Status of the algorithms for TAI

Gianna Panfilo

BIPM, Sèvres, France

Bureau
International des
Poids et
Mesures

Presentation Plan

- The uncertainties of [UTC-UTC(*k*)]
- Studies on the status of UTC
- Constrained Least Square solution applied to the calculation of TT

The uncertainties of the [UTC-UTC(*k*)]

Panfilo Gianna, Petit Gérard, Harmegnies Aurélie

Summary

1. The uncertainty calculation of [UTC-UTC(k)]

2. Proposed approach
   – Using original GNSS data expressed with respect to an auxiliary timescale (ATS)
   – Introduction of correlations

3. Example of the effect of redundant Time Links on the evaluation uncertainty

Present: Introduction

The differences [UTC - UTC(k)] with their uncertainties are published monthly in Section 1 of Circular T.

No redundant time links are used for UTC calculation at the moment.

The BIPM Time Department is planning to introduce redundant time links in the next future. This is an important improvement in time links calculation because it will allow to exploit all the data provided to the BIPM by the contributing laboratories.

A new method for [UTC-UTC(k)] uncertainty calculation is required for this new scenario.

The network of UTC time links

The network of UTC time links until is now supported by two independent techniques, the two-way satellite time and frequency transfer (TWSTFT, or shorter, TW) and GNSS observations.

Several types of measurements exist for GNSS time transfer, e.g. single frequency code, dual frequency code, dual frequency code and phase for GPS. Typically, for each laboratory, the most stable technique available is chosen and other techniques, when available, are kept as backup

For the TW in Europe, in North America and in Asia a nearly complete set of redundant measurements between all stations is often available, but only a non-redundant set of TW links is used for UTC.

Other Satellite systems as the European Galileo or the Chinese BEIDOU could integrate the calculation process.

The present status of the uncertainties of [UTC-UTC(k)]

The current results values of uA and uB, reported in Section 1 of Circular T are obtained by uncertainty propagation:

- They depend on the weights of the laboratories as the result of UTC algorithm and on the links uncertainties.

- All the time links connect each contributing laboratory to PTB, which plays the role of the pivot

- No correlations are taken into account for the uncertainty propagation algorithm

….Some observation about the current solution

With the improvement of time links performance the current algorithm for uncertainty propagation shows some limitations

- The uncertainties for the pivot laboratory PTB, are underestimated and unrealistic.
- the uncertainties of all laboratories are strictly correlated to USNO uncertainty (due to its large weight in UTC calculation);
- if the uncertainty assigned to laboratories with uncalibrated links (20 ns until now) is enlarged (to a more realistic picture like 1000-10000 ns) all the laboratories will be affected by the uncertainties of uncalibrated links: a non-realistic result.
- All these limitations can be overcome by a new uncertainty propagation algorithm which can take into account correlations.

Proposed algorithm

Estimating the correlation in the uncertainty propagation is very difficult in the current formalism and a different approach is proposed:

- To **facilitate the evaluations of correlations** the time link value is written split in two contributions ("measurement" and "bias")

- The GNSS measurements are considered with respect to an Auxiliary Time Scale (ATS) external to UTC calculation

→

- uCal is considered related to the receiver of the concerned laboratory (no correlations)
- uStb correlates the results (easier to evaluate because external)

- For the TWSTFT the picture will be unchanged: the time links the PTB laboratory will remain the pivot (**no correlations are considered**)

Proposed algorithm - Theory

The problem is solved in two steps considering the time links as a classical estimation problem:

Time links –GNSS: $L_k = (Lab_k - AST) + b_{Gk}$

Time links – TWSTFT: $L_{k,l} = (Lab_k - Lab_l) + b_{k,l}$

The solution: $(UTC - Lab_k) + b_k$

1 – First Step : the solution of **AX = L** gives the decoupled solution by the least square technique $\boldsymbol{(UTC - Lab_k)}$ and $\boldsymbol{b_k}$

$$X = \left(A^T \times S_L{}^{-1} \times A\right)^{-1} \times A^T \times S_L{}^{-1} \times L$$

$$S_X = \left(A^T \times S_L{}^{-1} \times A\right)^{-1}$$

2 – Second Step : the solution **CY = X** gives the final solution is found by the least square technique in **Y =** $\boldsymbol{(UTC - Lab_k) + b_k}$.

Bureau International des Poids et Mesures

Application to the real case – UTC calculation

This example uses the current links in UTC using IGRT as pivot in the case of GNSS equipment, and using PTB for the TWSTFT links. The correlation is set equal to 0.03 ns.

The uncertainty value of the PTB is still smaller then the other labs because TW redundant time links are not taken in to account.

Lab	Link	Weight (%)	uCal (ns)	uStb (ns)	uf (ns)	u (ns)
IT	TWGPPP	0.03	1.0	0.3	0.26	1.03
USNO	TWGPPP	0.33	1.0	0.3	0.2	1.02
NIST	TWGPPP	0.06	1.5	0.3	0.25	1.52
OP	TWGPPP	0.04	1.0	0.3	0.26	1.03
APL	GPSPPP	0.03	11.2	0.3	0.3	11.2
AUS	GPSPPP	0.0017	5.8	0.3	0.3	5.81
CAO	GPS MC	0.00	20.0	8.0	8.0	21.54
SG	GPS P3	0.01	5.8	0.7	0.69	5.84
SMD	GPSPPP	0.0016	7.3	0.3	0.3	7.31
MBM	GPS MC	0.00	20.0	5.0	5.0	20.62
PTB	-	0.02	-	-	0.15	0.38

Example of Uncalibrated Equipment

The uncertainty obtained with the current and the new method with USNO uncertainty uCal set to 200 ns.

Lab	Link	Weight (%)	uCal (ns)	u_{old} (ns)	u_{new} (ns)
IT	TWGPPP	0.03	1.0	66.9	1.03
USNO	TWGPPP	0.33	200	133.1	181.02
NIST	TWGPPP	0.06	1.5	66.9	1.52
OP	TWGPPP	0.04	1.0	66.9	1.03
APL	GPSPPP	0.03	11.2	67.8	11.2
AUS	GPSPPP	0.0017	5.8	67.2	5.81
CAO	GPS MC	0.00	20.0	69.8	21.54
SG	GPS P3	0.01	5.8	67.1	5.84
SMD	GPSPPP	0.0016	7.3	67.3	7.31
MBM	GPS MC	0.00	20.0	69.8	20.62
PTB	-	0.02	-	66.9	0.40

Even if USNO has more than 30% of the total weight, its uncertainty does not propagate massively the other participating laboratories.

Bureau International des Poids et Mesures

Results of tests with redundant TW measurements

uf and u for several laboratories depending on weights and link uncertainties. IGRT is used as pivot in the case of GNSS equipment and TWSTFT redundant links. The correlation is set equal to 0.03 ns.

When redundant time links are considered all the concerned laboratories have a smaller value for the uncertainty.

Lab	Link	Weight (%)	uCal (ns)	uStb (ns)	uf (ns)	u (ns)
IT	-	0.03	-	-	0.14	0.34
USNO	-	0.33	-	-	0.13	0.34
NIST	-	0.06	-	-	0.14	0.35
OP	-	0.04	-	-	0.14	0.35
APL	GPSPPP	0.03	11.2	0.3	0.27	11.2
AUS	GPSPPP	0.0017	5.8	0.3	0.28	5.81
CAO	GPS MC	0.00	20.0	8.0	8.0	21.54
SG	GPS P3	0.01	5.8	0.7	0.69	5.84
SMD	GPSPPP	0.0016	7.3	0.3	0.28	7.31
MBM	GPS MC	0.00	20.0	5.0	5.0	20.62
PTB	-	0.02	-	-	0.13	0.37

Bureau International des Poids et Mesures

Conclusions and Perspectives

The new algorithm for the calculation of the uncertainties of [UTC-UTC(*k*)] is presented and correctly takes into account correlations in uncertainty propagation

Two major changes are envisaged for its application

- the pivot for GNSS time links is an auxiliary time scale instead of the PTB
- the correlations are added.

It is planned to introduce this new algorithm within a few months, and to modify in consequence the information in Section 5 of Circular T.

The future development of this work is the introduction of redundant links in UTC calculation, starting from TWSTFT measurements.

Studies on the status of UTC

Status of UTC

A key point of the UTC algorithm is the prediction of clock frequency drift, a feature introduced recently.

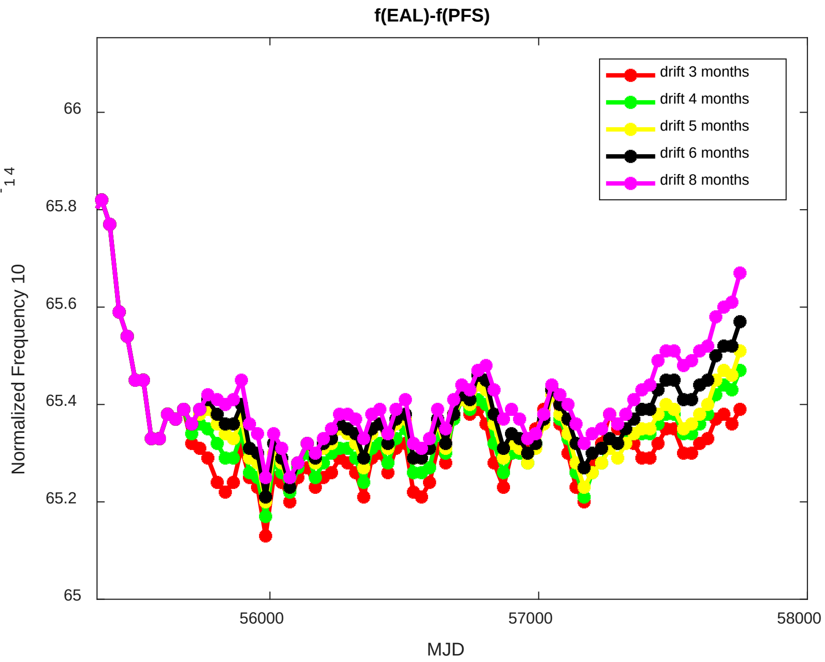
The least square technique is applied on the frequency difference ($TT\text{-}H_i$) to evaluate the drift. The least squares are evaluated on a 6 months interval.

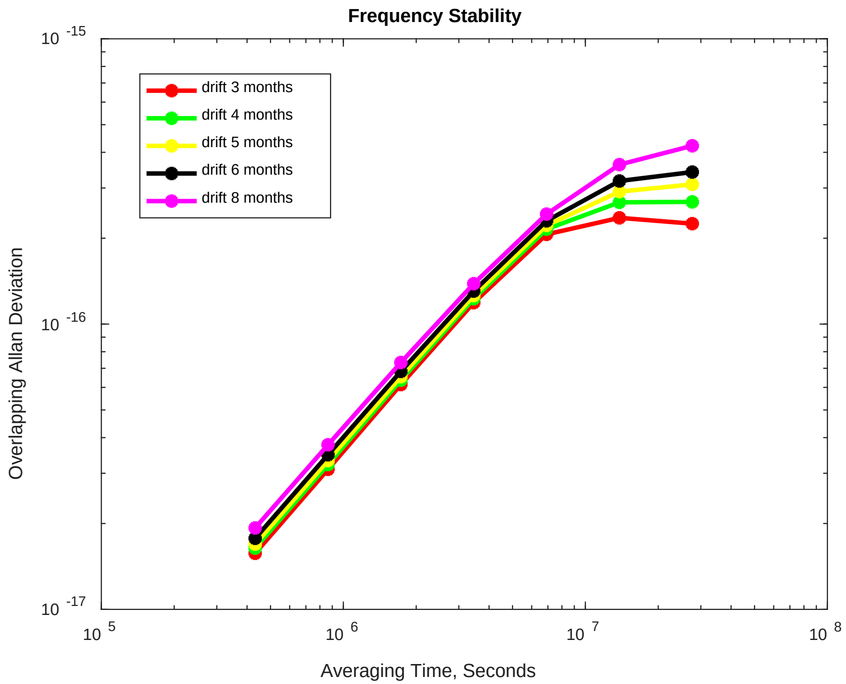
Two analysis have been done:

1) A different evaluation interval (3, 4, 5, 6 and 8 months)
2) A different interval for each clock, optimized with respect to clock predictability

Drift Estimation – Fixed interval for each clock

Different intervals (3, 4, 5, 6 and 8 months) are used for the evaluation of the frequency drift. From the results the use of **three months** is recommended.



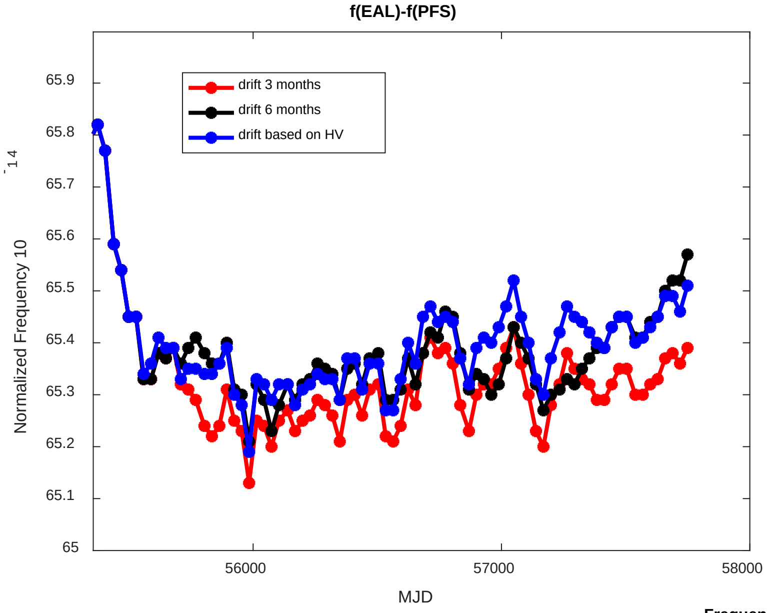


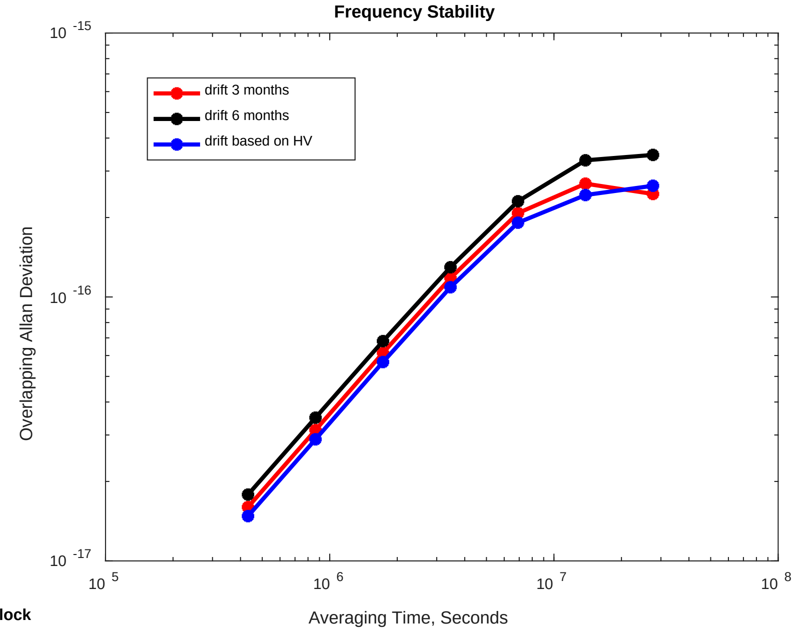
Bureau International des Poids et Mesures

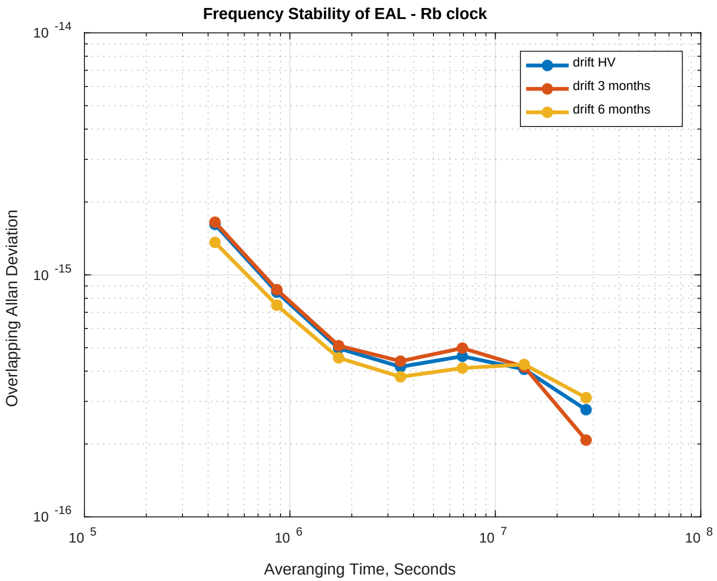
Drift Estimation – Optimized interval for each clock

- In this test a different drift evaluation interval for each clock is used.
- The Hadamard Variance is calculated for the data ($TT\text{-}H_i$) (3 years), its minimum value corresponds to the optimized interval for the ith clock (the longer predictable period for the drift estimation).
- The results show that 15% of the clocks are very predictable and the best drift evaluation interval is 6 months. For the rest of the clocks 3 months is the longer predictable period.

EAL calculated with different past periods for each clock







Bureau International des Poids et Mesures

Conclusions and perspectives

- The drift prediction interval has been set equal to 3 months to avoid some clocks with unpredictable drift could spoil EAL and UTC long term frequency stability.

- The Hadamard Variance has been used to evaluate the best prediction interval for each clock and the results are promising improving the stability of UTC.

- In the future the characterization of each single clock with respect to their drift predictability will be developed to rapidly detect clock anomalies and prevent their impact on UTC.

Bureau International des Poids et Mesures

Constrained Least Square

TT filters – Constrained least squares

- TT is calculated with the primary and secondary frequency standards, the algorithm solves a least square problem.

- The LS solutions give the “weights” (called “filters”) of the PFS and SFS in TT calculation. The filters can be negative and this has no meaning from the physical point of view.

- The Constrained Least Squares method can be use to overcome this problem.

TT filters calculated with the (CLS)

The goal of this new proposal is to reach a solution in close agreement with the physical reality.

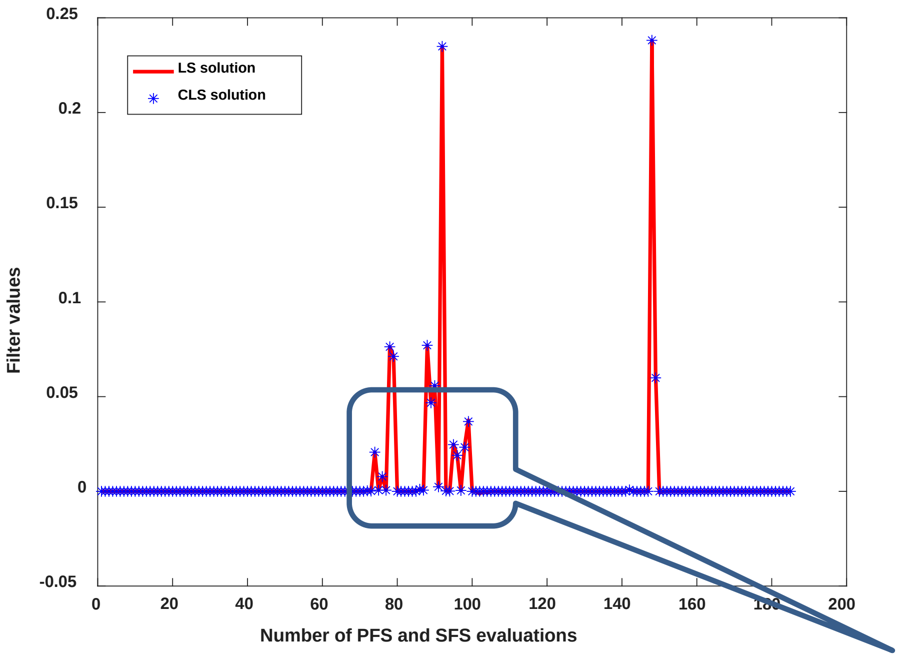
Minimize the functional: $f(x)$

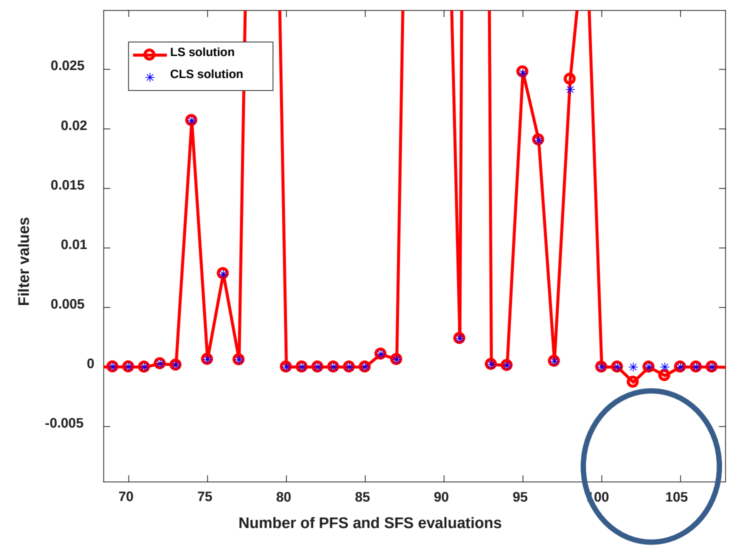
With constrained conditions:

$$h_i(x) = 0 \quad i = 1, \quad .., n$$
$$g_j(x) \geq 0 \quad j = 1, \quad .., m$$

For solving this problem the Kuhn-Tucker conditions are used that are a generalization of the method of Lagrange multipliers.

Example: comparison between the LS and CLS





The CLS algorithm allows only positive (or null) filters but more detailed studies are necessary to validate the results.

Thank you very much for your attention

Bureau International des Poids et Mesures

www.bipm.org