On choosing the number of digits for the numerical value of the new SI defining constant *k*

Joachim Fischer
Physikalisch-Technische Bundesanstalt (PTB), Abbestrasse 2-12, 10587 Berlin, Germany

The discussion in this document is expected to be relevant to the values of the constants resulting from the special CODATA adjustment to be carried out in the summer of 2017 that will determine the exact numerical values of h, e, k, and N_A to be used to define the revised SI and to be adopted by the 26th CGPM, here called for simplicity the "new" SI. Four of the seven exactly known defining constants of the present or "old" SI will become inexactly known constants in the new SI and must be determined experimentally. They are the mass of the international prototype of the kilogram $m(\mathcal{K}) = 1$ kg, the magnetic constant $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}, the triple point of water $T_{TPW} = 273.16$ K, and the molar mass of the carbon 12 atom $M(^{12}C) = 0.012$ kg/mol.

Resolution 1 of the 24th CGPM (2011) implies that a sufficient number of digits for each constant is required such that the numerical value of each constant $m(\mathcal{K})$, μ_0, T_{TPW}, and $M(^{12}C)$ expressed in the "old" and "new" SI units should be strictly equal, consistent with their standard uncertainties, at the time of the redefinition (cf. document CCU/16-29). Tables 1 and 2 summarize cases for the Boltzmann constant k all based on the 2014 CODATA least-squares adjustment. The first case represents exactly the recommended 2014 CODATA value of k with 9 digits, thus satisfying the present 26th CGPM Draft Resolution A. The case with 7 digits also satisfies the Draft Resolution A but uses the **minimum** number of digits and yields a consistency factor equal to 1 within its standard uncertainty. The case with 8 digits is in between.

Table 1. Exact numerical values chosen for k all based on the 2014 CODATA recommended value. 9 digits for k so that the consistency factor is exactly 1 (see Table 2). 7 digits for k so that the consistency factor is equal to 1 within its standard uncertainty.

Constant	**9 digits**	**8 digits**	**7 digits**
k / JK^{-1}	$1.380\,648\,52 \times 10^{-23}$	$1.380\,648\,5 \times 10^{-23}$	$1.380\,649 \times 10^{-23}$

Table 2. Consistency factors and their standard uncertainties for cases of Table 1. The unit K, in column 1 is the new SI unit defined by the exact numerical value of *k*, given in Table 1 for the corresponding case.

“New” SI to “old” SI consistency factor	**9 digits**	**8 digits**	**7 digits**
T_{TPW}/(273.16 K)	1.000 000 00(57)	0.999 999 99(57)	0.999 999 65(57)
Corresponding difference in μK	0	4.0	95.6

The temperature difference would be evident if one remeasures in a few years the triple point of water temperature using a primary thermometer with sufficiently low uncertainty. This could well happen considering the reproducibility of the realization of the water triple point temperature which can be estimated to be as low as 10 μK (3.7×10^{-8} in relative units). Compared to the case with 7 digits for *k*, the temperature difference is nearly one order of magnitude higher than the reproducibility of a well operated triple point cell.

Also in future the triple point of water temperature is important in practical thermometry. To avoid such inconsistent results the case with 7 digits is not recommended for fixing the numerical value of the Boltzmann constant *k* on the occasion of the new definition of the kelvin.