ &= \mathbf{E}_r\cos\omega t - \mathbf{E}_i\sin\omega t\end{aligned} \tag{1}$$

where the real part of $\hat{\mathbf{E}}$ is $\mathbf{E}_r$ and the imaginary part is $\mathbf{E}_i$, with similar notation for $\hat{\mathbf{H}}$. Poynting´s vector representing the energy flow at instant t can now be expressed as

$$\begin{aligned}\mathbf{E}(t)\times\mathbf{H}(t) = {} & \left(\mathbf{E}_r\times\mathbf{H}_r\right)\cos^2\omega t + \left(\mathbf{E}_i\times\mathbf{H}_i\right)\sin^2\omega t \\ & - \left[\left(\mathbf{E}_r\times\mathbf{H}_i\right) + \left(\mathbf{E}_i\times\mathbf{H}_r\right)\right]\sin\omega t\cos\omega t\end{aligned} \tag{2}$$

Recall that the **E**(t) is in volts per meter and **H**(t) in amperes per meter, which makes **E**(t) × **H**(t) a quantity in watts per square meter.

Usually, the average energy flow per unit time is of more practical importance. If we average the instantaneous Poynting´s vector over an integer number of signal periods T (= $2\pi/\omega$), we obtain

$$\langle \mathbf{E}(t) \times \mathbf{H}(t) \rangle = \frac{1}{2}(\mathbf{E}_r \times \mathbf{H}_r + \mathbf{E}_i \times \mathbf{H}_i) \quad (3)$$

This gives the average power flow in steady-state ac problems. It can be written in a more convenient form using complex notation as

$$\langle \mathbf{E}(t) \times \mathbf{H}(t) \rangle = \frac{1}{2} \mathrm{Re}\, \hat{\mathbf{E}} \times \hat{\mathbf{H}}^* \quad (4)$$

where the asterisk * denotes the complex conjugate. The time-average value for the cross product of **E**(*t*) and **H**(*t*) is equal to half the real part of the cross product of $\hat{\mathbf{E}}$ and the complex conjugate of $\hat{\mathbf{H}}$, $\hat{\mathbf{H}}^*$.

The vector $\hat{\mathbf{E}} \times \hat{\mathbf{H}}^*$ represents the amount of complex energy per unit time crossing a unit area. This power density is called the *complex Poynting vector*

$$\hat{\mathbf{S}} = \hat{\mathbf{E}} \times \hat{\mathbf{H}}^* \quad (5)$$

Such a vector is perpendicular to the plane determined by $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}^*$, and is in the direction of energy flow. Thus, a negative value for the complex Poynting vector represents inward energy flow through a unit area.

3. Time-averages of Sinusoidal Voltages and Currents

For sinusoidal time variation it is also convenient to use the complex notation (phasor method) to represent the instantaneous values of the voltage *v*(*t*) and the current *i*(*t*) as the real parts of complex exponentials $\hat{V} e^{j\omega t}$ and $\hat{I} e^{j\omega t}$, respectively, where $\hat{V}$ and $\hat{I}$ are complex scalars.

The instantaneous electric power at a single terminal pair is

$$\begin{aligned} v(t)i(t) = {} & V_r I_r \cos^2 \omega t + V_i I_i \sin^2 \omega t \\ & - (V_r I_i + V_i I_r) \sin \omega t \cos \omega t \end{aligned} \tag{6}$$

where the real part of $\hat{V}$ is V_r and the imaginary part is V_i, with similar notation for $\hat{I}$. Recall that the $v(t)$ is in volts and $i(t)$ in amperes, which makes $v(t)i(t)$ a quantity in watts.

If we average $v(t)i(t)$ over an integer number of signal periods T $(= 2\pi/\omega)$, we obtain

$$\langle v(t)i(t) \rangle = \frac{1}{2}(V_r I_r + V_i I_i) \tag{7}$$

This gives the average power flow in steady-state ac problems using the circuit theory approximation. It can be written in a more convenient form using complex notation as

$$\langle v(t)i(t) \rangle = \frac{1}{2} \mathrm{Re} \hat{V} \hat{I}^* \tag{8}$$

The time-average value for the voltage-current product of $v(t)$ and $i(t)$ is equal to half the real part of the product of the voltage phasor $\hat{V}$ and the complex conjugate of the current phasor $\hat{I}$, $\hat{I}^*$.

The scalar $\hat{V}\hat{I}^*/2$ represents the amount of complex energy per unit time flowing in a given point of a network. It is referred to as the *complex power*

$$\hat{S} = \hat{V}\hat{I}^*/2 \tag{9}$$

The magnitude of the complex quantity $\hat{S}$ is called *apparent power* and is expressed in volt-amperes (unit symbol V A) to distinguish from the real part of the complex power which is expressed in watts (unit symbol W). According to [2], volt-ampere and V A are respectively the IEC name and symbol for the SI unit of apparent power (the reader should also consult [3]).

4. Complex Poynting Vector Theorem

Attention is focused on a network excited through a single external terminal pair as illustrated in Fig. 1. This circuit may comprise any number of inductive, capacitive and resistive elements, including distributed reactance and resistance as well as discrete components.

We can formulate Poynting´s theorem for time-averaged quantities by using the complex Poynting vector $\hat{\mathbf{S}}$.

Consider the identity

$$\nabla \cdot \hat{\mathbf{E}} \times \hat{\mathbf{H}}^* = \hat{\mathbf{H}}^* \cdot \nabla \times \hat{\mathbf{E}} - \hat{\mathbf{E}} \cdot \nabla \times \hat{\mathbf{H}}^* \tag{10}$$

Using Maxwell´s equations for time-harmonic fields,

$$\nabla \times \hat{\mathbf{E}} = -j\omega \hat{\mathbf{B}} \tag{11}$$

$$\nabla \times \hat{\mathbf{H}}^* = \hat{\mathbf{J}}^* - j\omega \hat{\mathbf{D}}^* \tag{12}$$

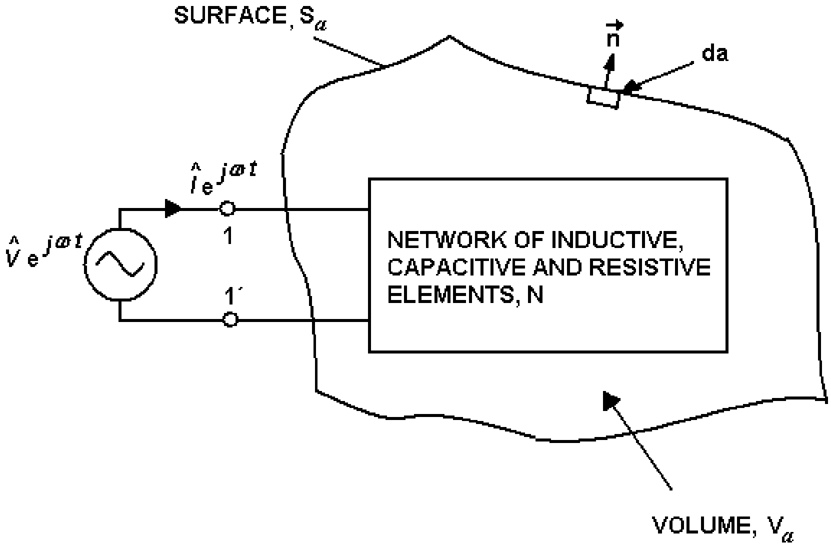

Fig. 1. Network "N" comprising an arbitrary number of discrete or distributed elements with a single frequency external source of excitation.

the above identity becomes

$$\nabla \cdot \hat{\mathbf{S}} = -j\omega \hat{\mathbf{B}} \cdot \hat{\mathbf{H}}^* + j\omega \hat{\mathbf{D}}^* \cdot \hat{\mathbf{E}} - \hat{\mathbf{E}} \cdot \hat{\mathbf{J}}^* \qquad (13)$$

Integrating this expression over the volume V_a which is bounded by a surface S_a and applying the divergence theorem to the left-hand side gives

$$-\oint\!\!\oint_{S_a} \hat{\mathbf{S}} \cdot d\mathbf{a} = j\omega \iiint_{V_a} \left(\hat{\mathbf{B}} \cdot \hat{\mathbf{H}}^* - \hat{\mathbf{E}} \cdot \hat{\mathbf{D}}^* \right) dV + \iiint_{V_a} \hat{\mathbf{E}} \cdot \hat{\mathbf{J}}^* \, dV \qquad (14)$$

which is the complex Poynting vector theorem.

5. Poynting Power Density Related to Circuit Power Input

Referring again to Fig. 1, the network "N" is assumed to be enclosed by a surface S_a which contains the passive elements but not the external source. Under what circunstances is the surface integral over $\hat{\mathbf{S}}$ equivalent to the voltage-current product of the terminals of the wires connected to the network?

Two attributes of the fields on the surface S_a enclosing the network are required. *First, the contribution of the magnetic induction to* $\hat{\mathbf{E}}$ *must be negligible on the surface* S_a. If this is so, then regardless of what is inside the surface S_a, *on that surface*, the electric field can be taken as irrotational. Thus,

$$\hat{\mathbf{E}} = -\nabla\hat{\Phi} \tag{15}$$

where $\hat{\Phi}$ is a complex scalar function of position and the power input term on the left in the integral conservation law, (14), can be expressed as

$$-\oint\!\!\oint_{S_a} \hat{\mathbf{S}} \cdot d\mathbf{a} = \oint\!\!\oint_{S_a} \nabla\hat{\Phi} \times \hat{\mathbf{H}}^* \cdot d\mathbf{a} \tag{16}$$

Next, the vector identity

$$\nabla \times \left(\hat{\Phi}\hat{\mathbf{H}}^*\right) = \nabla\hat{\Phi} \times \hat{\mathbf{H}}^* + \hat{\Phi}\nabla \times \hat{\mathbf{H}}^* \tag{17}$$

is used to write the right-hand side of (16) as

$$-\oint\!\!\oint_{S_a} \hat{\mathbf{S}} \cdot d\mathbf{a} = \oint\!\!\oint_{S_a} \nabla \times \left(\hat{\Phi}\hat{\mathbf{H}}^*\right) \cdot d\mathbf{a} - \oint\!\!\oint_{S_a} \hat{\Phi}\nabla \times \hat{\mathbf{H}}^* \cdot d\mathbf{a} \tag{18}$$

The first integral on the right is zero because the curl of a vector is divergence free and a field with no divergence has zero flux through a closed surface. Equation (12) can be used to eliminate *curl* $\hat{\mathbf{H}}^*$ from the second. It follows that in taking the integral over a closed surface of the complex Poynting´s vector, we can just as well write

$$-\oint\!\!\!\oint_{S_a} \hat{\mathbf{S}} \cdot d\mathbf{a} = -\oint\!\!\!\oint_{S_a} \hat{\Phi}\left(\hat{\mathbf{J}}^* - j\omega\hat{\mathbf{D}}^*\right) \cdot d\mathbf{a}\,, \tag{19}$$

Note that this expression holds only on the surface which encloses the network, not necessarily on surfaces inside the volume enclosed by that surface.

Second, on the surface S_a, the contribution of the displacement current must be negligible. This is equivalent to requiring that S_a is chosen parallel to the displacement flux density. In this case, the total power into the system reduces to

$$-\oint\!\!\!\oint_{S_a} \hat{\mathbf{S}} \cdot d\mathbf{a} = -\oint\!\!\!\oint_{S_a} \hat{\Phi}\hat{\mathbf{J}}^* \cdot d\mathbf{a} \tag{20}$$

The integrand has value only where the surface S_a intersects a wire. If taken as perfectly conducting (but nevertheless in a region where $j\omega\hat{\mathbf{B}}$ is zero and hence $\hat{\mathbf{E}}$ is irrotational), the wires have potentials that are uniform over their cross-sections. Thus, in (20), $\hat{\Phi}$ is equal to the voltage phasor of the terminal. In integrating the current density over the cross-section of the wire, note that $d\mathbf{a}$ is directed out of the surface, while a positive terminal current phasor is directed into the surface. Thus, the input complex power expressed by (20) is equivalent to what would be expected from ac circuit theory, that is

$$-\oiint_{S_a} \hat{\mathbf{S}} \cdot d\mathbf{a} = \hat{V}\hat{I}^* \tag{21}$$

6. Active and Reactive Power; Apparent Power

Expressing $\hat{V}$ and $\hat{I}^*$ in polar coordinates in (21), that is, using $\hat{V} = |\hat{V}|\angle\theta_V$ and $\hat{I}^* = |\hat{I}|\angle -\theta_I$, gives

$$-\oiint_{S_a} \hat{\mathbf{S}} \cdot d\mathbf{a} = \left|\hat{V}\right|\left|\hat{I}\right|\angle\theta_V - \theta_I \tag{22}$$

From (21) and (22), the integral conservation law, (14), becomes

$$\begin{aligned} \hat{V}\hat{I}^* &= \left|\hat{V}\right|\left|\hat{I}\right|\angle\theta_V - \theta_I \\ &= j\omega \iiint_{V_a} \left(\hat{\mathbf{B}} \cdot \hat{\mathbf{H}}^* - \hat{\mathbf{E}} \cdot \hat{\mathbf{D}}^*\right) dV + \iiint_{V_a} \hat{\mathbf{E}} \cdot \hat{\mathbf{J}}^* \, dV \end{aligned} \tag{23}$$

If we now divide by 2 and take the real part of (23), assuming the volume integrands are real we obtain

$$\begin{aligned} \frac{1}{2}\mathrm{Re}\hat{V}\hat{I}^* &= \left|\frac{\hat{V}}{\sqrt{2}}\right|\left|\frac{\hat{I}}{\sqrt{2}}\right|\cos(\theta_V - \theta_I) \\ &= \frac{1}{2}\iiint_{V_a} \hat{\mathbf{E}} \cdot \hat{\mathbf{J}}^* \, dV \end{aligned} \tag{24}$$

where $|\hat{V}|/\sqrt{2}$ and $|\hat{I}|/\sqrt{2}$ are respectively the root-mean-square (rms) values of the voltage and current applied to the network at a single terminal pair (their product is the *apparent power*). The term cos ($\theta_V - \theta_I$) characteristically indicates the extent to which the network absorbs power from the source and it is called *power factor*. The product of the rms value of the voltage and the rms value of the in-phase component of the current is the *active power*. This term is used as a means of reference to the average energy supplied by the source and consumed in the network per unit time. The unit name of active power is watt (unit symbol W).

This equation expresses the balance for real power flow. The left-hand side of (24) represents the average flow of power through the surface S_a which bounds volume V_a. The right-hand side of (24) expresses the power dissipated as heat and the rate of work done inside the volume V_a. Note that the energy–storage terms are absent because, for sinusoidal time dependence, there can be no average increase in energy stored.

The remaining imaginary part of (23) which is

$$\begin{aligned} \frac{1}{2}\mathrm{Im}\hat{V}\hat{I}^{*} &= \left|\frac{\hat{V}}{\sqrt{2}}\right|\left|\frac{\hat{I}}{\sqrt{2}}\right|\sin\left(\theta_V - \theta_I\right) \\ &= 2\omega\iiint_{V_a}\left(\hat{\mathbf{B}}\cdot\hat{\mathbf{H}}^{*} - \hat{\mathbf{E}}\cdot\hat{\mathbf{D}}^{*}\right)dV \end{aligned} \tag{25}$$

shows that the imaginary part of the input power is equal to 2ω times the difference in the time-averaged values of the magnetic and electric energy stored inside the volume V_a. The product of the rms value of the voltage and the rms value of the quadrature component of the current is called the *reactive power*. This term is used as a means of reference to the average net energy stored in the network. According to [2], var is both the IEC name and symbol for the SI unit of reactive power (the reader should also consult [3]).

The average power supplied by the source is given by the real part of $\hat{V}\hat{I}^*/2$. Since the reactive power, as just defined, is equal to the imaginary part of $\hat{V}\hat{I}^*/2$, it seems logical to regard the quantity $\hat{V}\hat{I}^*/2$ as the *complex power*. If the active or average power is denoted by the symbol *P*, and the reactive power by *Q*, we have

$$\hat{S} = P + jQ \qquad (26)$$

7. Discussion on the Role of Reactive Power

If the two fields associated with a given network store, on the average, equal amounts of energy, then they merely swap a certain amount of energy back and forth between them inside the volume V_a, and the source is *not* called upon to enter into this interplay once it has reached the steady state. It is only when the average energy stored in the magnetic field differs from the corresponding one in the electric field that some of the stored energy is continuously played back and forth between the source and the network. The reactive power is thus seen to be a measure of the extent to which the *source* participates in the interplay of stored energy, because it is proportional to the excess in the average value of magnetic as compared with electric stored energy.

One might argue that, so long as these storage elements absorb no net energy on the average, their presence or their effect upon the network behavior implies no net cost to the one who has to pay for the energy consumed (and only energy actually consumed means work done and heat dissipated by some facility). The fallacy of this argument lies in the assumption that the only energy consumed has to be paid for, or that energy that is stored should cost nothing. On the contrary, utilities who are in the business of supplying electric energy at a price justifiably feel that they are entitled to some fee for energy that is only stored by the costumer and returned in good condition, because the utility has to go the same trouble and expense to generate and distribute the energy, whether stored or consumed, notwithstanding the fact that only the consumed energy materially diminishes the coal pile.

Thus it is seen that somehow the energy that is swapped back and forth between source and the network has to be tracked of. Although the term “power” is not well suited as a designation for these “swappage” of energy, since power means energy flow and on the average there isn´t any, nevertheless the term *reactive power* or wattless power is used as a means of reference to the phenomenon we are discussing.

The fact that the excess in the average value of magnetic as compared with electric stored energy may be numerically negative as well as positive makes it physically possible for one passive network to supply the reactive power called for in another. Such a process, which relieves the source from the burden of entering into the role of an energy lending agency (and relieves the customer of the burden of paying an additional fee) is referred to as “power-factor correction”, a term that evidently is appropriate since the reactive power is zero when the power factor is unity, and vice versa. Reactive power is thus seen as something that a passive network can supply, and we note again the need for appropriated interpreting the term “power” in this connection.

8. Power Factor in AC Machines

The power factor at which ac machines operate is an economically important feature because of the cost of reactive power. Low power factor adversely affects electrical system operation in three principal ways. (a) Generators, transformers, and transmission equipment are rated in terms of kilovolt-amperes rather than kilowatts because their losses and heating are very nearly determined by voltage and current regardless of power factor. The physical size and cost of ac apparatus are roughly proportional to its kilovolt-ampere rating. The investment in generators, transformers, and transmission equipment for supplying a given useful amount of active power therefore is roughly inversely proportional to the power factor. (b) Low power factor means more current and greater dissipation losses in the generating and transmitting equipment. (c) A further disadvantage is poor voltage regulation.

Factors influencing kilovar requirements in motors can be visualized readily in terms of the relationship of these requirements to the establishment of magnetic flux. As in any electromagnetic device, the resultant flux necessary for motor operation must be established by a magnetizing component of current. It makes no difference either in the magnetic circuit or in the fundamental energy conversion process whether this magnetizing current is carried by the rotor or stator winding, just as it makes no basic difference in a transformer which winding carries the exciting current. In some cases, part of it is supplied from each winding. If all or part of the magnetizing current is supplied by an ac winding, the input to that winding must include lagging kilovars, because magnetizing current lags voltage drop by 90°. In effect, the lagging reactive power sets up flux in the motor.

The only possible source of excitation in an induction motor is the stator input. The induction motor therefore must operate at a lagging power factor. This power factor is very low at no load and increases to about 85 to 90 percent at full load, with improvement being caused by the increased real power requirements with increasing load.

With a synchronous motor, there are two possible sources of excitation: alternating current in the armature or direct current in the field winding. If the field current is just sufficient to supply the necessary magnetomotive force (mmf), no magnetizing current component or kilovars are needed in the armature and the motor operates at unit power factor. If the field current is less, i.e., the motor is *underexcited*, the deficit in mmf must be made up by the armature and the motor operates at a lagging power factor. If the field current is greater, i.e., the motor is *overexcited*, the excess mmf must be counterbalanced in the armature and a leading component of current is present; the motor then operates at a leading power factor.

Because magnetizing current must be supplied in inductive loads such as transformers and induction motors, the ability of overexcited synchronous motors to supply lagging current is a highly desirable feature which may have considerable economic importance. In effect, overexcited synchronous motors act as generators of lagging kilovars and thereby relieve the power source of the necessity for supplying this component. They thus may perform the same function as a local capacitor installation. Sometimes, unloaded synchronous machines are installed in power systems solely for power-factor correction or for control of reactive power flow. Such machines, called *synchronous condensers*, may be more economical in the larger sizes than static capacitors.

Both synchronous and induction machines may become self-excited when a sufficiently heavy capacitive load is present in their stator circuits. The capacitive current then furnishes the excitation and may cause serious overvoltage or excessive transient torques. Because of the inherent capacitance of transmission lines, the problem may arise when synchronous generators are energizing long unloaded or lightly loaded lines. The use of shunt reactors at the sending end of the line to compensate the capacitive current is sometimes necessary. For induction motors, it is normal practice to avoid self-excitation by limiting the size of any parallel capacitor when the motor and capacitor are switched as a unit.

9. Impact on Appendices B and C of the MRA

The CCEM-K5 key comparison on 50/60 Hz electric power [4] run from 1996 to 1999 included measurements of active power only. The commercial traveling standard used in the CCEM-K5 comparison was only able to measure active power. The test points were: rms voltage of 120 V, rms current of 5 A with power factor 1.0, 0.5 lead, 0.5 lag, 0.0 lead and 0.0 lag.

RMO comparisons on 50/60 Hz electric power were implemented subsequently using similar protocols. Nevertheless, the key comparison implemented in SIM region from 2010 to 2012, namely, SIM.EM-K5 [5], modified the original protocol by adding test points for reactive power. This was made possible with the use of stable commercial traveling standards which can measure both active and reactive power. The test points were then: rms voltage of 120 V, rms current of 5 A, power factor 1.0, 0.5 lead and 0.5 lag for active power measurements, and phase angles 30° lead, 30° lag, 90° lead and 90° lag for reactive power measurements. Note however that active power measurement with null power factors were eliminated from the protocol. The argument for that decision was that the measurement of active power is prone to low resolution at power factor close to zero, either lead or lag.

Though the document ´Classification of Services in Electricity and Magnetism´ issued and periodically updated by the CCEM does not mention explicitly the terms active power and reactive power in Service Category 7 dedicated to ac power and energy, all NMIs who have CMCs published in Appendix C of the MRA and which are capable to perform reactive power measurements publish such services in their CMCs and use the unit symbol var when declaring the related CMC entries.

10. Alternative Polarity Convention

The current phasor was assumed as the reference phasor in all the previous sections. The reactive power is then positive when the time-averaged value of magnetic energy exceeds the time-averaged value of electric energy in the network and being negative when the reverse occurs. When the reactive power is positive, we say that the network operates at a lagging power factor as an inductive load where the current lags the voltage in time or, in terms of phase angles, $\theta_V > \theta_I$. In this case, the source needs to supply the lagging reactive power demanded by the network. When the reactive power is negative, we say that the network operates at a leading power factor as a capacitive load where the current leads the voltage in time or, in terms of phase angles, $\theta_V < \theta_I$. In this case, the network can supply the lagging reactive power called for in another network connected to the same source, thus relieving the source from this task.

This sign convention agrees with most European manufactured meters and is adopted by IEC. But most American meters use the opposite sign convention (simply by using the voltage phasor as the reference) [6]. The latter convention results from defining the complex Poynting vector as $\hat{\mathbf{E}}^* \times \hat{\mathbf{H}}$ and the complex power as $\hat{V}^*\hat{I}/2$, so that the power factor becomes cos $(\theta_I - \theta_V)$ and all the reactive power signs referred to in the above paragraph are reversed accordingly.

One additional note: it was assumed here that the source delivers active power to a linear passive network. The active power is then always positive. The active power may become negative when the network contains sources inside the volume V_a, so that it supplies average power to the external source.

11. Conclusions

It was shown here that indeed the quantities active power, reactive power and apparent power should have different unit names and symbols. The measurement of each of these quantities and the correct interpretation of their meaning are relevant for the society. They have played a significant role in the last century since the advent of electricity and will continue to play a significant role in the future.

Based on all information provided above, the Consultative Committee on Electricity and Magnetism (CCEM) requests the Consultative Committee on Units (CCU) to include the unit var in Table 8 of the SI Brochure which lists the non-SI units accepted for use with the SI.

12. Acknowledgments

This draft proposal was compiled by Dr. Gregory Kyriazis from Inmetro, Brazil, based on a request from Dr. Gert Rietveld, from VSL, The Netherlands, president of the Consultative Committee on Electricity and Magnetism (CCEM), as working document for discussion at the next CCEM meeting in 2017.

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