The kelvin redefinition and the *MeP*-K

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The kelvin redefinition and the *Mise an Pratique* of the realization of the kelvin (*MeP*-K)

Mise en Pratique of the definition of the kelvin (*MeP*-K)

Mise en Pratique (MeP): Practical realisation

The CIPM foresaw desirability of a *MeP* for each base unit on redefinition of the SI

MeP should include only top-level realisation methods

MeP-K > flexible path for expanding the range of thermometric methods

SI Brochure: The International System of Units (SI) [8th edition, 2006; updated in 2014]

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We are pleased to present the updated (2014) 8th edition of the SI Brochure, which defines and presents the Système International d'Unités, the SI (known in English as the International System of Units).

- Preface to the 8th edition

Chapter 1: Introduction

- Quantities and units
- The International System of Units (SI) and the corresponding system of quantities
- Dimensions of quantities
- Coherent units, derived units with special names, and the SI prefixes
- SI units in the framework of general relativity
- Units for quantities that describe biological effects
- Legislation on units
- Historical note

Chapter 2: SI units

Chapter 3: Decimal multiples and submultiples of SI units

Chapter 4: Units outside the SI

Chapter 5: Writing unit symbols and names, and expressing the values of quantities

Appendix 1: Decisions of the CGPM and the CIPM

Appendix 2: Practical realization of the definitions of some important units

Appendix 3: Units for photochemical and photobiological quantities

- List of acronyms used

First version of the *MeP*-K (approved in 2011)

Text of the defined ITSs

- International Temperature Scale of 1990 (ITS-90)
- Provisional Low Temperature Scale from 0.9 mK to 1 K (PLTS-2000)

Technical Annex for the ITS-90 (Progress!)

- Prescription of the isotopic composition for H_2, Ne, and H_2O
- Correction equations for samples having other isotopic compositions

Guides for the realisation of the ITS-90 and PLTS-2000

$T - T_{90}$ and $u(T - T_{90}) \rightarrow$ conversion of values (Progress!)

Technical Annex for the ITS-90

Isotopic composition and corrections for the TPW

- Vienna Standard Mean Ocean Water (V-SMOW2)
- $T_{meas} = T_{90}(\mathrm{TPW}) + A_D\,\delta\mathrm{D} + A_O\,\delta^{18}\mathrm{O}$
- $u_{max}(T_{meas} - T_{90}) < 5\ \mu\mathrm{K}$

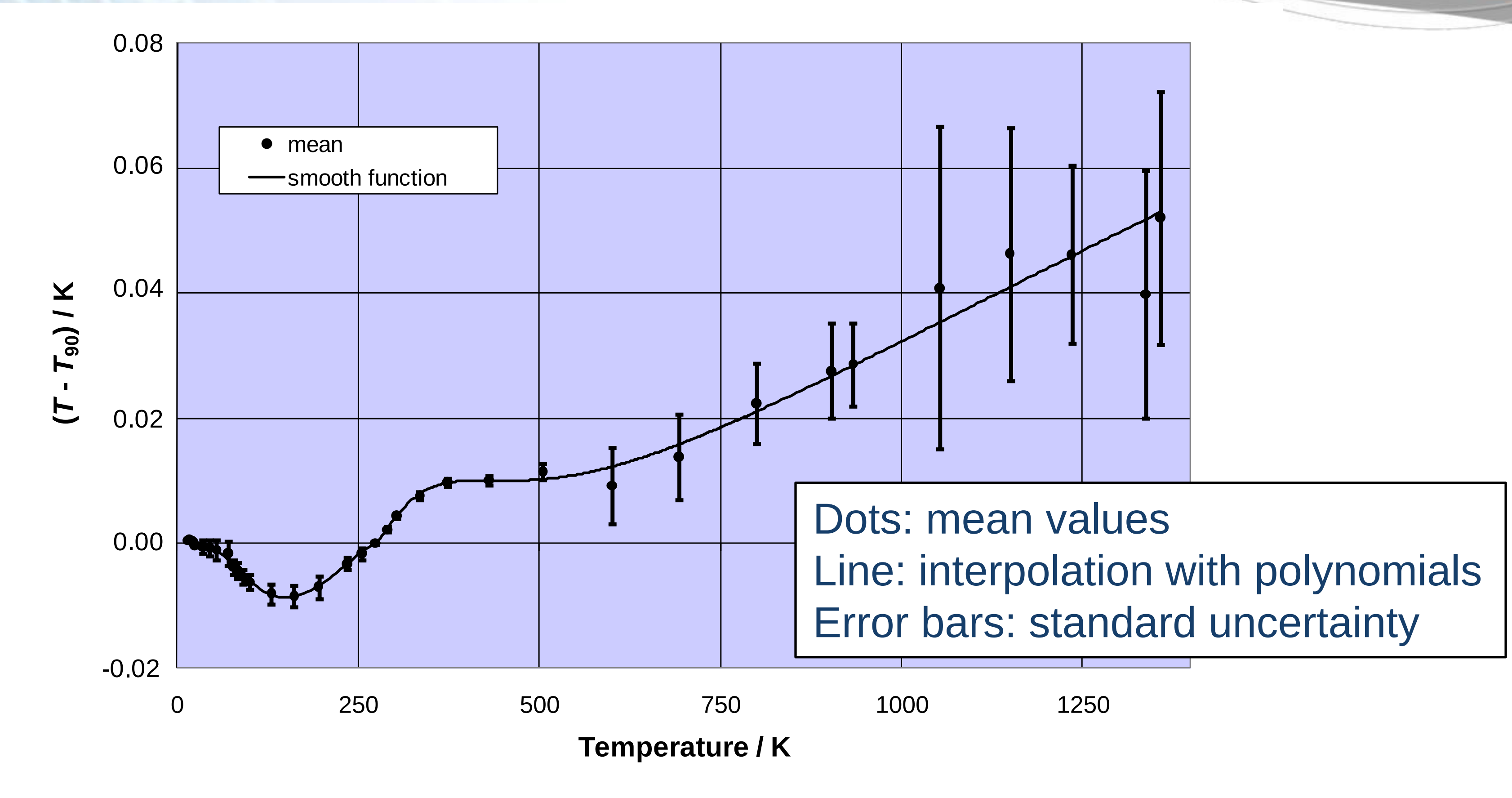
Isotopic composition and corrections for Neon

- IUPAC (International Union of Pure and Applied Chemistry) Composition
- $T_{meas} = T_{90}(\mathrm{Ne\ TP}) + k_0 + k_1\,({}^{22}x + {}^{21}x/2) + k_2\,({}^{22}x + {}^{21}x/2)^2$
- $u_{max}(T_{meas} - T_{90}) < 5\ \mu\mathrm{K}$

Isotopic composition and corrections for Hydrogen

- Standard Light Antarctic Precipitation (SLAP)
- $T_{meas} = T_{90}(\mathrm{e\text{-}H_2\ TP}) + k_D\,(x - x_0)$
- $u_{max}(T_{meas} - T_{90}) < 20\ \mu\mathrm{K}$

$T - T_{90}$ and $u(T - T_{90})$

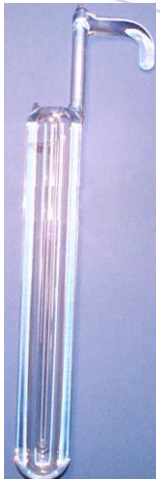


References:

Fischer *et al.*: *Int. J. Thermophys*. **32,** 12-25 (2011)

Engert *et al.*: *Metrologia* **44**, 40-52 (2007)

Redefinition of the SI base unit kelvin in 2018



Actual definition:
1/273.16 of the temperature of the triple point of water

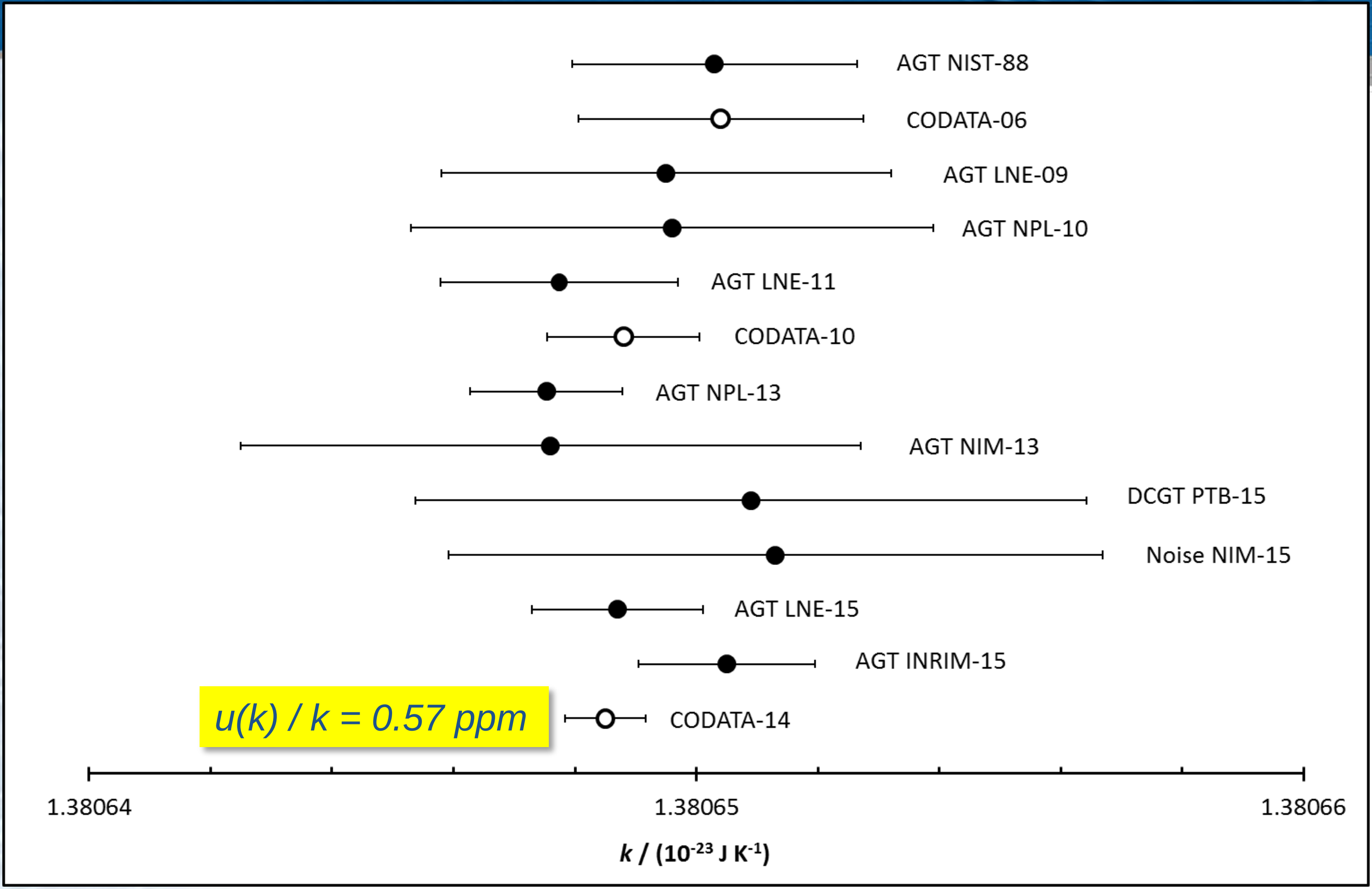
Weakness in the actual definition:
Dependence on the properties of the water sample, especially the isotopic composition

Planned explicit-constant definition:
The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380 6X × 10^{-23} when expressed in the unit J K^{-1}, which is equal to kg m^2 s^{-2} K^{-1}, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

$$k = 1.380\,6X \times 10^{-23}\ J/K$$

This means, the kelvin will be defined in terms of the SI derived unit of energy, the joule.

Values of *k* considered by CODATA 2014



Redefinition of the SI base unit kelvin in 2018

RECOMMENDATION T1 (2014)
On a new definition of the kelvin

The Consultative Committee for Thermometry (CCT)

recalling

- the CCT Report to the CIPM in 2007, "Report to the CIPM on the implications of changing the definition of the base unit kelvin";
- the CCT Recommendation to the CIPM in 2010, "Considerations for a new definition of the Kelvin", CCT T 2 (2010);

welcoming

- the Resolution 1 (2011) of the CGPM, "On the possible future revision of the International System of Units, the SI" which, when accomplished, will link the unit of temperature to the Boltzmann constant;
- the CCU Recommendation to the CIPM, "Revision of the International System of Units, the SI", CCU U 1 (2013);

recognizing

the need to confirm and clarify Recommendation CCT T 2 (2010) in the light of Resolution CCU U 1 (2013);

noting that

- experiments such as acoustic gas thermometry, dielectric constant gas thermometry, Johnson noise thermometry, and Doppler broadening thermometry represent fundamentally different methods to determine the Boltzmann constant *k*;
- the CODATA recommended a value for *k* with a relative standard uncertainty equal to 9.1 parts in 10^7 in its 2010 adjustment of fundamental constants, however based on only one experimental method;
- a relative standard uncertainty in *k* of 9.1 parts in 10^7 would correspond to a standard uncertainty of about 0.25 mK of the temperature of the triple point of water after the redefinition;

Redefinition of the SI base unit kelvin in 2018

considering

- the discussions held at the 26th and 27th meetings of the CCT in 2012 and 2014;
- the considerable progress recently achieved in experimental determinations of the Boltzmann constant to improve confidence in the 2010 value, as reported at the CCT “Task Group on the SI” meetings held in 2013 and 2014;
- that additional results are anticipated before the end of 2015;
- that experimental progress has allowed the development of a *mise en pratique* for the new definition of the kelvin, which has been extended to cover direct measurement of thermodynamic temperature after the new definition of the kelvin;

recommends

that the CIPM request the CODATA to adjust the values of the fundamental physical constants, from which a fixed numerical value of the Boltzmann constant will be adopted, when the following two conditions are met:

1. the relative standard uncertainty of the adjusted value of k is less than one part in 10^6; ✓
2. the determination of k is based on at least two fundamentally different methods, of which at least one result for each shall have a relative standard uncertainty less than 3 parts in 10^6.

Condition 2 fulfilled for AGT and DCGT, further method: noise thermometry

Effects of the redefinition of the kelvin

Direct realisation by primary thermometry possible

Benefits particularly below ≈ 20 K and above ≈ 1300 K

Definition independent of any material, no favoured fixed point, no favoured measurement method, no error propagation from TPW

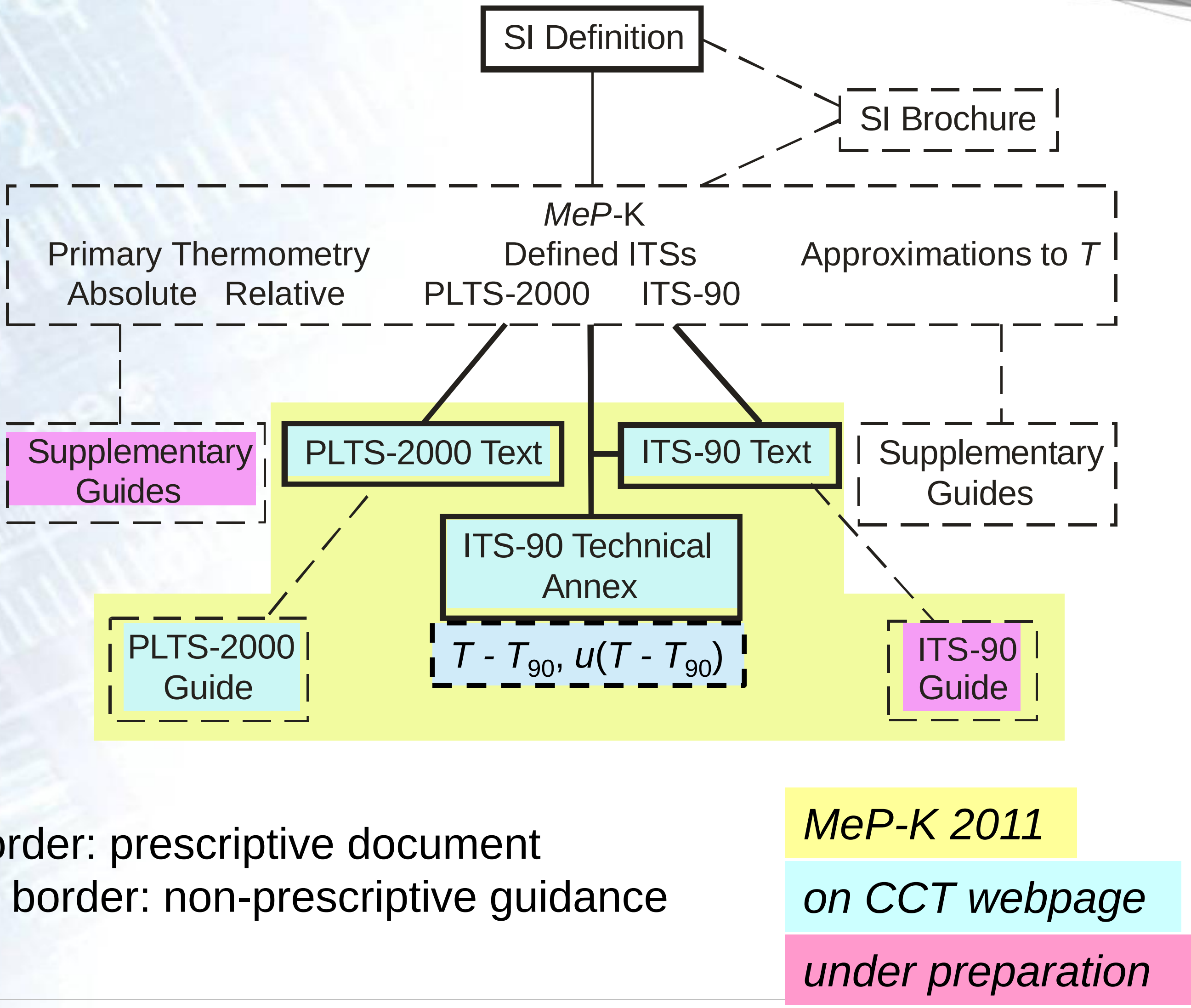
No immediate impact on the status of ITS-90 and PLTS-2000

Most precise temperature measurements from about 25 K to 1235 K will continue to be traceable to SPRTs calibrated according to the ITS-90

Relative uncertainty in the determination of k is transferred to T_{TPW}

The T_{TPW} value will not change in the foreseeable future

Schematic representation of relationship between *MeP*-K and other documents



Solid border: prescriptive document
Dashed border: non-prescriptive guidance

Second version of the *MeP*-K (after redefinition of K)

Nomenclature: Primary Thermometry

Thermometers based on well-understood physical systems, for which the equation of state describing the relation between T and other independent quantities can be written down explicitly without unknown constants

Absolute ↔ relative primary thermometry

- Absolute primary thermometry: Measurement directly in terms of the definition using the value of k, no reference to any fixed point.
- Relative primary thermometry: Use of fixed points, for which values of T and their uncertainties are known from previous primary thermometry.

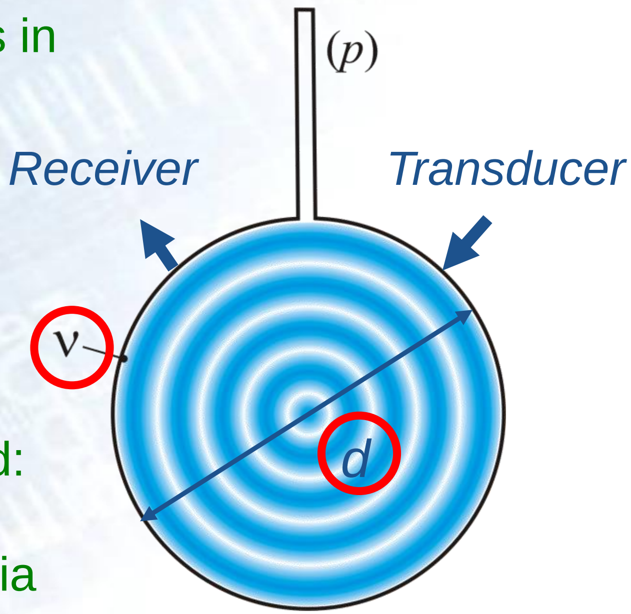
Brief descriptions for AGT, RT, PGT (DCGT, RIGT), JNT (SQUID, QVNS)

Appendices/references for AGT, RT, PGT (DCGT, RIGT), JNT (SQUID, QVNS)

Primary-thermometry methods: AGT

Acoustic Gas Thermometry

Standing waves in a resonator



To be measured:
- frequency ν_a
- dimensions via microwaves ν_m, pyknometry, or CMM

Equation of state for an ideal gas

$$u^2 = \frac{\gamma k T}{m}$$

u	Speed of sound in the limit of zero ν
γ	Heat-capacity ratio (c_p / c_V)
k	Boltzmann constant
T	Temperature
m	mass of a gas particle
ν	Frequency

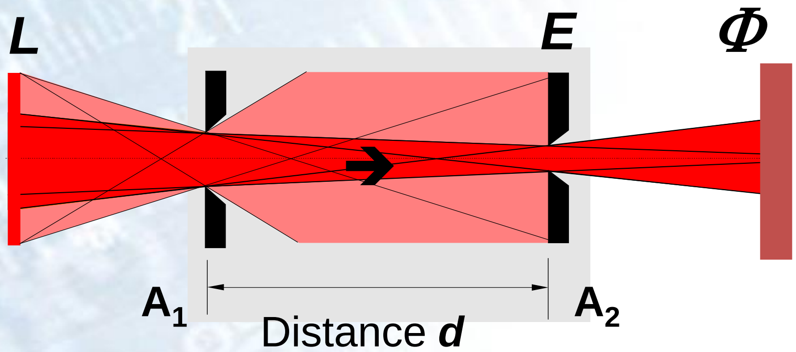
Quasi spheres and microwaves:

$$k = \frac{M}{\gamma_0 T_{\mathrm{TPW}} N_{\mathrm{A}}} c_0^2 \lim_{p \to 0} \left(\frac{\nu_{\mathrm{a}}(p)}{\langle \nu_{\mathrm{m}}(p) \rangle} \right)^2$$

- **Absolute AGT: *u*(*T*) / *T* of order 1 ppm**
- **Relative AGT: measurement of *u* ratios**
- **Review paper: Metrologia 2014**

Primary-thermometry methods: RT

Spectral-band Radiometric Thermometry



Source (e.g. **blackbody**)

Geometrical system

Detector (e.g. **diode**)

Planck law

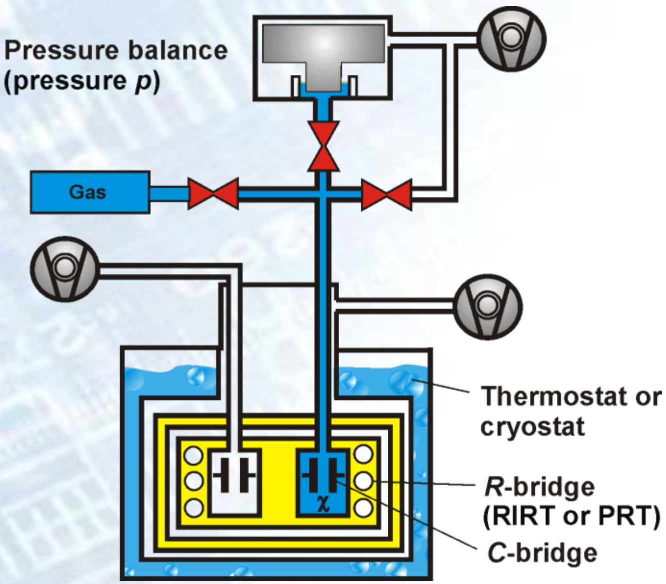
$$L_{b,\lambda}(\lambda, T) = \left(\frac{2hc^2}{\lambda^5}\right)\frac{1}{\exp(hc/\lambda kT) - 1}$$

L_λ	Spectral radiance
λ	Wavelength in vacuo
T	Temperature
h	Planck constant
k	Boltzmann constant
c	Speed of light in vacuo
E	Irradiance ($E = \Phi / A_2 = A_1 L / d^2$)
Φ	Radiant power ($\Phi = A_1 A_2 L / d^2$)

- Absolute RT: absolute spectral responsivity, geometric factors defining the solid angle
- $u(T)$ of order 0,1 K at 2800 K
- Appendices prepared

Primary-thermometry methods: PGT

Polarizing Gas Thermometry: Dielectric-Constant Gas Thermometry



Clausius-Mossotti equation combined with the ideal-gas law:

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{p}{kT}\frac{\alpha_0}{3\varepsilon_0}$$

Measuring quantity :

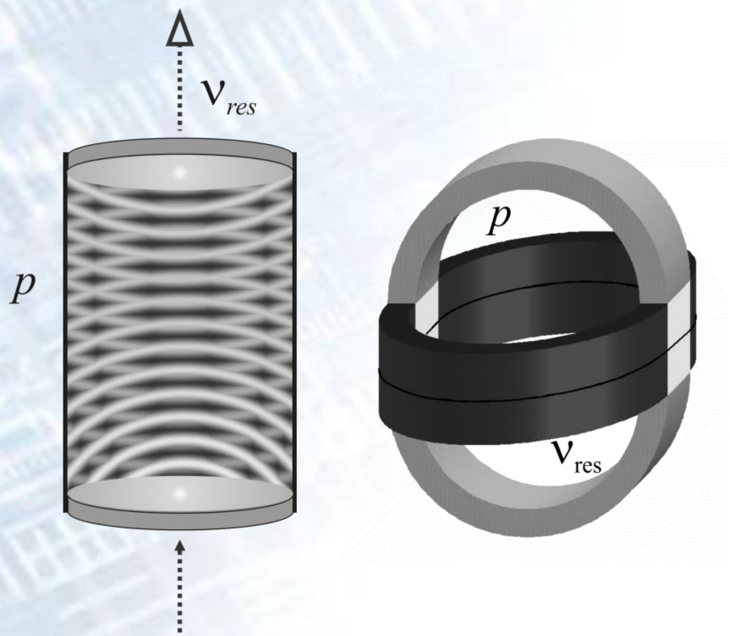
$$\frac{C(p) - C(0)}{C(0)} = \underbrace{\varepsilon_r - 1}_{\chi} + \varepsilon_r \kappa_{eff}\, p$$

- ε_r dielectric constant
- ε_0 electric constant
- α_0 atomic polarizability
- κ_{eff} effective compressibility
- χ electric susceptibility
- p pressure
- T temperature

- **Main uncertainty components completely different from AGT**
- ***u*(*k*)/*k* ≈ 2 ppm 2017**
- **Review paper: Metrologia 2015**

Primary-thermometry methods: PGT

Polarizing Gas Thermometry: Refractive-Index Gas Thermometry



Lorentz-Lorenz equation combined with the ideal-gas law:

$$\frac{n^2-1}{n^2+2}=\frac{p}{RT}(A_\varepsilon+A_\mu)$$

Measuring quantity :

$$n^2=\frac{f_m^2(0)}{f_m^2(p)(1-\kappa_{eff}\,p)^2}\approx\frac{f_m^2(0)}{f_m^2(p)}(1+2\kappa_{eff}\,p)$$

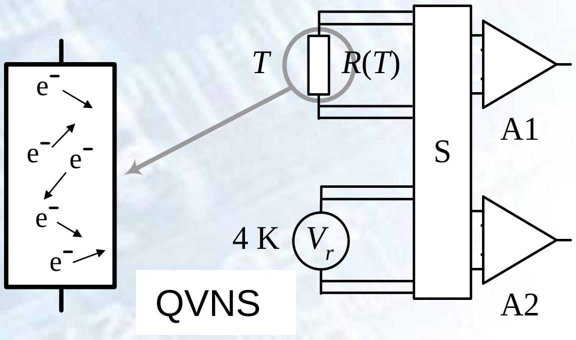
- ❖ **Main uncertainty components completely different from AGT**
- ❖ ***u*(*k*)/*k* ≈ 10 ppm 2007**
- ❖ ***MeP*-K Appendix 2017**

n	refractive index
p	pressure
R	molar gas constant
T	temperature
A_ε	molar electric polarizability
A_μ	molar magnetic polarizability
f_m	frequency of resonance mode *m*
κ_{eff}	effective compressibility

Primary-thermometry methods: JNT

Johnson Noise Thermometry

Determination of *k* at the TPW



(Quantum-accurate pseudo-random Voltage-Noise Source traceable to voltage standard)

Nyquist law

$$\overline{U^2} = 4\,kT\,R\,\Delta f$$

U	Voltage
k	Boltzmann constant
T	Temperature
R	Resistance
Δf	Bandwidth
Δt	measurement time

- ***U* extremely small, typically < 2 µV rms → cross correlation of two channels**
- **Statistical uncertainty ~ 1 / $\sqrt{\Delta t}$**
- **ac-Josephson voltage synthesizers → QVNS → JNT is operated as a comparator**
- **ADC + digital signal processing → Δf**
- ***u*(*k*)/*k* ≈ 3 ppm (Qu *et al.* 2017)**

Thermometry below 1 K

Basis: Superconducting QUantum Interferometer Devices (SQUIDs) with resolution near to the quantum limit

Voltage *to frequency conversion*
Current *Sensing Noise Thermometry*
Magnetic Field *Fluctuation Thermometry*

Relative uncertainty (0.1 – 1) %

Second version of the *MeP*-K (after redefinition of K)

Nomenclature: Defined temperature scales

Approximation to *T*, highly prescriptive, new quantities T_{XX}

Exact temperature values, based on primary thermometry, assigned to fixed points

Interpolating or extrapolating instruments

Interpolating or extrapolating equations

ITS-90: 17 fixed points (He-VP, H_2-VP, TP, MP, FP), CVGT, SPRT, RT

PLTS-2000: ^{3}He melting pressure, 4 intrinsic fixed points (T_{2000}, p_{2000})

Defined temperature scales

International Temperature Scale of 1990 (ITS-90)

- $T_{90} \geq 0.65$ K, lower limit caused by technical reasons (pressure measurement)
- T - T_{90} larger than originally expected, see appendix of the *MeP*-K
- Prescription of the isotopic composition for H_2, Ne, and H_2O in Technical Annex
- $p_{vp}(T_{90})$ for He (0.65 K – 5.0 K) and H_2 (17.025 K – 17.045 K; 20.26 K – 20.28 K)
- Fixed points: 6 triple points, 1 melting point, 7 freezing points
- Uncertainty of fixed-point realisation from comparisons: 0.03 mK (H_2) – 4 mK (Ag)
- Interpolation / extrapolation: ICVGT (3 K – 25 K), SPRT (14 K – 1235 K), RT

Provisional Low Temperature Scale from 0.9 mK to 1 K (PLTS-2000)

- $p_{mp}(T_{2000})$ for ^{3}He (3 MPa – 4 MPa)
- 4 intrinsic fixed points (p_{2000}, T_{2000})
- Relative thermodynamic uncertainty: 2% (0.9 mK) – 0.05% (1 K)
- $T_{2000} - T_{90}$ at 0.65 K: -1.6 mK
- PTB-2006: $p_{vp}(T_{90})$ (Metrologia 2007) is compatible with PLTS-2000

Second version of the *MeP*-K (after redefinition of K)

Criteria for the inclusion of a method in the *MeP*-K

- *Primary thermometry: Well derived equation of state*
- *Approximation to T: well derived formulas or empirical relations*
- *A complete uncertainty budget must be approved by CCT*
- *Uncertainty acceptable small*
- *At least two independent realisations*
- *Comparison with the results of already accepted methods*
- *Applicable over acceptable temperature ranges*
- *Detailed documentation in the open literature*

Summary

1st Version of the *Mise en Pratique* of the definition of the kelvin (*MeP*-K-11)

- Technical Annex for the ITS-90.
- $T - T_{90}$ and $u(T - T_{90})$ $\rightarrow$ conversion of values.

Redefinition of the SI base unit kelvin

- Determination of k with $u(k)/k < 1$ ppm, at least two independent methods
- Explicit-constant definition: $k = 1.3806X \times 10^{-23}$ J/K.

2nd Version of the *Mise en Pratique* of the realization of the kelvin (*MeP*-K-19)

- Nomenclature (taxonomy of methods)
- Criteria for the inclusion of a method
- Primary thermometry: Acoustic gas thermometry, radiometric thermometry, Johnson noise thermometry (SQUID, QVNS), polarizing gas thermometry (DCGT, RIGT)
- Defined ITSs: ITS-90 and PLTS-2000
- Technical Annex for the ITS-90 and $T-T_{90}$ together with $u(T-T_{90})$