Comments on the proposed wording for the unit definitions in the next SI

Walter Bich

12° CCM WG-M, BIPM, Sèvres, 24 March 2010

A warning

All definitions have the same format. Therefore, I will mostly discuss them in general, with some specific remarks concerning the kilogram.

A reminder on possible definitions (largely based on CCU/09-08)

- Explicit-unit
- Explicit-constant

Explicit-unit definitions

Traditional, used so far in the SI Brochures.

Example:
The metre is the length of path travelled by light in vacuum during a time interval of 1/299 792 458 of a second

Advantages: •Familiarity

•Easy to understand and visualize

Disadvantage: Implicitly suggests a specific experiment for the realization

Comment: The present SI survived for a long time with this disadvantage.

Explicit-constant definitions

➢A novelty of the draft Ch 2 SI Brochure (CCU/10-3.1)

Example:
The metre, unit of length, is such that the speed of light in vacuum is equal to exactly 299 792 458 metres per second.

Claimed advantages:

•Simplicity

•"... draws attention to the implications of the definition for fundamental physics"

•No reference to, or suggestion of a specific experiment for the realization

Explicit-constant definitions. Difficulties

- A good step towards rationalization (all definitions have the same format).
- However, there are some difficulties:

  - Incompleteness
  - Circularity
  - Bi-univocal association of a unit with a constant
  - Mysterious wording

All these difficulties stem from a common weakness, intrinsic in the choosen kind of definition (more on this later).

Explicit-constant definitions. Incompleteness

Incompleteness is the main concern. Excluding the second, which is the first definition in the list, and the mole, which is now independent of the kilogram, each definition implies knowledge of at least one of the preceding definitions.

Examples: **The metre, unit of length, is such that the speed of light in vacuum is equal to exactly 299 792 458 metres per second.**

This definition is not self-consistent, as requires the second to be separately defined.

Taken alone, the definition establishes a **necessary** and not **sufficient** condition.

There are infinitely many metres satisfying the definition, depending on how the second is defined.

Explicit-constant definitions.
Incompleteness - further example

The kilogram, unit of mass, is such that the Planck constant is equal to exactly $6.626\,068\,96 \times 10^{-34}$ joule second.

Also this definition is not self-consistent, as it implies knowledge of the dimension of the joule and the definition of the second.

Again, taken alone, the definition establishes a **necessary** and not **sufficient** condition.

There are ∞^2 kilograms satisfying the definition, depending on how the second **and** the metre are defined.

The change in the order of definitions, intended to avoid defining a unit in terms of another which is defined later, is a necessary but not sufficient measure. The result is still unsatisfactory.

Explicit-constant definitions. Circularity

A base unit/quantity is defined in terms of a unit/quantity derived from itself.

Examples:

The metre, unit of length, is defined in terms of metre per second, unit of speed, the time derivative of length.

The kilogram, unit of mass, is defined in terms of joule second, unit of action, thus involving the joule, a unit derived from the kilogram.

This is a serious concern, especially in a system in which the distinction is maintained between base and derived units.

Explicit-constant definitions.
Bi-univocal association of a unit with a constant

With the present form of explicit-constant definitions, an unnecessary and potentially misleading bi-univocal relationship between a given unit and a constant is established.

The kilogram and the Planck constant, the mole and the Avogadro constant, the ampere and the charge of the electron etc.

This might even implicitly suggest a possible experiment for the realization, similarly to the old-fashioned explicit-unit definitions.

Explicit-constant definitions. Mysterious wording

(a comparatively minor concern, half aesthetic, half semantic)

The international system of units, the SI, is the system of units *scaled so that* (from CCU/10-3.1)

And, again from CCU/10-3.1:

The …, unit of …, *is such that* …., is equal to exactly ….

… a format common to all definitions.

Explicit-constant definitions. Mysterious wording

(a comparatively minor concern, half aesthetic, half semantic)

I personally have problems in understanding how a system can be "scaled", and how a unit alone can be "such that"… something else has a given value.

These sentences can perhaps be grasped intuitively, but would not stand a rational or semantic analysis.

Not casually, they cannot be written as explicit mathematical relations.

I anticipate considerable problems in teaching.

Comments

Not a new situation, as both incompleteness and circularity are also in the present SI.

However, this is not a good argument to continue.

Rather, we have a good occasion to solve the problem.

Why these difficulties?

All the difficulties arise because in the definitions the base units $[Q_B]_i$ and the associated constants C_i refer to different quantities. For example, the kilogram is defined in terms of an action.

It might be said that the international system of units SI (or perhaps the underlying system of quantities ISQ) and the international system of constants are based on two different sets of base quantities.

This causes incompleteness and circularity.

The obscure wording reflects the complication involved in defining a unit of a quantity in terms of a different quantity.

Is there a possible way out?

A so-called *generalized* explicit-constant set of definitions was proposed, in which there is no association of a unit with a constant, and the distinction between base and derived quantities is dropped.

In this set of definitions, the constants which are “fixed” - the elementary action, the elementary charge, etc. - are simply listed with their values in SI units. This is sufficient to fix all units.

This view is partially adopted in the draft Brochure. An integral application is probably considered too radical, an understandable concern which can be shared.

Proposal

I agree to keep the distinction between base and derived units, and the individual definitions of the base units, as in the present draft Ch 2.

But:

Each unit shall be defined in terms of a constant of the same kind.

The second shall be a multiple of an elementary time interval, the metre of an elementary length, the kilogram of an elementary mass and so on.

Proposal

Whichever set of base constants is selected, a monomial combination of them can easily be found which has the dimension of an elementary length, an elementary mass and so on.

Accordingly, the numerical coefficient linking the SI unit to the corresponding elementary (monomial combination of) constant(s) can easily be found. (See Cabiati Bich, *Metrologia* **46**, 2009).

Proposal

For example:

The metre, unit of length, is equal to exactly 30,663 318 988 5 $c\,/\,\nu_{\mathrm{Cs}}$

The kilogram, unit of mass, is equal to exactly 1,475 521 665 × 10^{40} $h\,\nu_{\mathrm{Cs}}/c^2$

And so on

Comments

In this way:

- Each definition is self-consistent (no need for other definitions)
- Circularity is avoided
- The constants involved in each unit definitions are clearly shown
- The definitions are rational and intuitive at the same time. This greatly helps understanding and teaching them

Straying on forbidden grass…

This way of defining cruelly brings into light how all units except the mole depend on ν_{Cs}, definitely not a universal constant, rather an invariant of nature which is ultimately nothing else than a standard of the same kind of the Prototype kilogram (the former natural, the latter artificial).

Should a better transition be found, all these definitions would need to be changed accordingly. This is not a proof of the superiority of the formulation given in CCU/10-3.1. On the contrary, the latter simply hides the problem, does not solve it.

Straying on forbidden grass…

The natural invariant ν_{Cs} is inadequate as base constant, and would be better placed as a recommended transition frequency in a *mise en pratique*.

This arrangement would reproduce for the time unit the present situation of the length unit.

An indirect proof of the inadequacy of ν_{Cs} as base constant is the large value of the numerical coefficient for the kilogram. This value is questioned in some quarters as unphysical.

Clearly, it is not appropriate to attach a special physical meaning to combinations of universal constants **and** a specific natural invariant.

Straying on forbidden grass…

The adequate base constant to complete the transformation of the SI would be the electron mass m_e.

In this way, the definition of the kilogram would very naturally be

The kilogram, unit of mass, is equal to exactly 1,097 769 29 × 10^{30} m_e

All the other definitions would equally benefit from the introduction of the electron mass as base constant. Contrary to their belief, the time&frequency community would not even notice the change in the definition of the second. But this is a different story…

Thank you for your attention