11th Meeting of the Consultative Committee for Acoustics, Ultrasound and Vibration (CCAUV)

Primary Method for the Complex Calibration of a Hydrophone from 1 Hz to 2 kHz

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Acknowledgment

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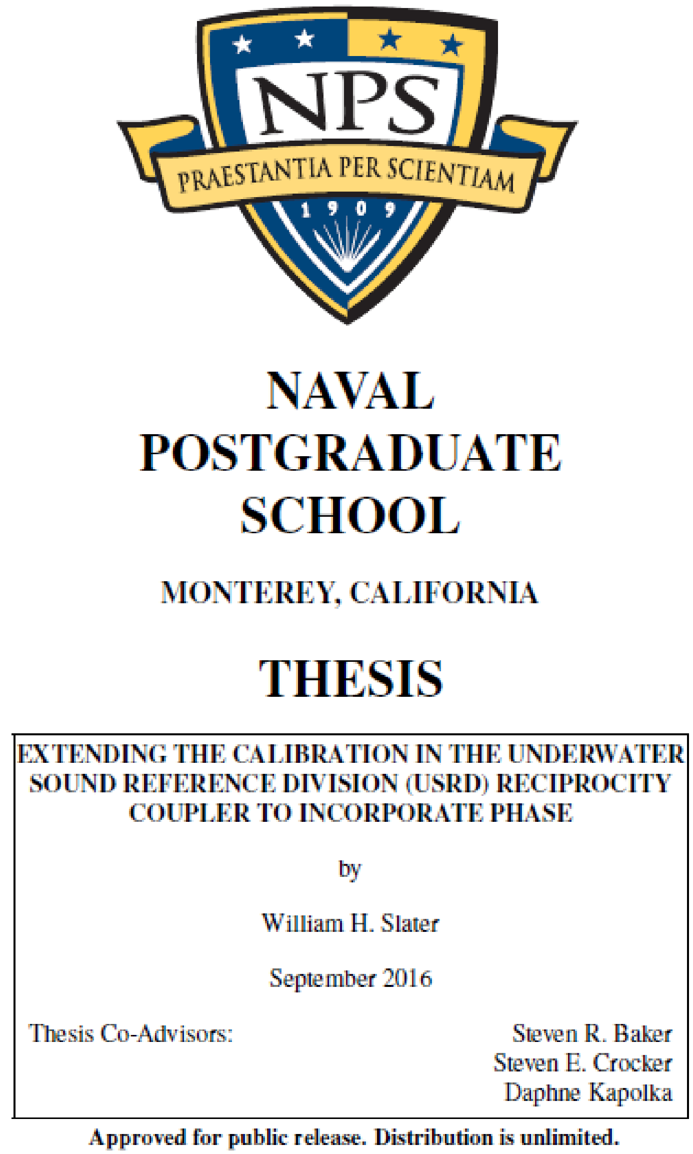
NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

EXTENDING THE CALIBRATION IN THE UNDERWATER SOUND REFERENCE DIVISION (USRD) RECIPROCITY COUPLER TO INCORPORATE PHASE

by

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September 2016

Thesis Co-Advisors: Steven R. Baker, Steven E. Crocker, Daphne Kapolka

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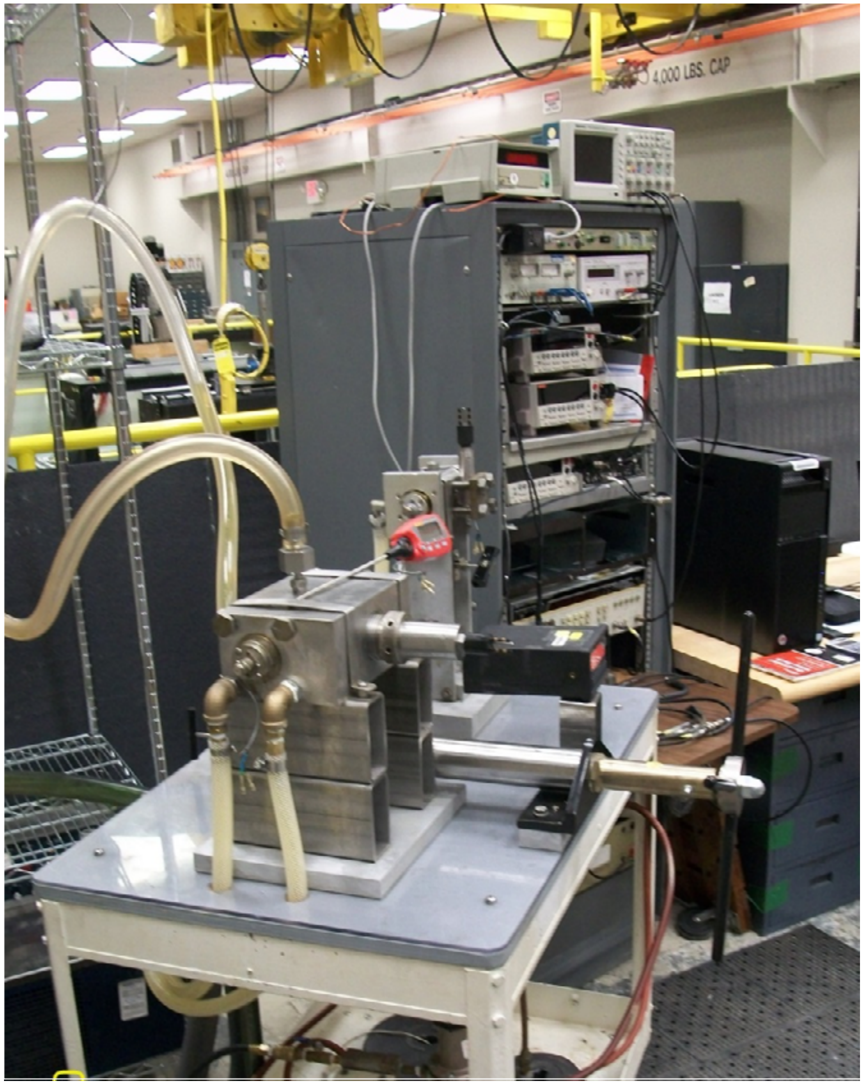
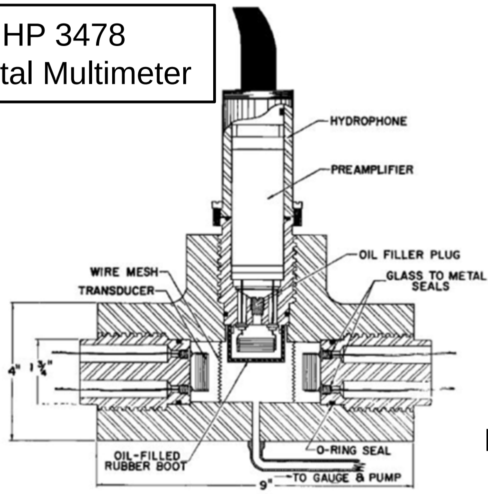
Outline

- Traceability at Low Frequency
- Theory
- Practice
- Systematic Errors & Corrections
- Uncertainty
- Results
- Conclusions



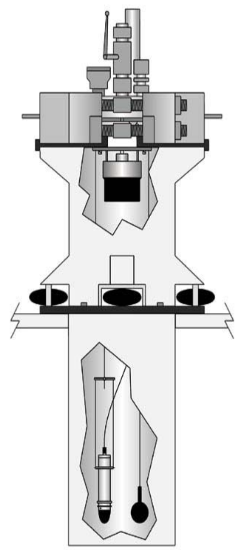
Traceablility at Low Frequency

HP 3478
Digital Multimeter

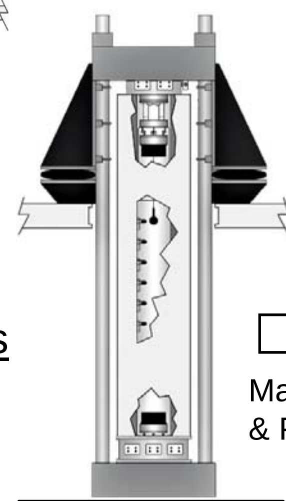


Magnitude

HP 3562
Signal Analyzer



Magnitude
& Phase



Magnitude
& Phase

Custom
Measurement
System

<u>Primary Reference Standard</u>
Reciprocity Coupler
Type H48 and Type A47

<u>Secondary Reference Standards</u>
Standing Wave Tube
Traveling Wave Tubes

<u>User Applications</u>
USRD Facility Standards
USRD Standards Leasing
Other Navy Labs
Gov't Agencies
Non-Gov't Orgs.
Academia
Industry

Theory

$$M_H = \left(\frac{e_{HY} e_{HX}}{e_{YX} i_Y} J \right)^{1/2}$$

Table 1. Reciprocity calibration measurement groupings, adapted from [1].

Configuration	Input Current Measurement	Transmitting Transducer	Receiving Transducers	Output Voltage Measurements
(a)	i_X	Reciprocal Transducer X	Hydrophone, Reciprocal Transducer Y	e_{HX}, e_{YX}
(b)	i_Y	Reciprocal Transducer Y	Hydrophone, Reciprocal Transducer X	e_{HY}, e_{XY}

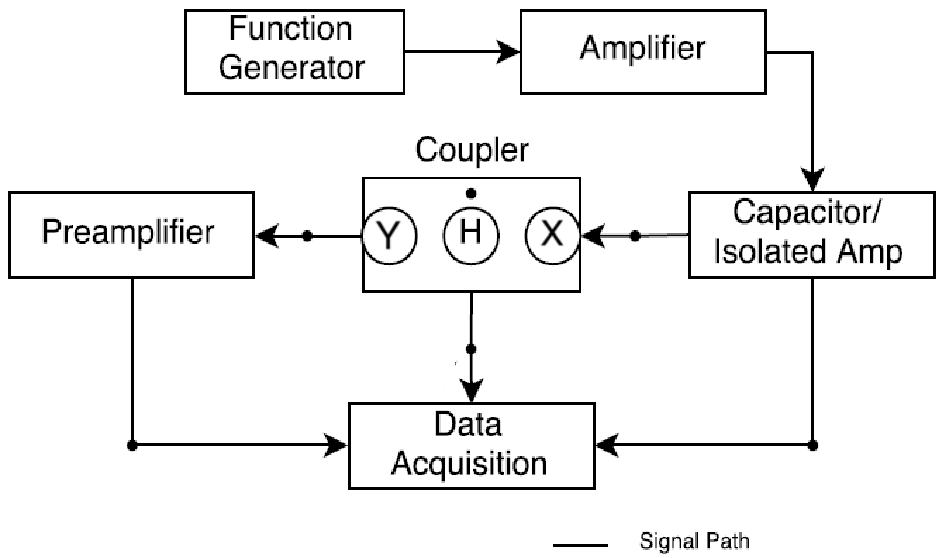
Configuration (a)

$$e_{HX} = M_H p_X = M_H i_X S_X$$

$$e_{YX} = M_Y i_X S_X$$

Configuration (b)

$$e_{HY} = M_H i_Y S_Y$$



Reciprocity Parameter

$$J = \frac{M_Y}{S_Y} = \frac{j\omega V}{\rho c^2} = j\omega C$$

C is the acoustical compliance of the coupler fluid medium

ρc^2 is the adiabatic bulk modulus of the fill fluid (castor oil)

V is the coupler volume

Practice

$$M_H = \sqrt{\frac{e_{HX} e_{HY}}{\sqrt{e_{YX} e_{CX} \times e_{XY} e_{CY}}} \left(\frac{V}{C_p \rho c^2} \right)}$$

C_p is capacitance

Transmitting Transducer	Receiving Transducers	Output Voltage Measurements
Reciprocal Transducer X	Hydrophone, Reciprocal Transducer Y	e_{HX}, e_{YX}, e_{CX}
Reciprocal Transducer Y	Hydrophone, Reciprocal Transducer X	e_{HY}, e_{XY}, e_{CY}

<u>*Configuration (a)*</u>

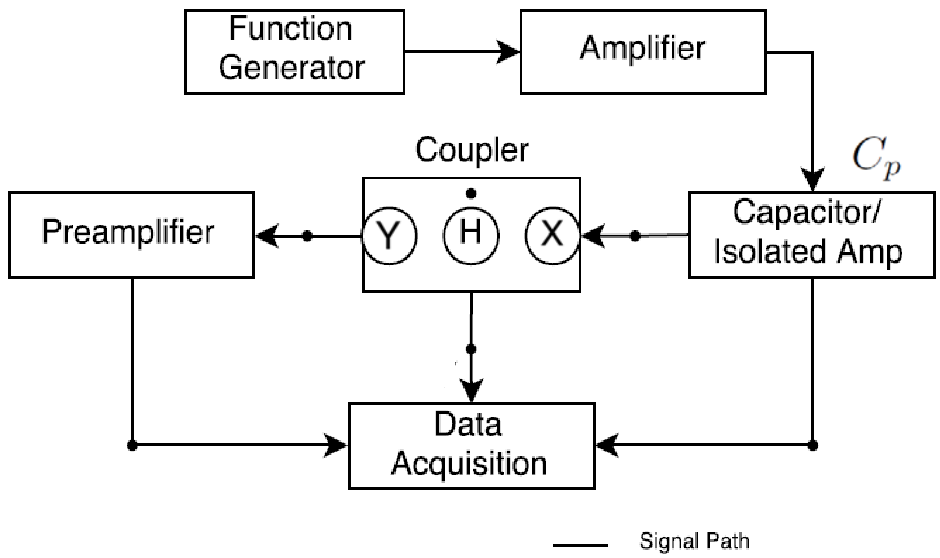
$$e_{HX} = M_H p_X = M_H i_X S_X$$

$$e_{YX} = M_Y i_X S_X$$

<u>*Configuration (b)*</u>

$$e_{HY} = M_H i_Y S_Y$$

$$e_{XY} = M_X i_Y S_Y$$



<u>*Reciprocity Parameter*</u>

$$J = \frac{M_Y}{S_Y} = \frac{j\omega V}{\rho c^2} = j\omega C$$

C is the acoustical compliance of the coupler fluid medium

ρc^2 is the adiabatic bulk modulus of the fill fluid (castor oil)

V is the coupler volume

Systematic Errors & Corrections

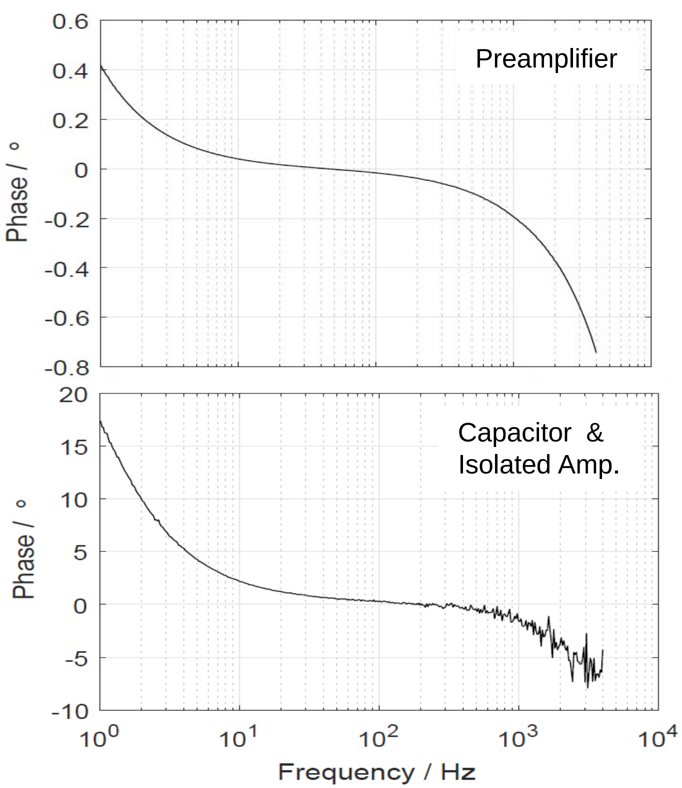
Phase

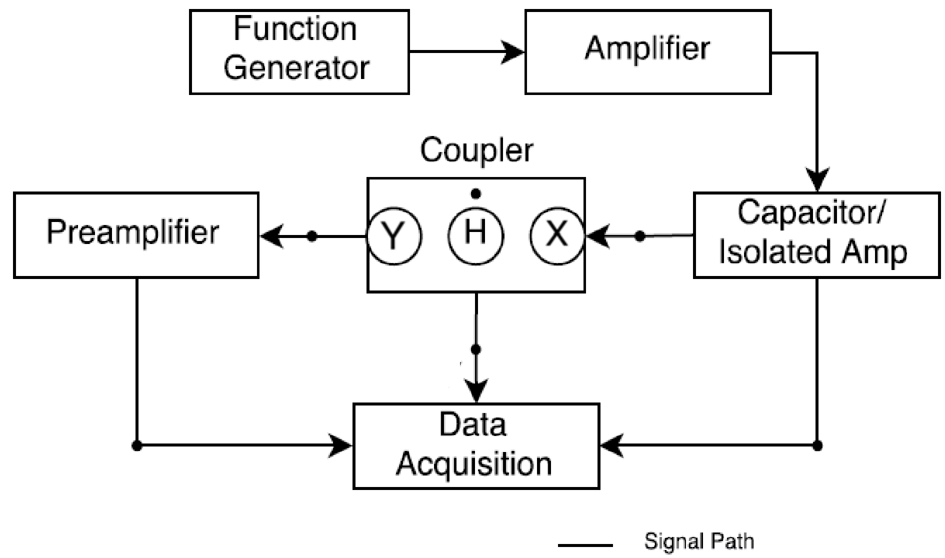
- Preamplifier (Itaco 1201)
- Capacitor / Isolated Amplifier
- Sequential sampling skew

Magnitude

- Coupler compliance
- Transducer compliance
- Acoustic wavelength

(Coupler compliance and transducer compliance: 0.25 to 0.33 dB)





Uncertainty

<u>Type A</u>

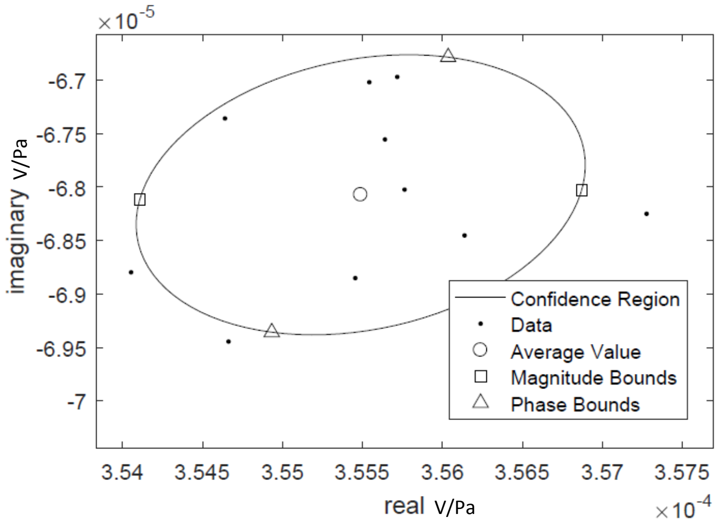
- Random errors modeled using a bivariate normal distribution for the real and imaginary parts of the complex sensitivity.

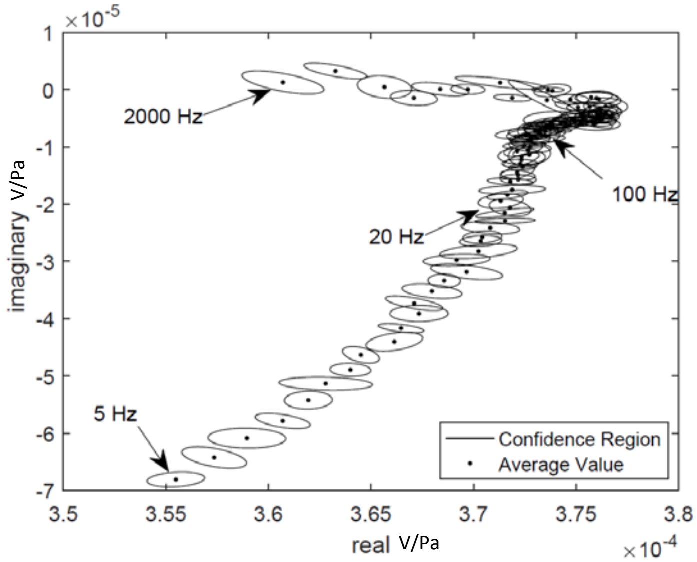
- Semi-major a and semi-minor b axes of elliptical uncertainty bound given by largest and smallest eigenvalues of covariance matrix,

$$a = \sqrt{s\lambda_{\max}} \qquad b = \sqrt{s\lambda_{\min}}$$

where s is a coverage factor given by

$$s = \left(\frac{2(n-1)}{n-2} F_{2,n-2}(\alpha) \right)^{1/2}$$

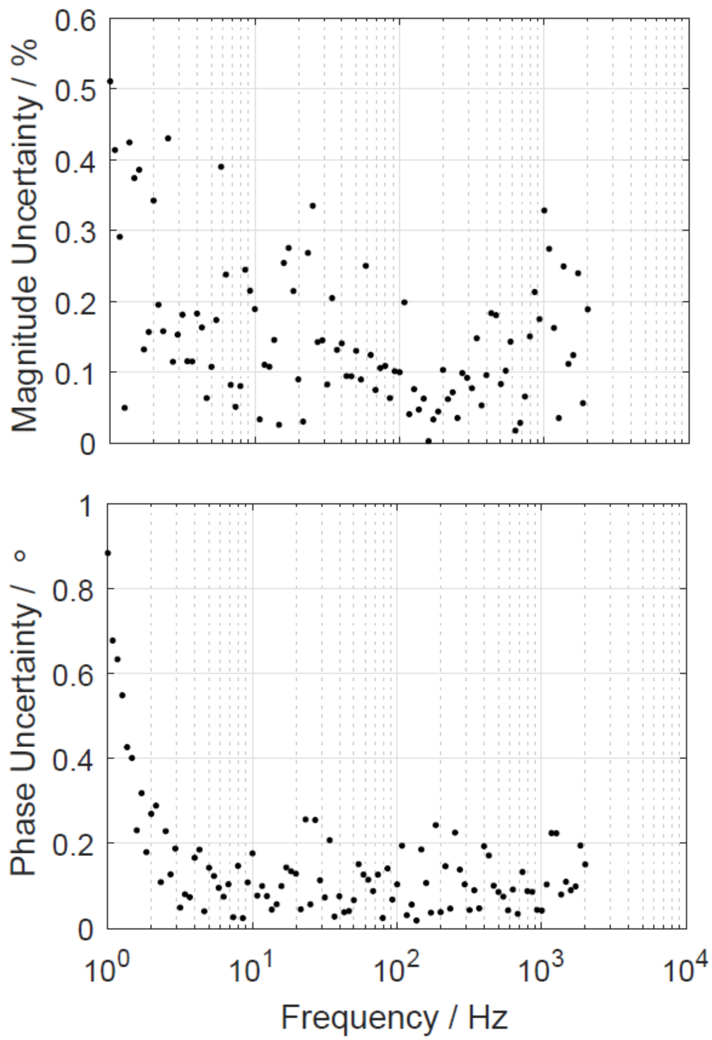




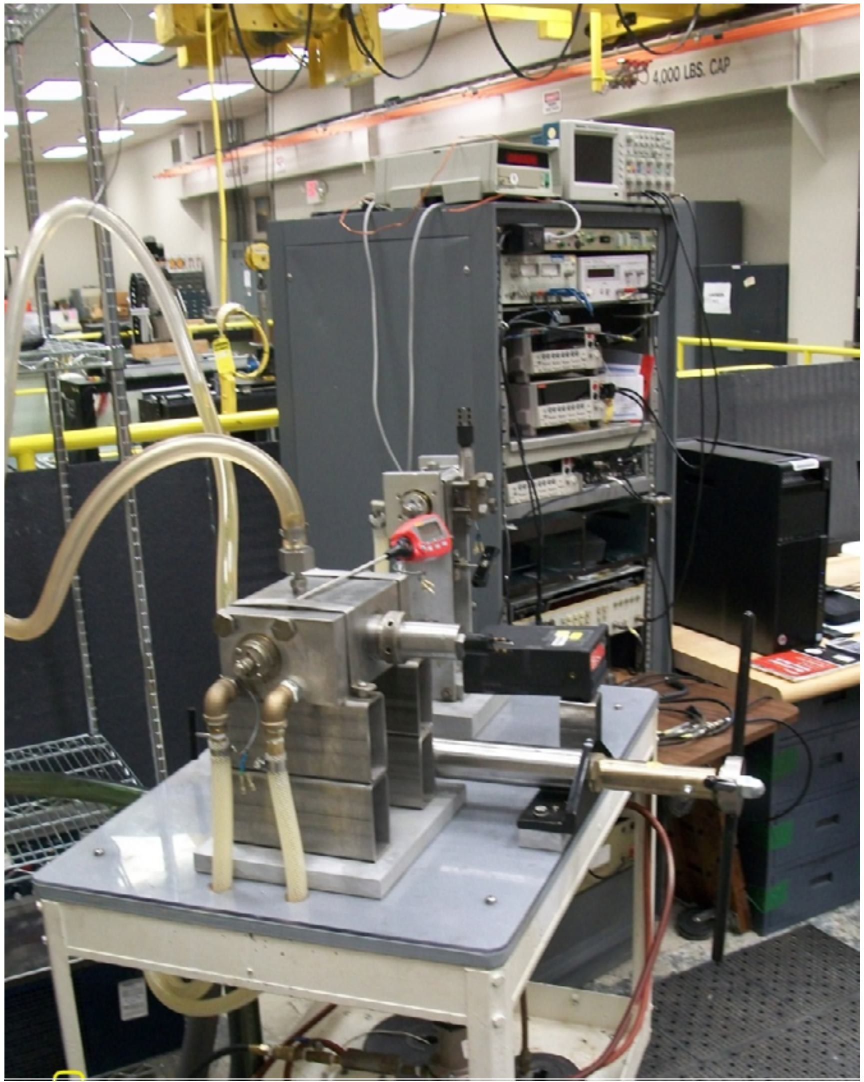
Uncertainty

<u>Type A</u>

- Rotation angle of covariance ellipse given by correlation of real and imaginary parts of measurements.
- Type A component of uncertainties in magnitude and phase of the complex sensitivity calculated from covariance ellipse boundaries.



Uncertainty



Type B

- Magnitude
  - Speed of sound in castor oil
  - Volume of reciprocity coupler
  - Voltage measurements
  - Capacitance
  - Density of castor oil

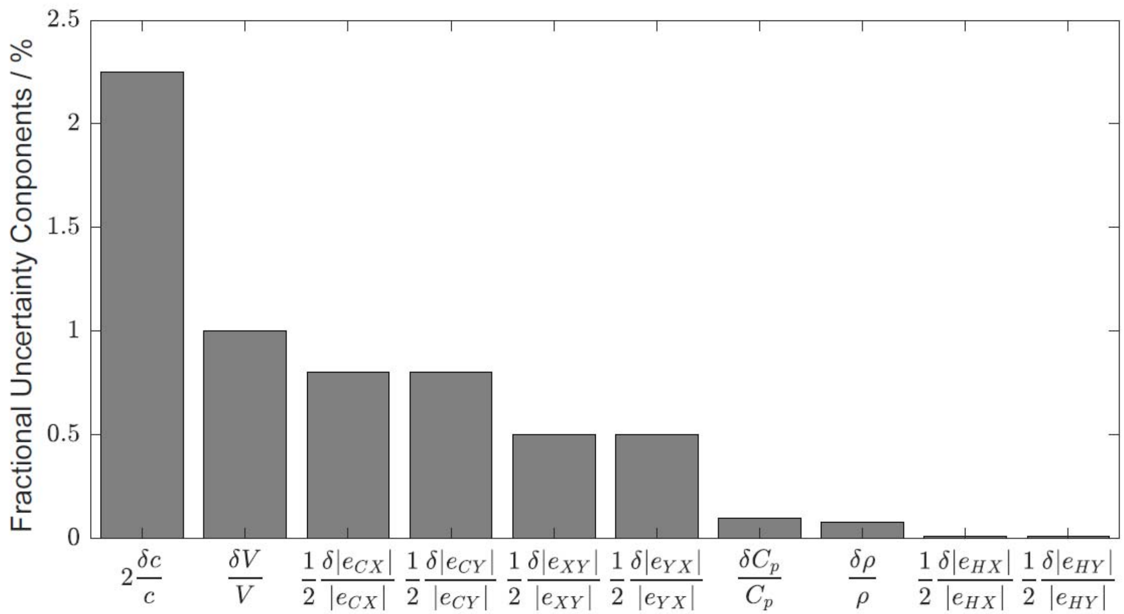
- Phase
  - ±0.5° for HP35565 signal analyzer used to correct phase shift in the differential amplifier.

Uncertainty

<u>Type B</u>

- Magnitude

$$\frac{\delta|\boldsymbol{M}_\mathrm{H}|}{|\boldsymbol{M}_\mathrm{H}|}_\mathrm{B} = \frac{1}{2}\Bigg[\left(2\frac{\delta c}{c}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{1}{2}\frac{\delta|\boldsymbol{e}_\mathrm{CX}|}{|\boldsymbol{e}_\mathrm{CX}|}\right)^2 + \left(\frac{1}{2}\frac{\delta|\boldsymbol{e}_\mathrm{CY}|}{|\boldsymbol{e}_\mathrm{CY}|}\right)^2 + \left(\frac{1}{2}\frac{\delta|\boldsymbol{e}_\mathrm{YX}|}{|\boldsymbol{e}_\mathrm{YX}|}\right)^2 + \left(\frac{1}{2}\frac{\delta|\boldsymbol{e}_\mathrm{XY}|}{|\boldsymbol{e}_\mathrm{XY}|}\right)^2 + \left(\frac{\delta C_\mathrm{p}}{C_\mathrm{p}}\right)^2 + \left(\frac{\delta\rho}{\rho}\right)^2 + \left(\frac{\delta|\boldsymbol{e}_\mathrm{HX}|}{|\boldsymbol{e}_\mathrm{HX}|}\right)^2 + \left(\frac{\delta|\boldsymbol{e}_\mathrm{HY}|}{|\boldsymbol{e}_\mathrm{HY}|}\right)^2\Bigg]^{1/2}$$

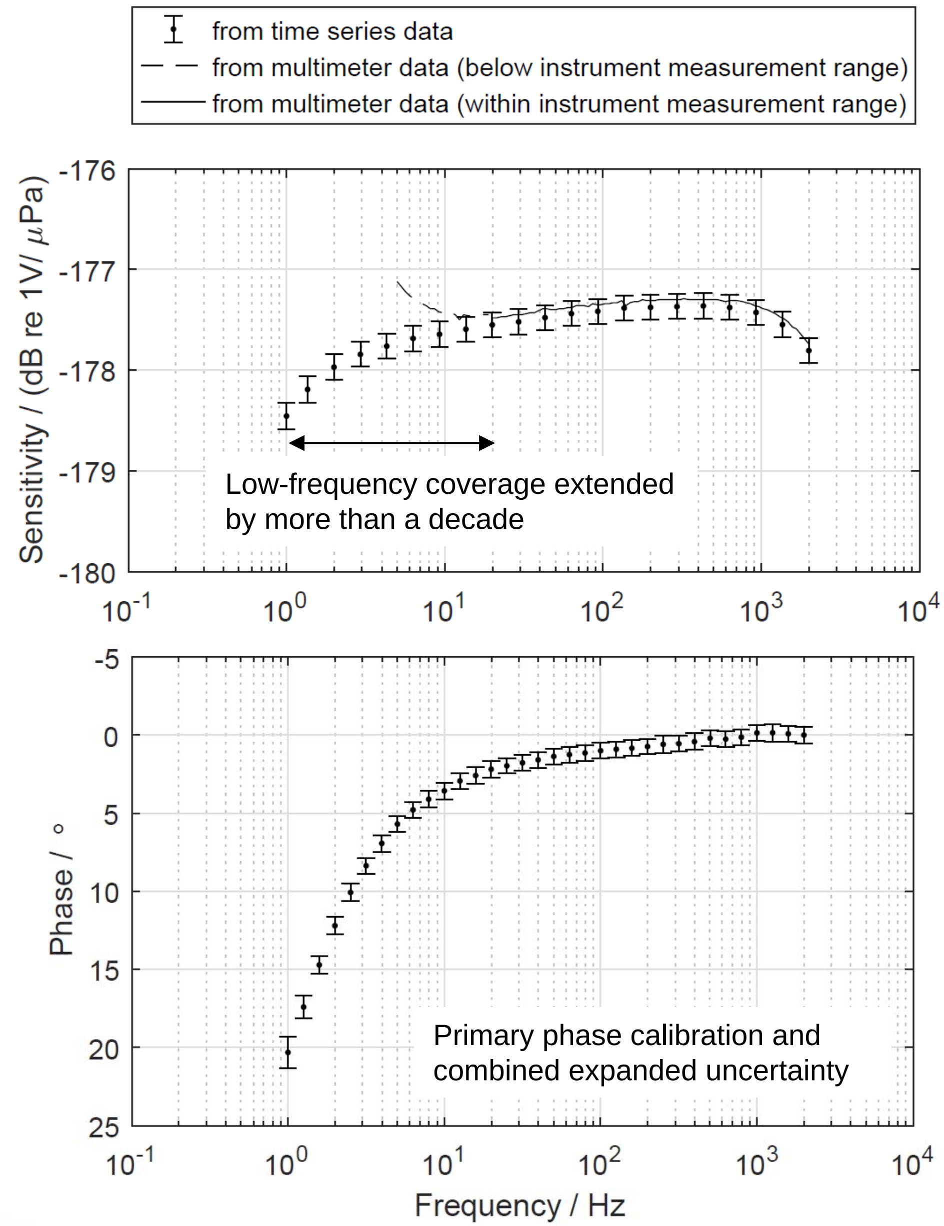


Results

$$\frac{\delta|\boldsymbol{M}_\mathrm{H}|}{|\boldsymbol{M}_\mathrm{H}|}_{\text{total}} = \sqrt{\left(\frac{\delta|\boldsymbol{M}_\mathrm{H}|}{|\boldsymbol{M}_\mathrm{H}|}\right)_\mathrm{A}^2 + \left(\frac{\delta|\boldsymbol{M}_\mathrm{H}|}{|\boldsymbol{M}_\mathrm{H}|}\right)_\mathrm{B}^2}$$

$$\delta\phi_{\text{tot}} = \sqrt{\delta\phi_\mathrm{A}^2 + 0.5^2}$$

Frequency/Hz	FFVS/dB re 1V/μPa	Phase/°
1	-178.5 ± 0.1	$20. \pm 1$
2	-178.0 ± 0.1	12.2 ± 0.6
4	-177.8 ± 0.1	6.9 ± 0.5
8	-177.7 ± 0.1	4.1 ± 0.5
16	-177.6 ± 0.1	2.6 ± 0.5
32	-177.5 ± 0.1	1.8 ± 0.5
64	-177.3 ± 0.1	1.2 ± 0.5
128	-177.3 ± 0.1	0.9 ± 0.5
256	-177.3 ± 0.1	0.5 ± 0.5
512	-177.3 ± 0.1	0.2 ± 0.5
1024	-177.4 ± 0.1	-0.2 ± 0.5
2000	-177.8 ± 0.1	0.0 ± 0.5



Conclusions

A primary calibration method was demonstrated to obtain both the magnitude and phase of the complex sensitivity for a Type H48 hydrophone at frequencies between 1 Hz and 2 kHz.

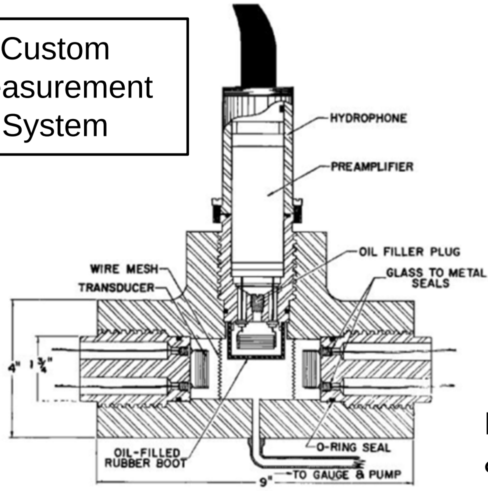
The combined expanded uncertainties of the magnitude and phase of the complex sensitivity at 1 Hz were 0.1 dB re 1V/μPa and 1°, with the uncertainty in phase decreasing to 0.5° at 4 Hz (and above).

The combined expanded uncertainty in magnitude was controlled by the (Type B) fractional uncertainties for sound speed in castor oil, volume of the reciprocity coupler and reciprocal transducer voltage measurements.

The combined expanded uncertainty in phase (above 4 Hz) was controlled by the specified phase tolerance (Type B) of the HP35565 signal analyzer used to measure the phase response of the differential amplifier and to correct the measured data.

Conclusions

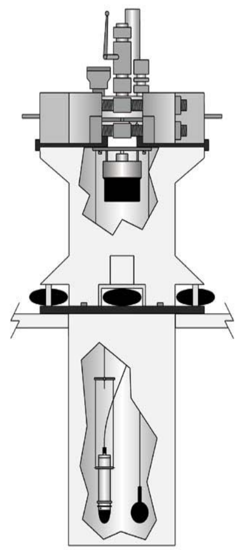
Custom Measurement System



<u>Primary Reference Standard</u>
Reciprocity Coupler
Type H48 and Type A47

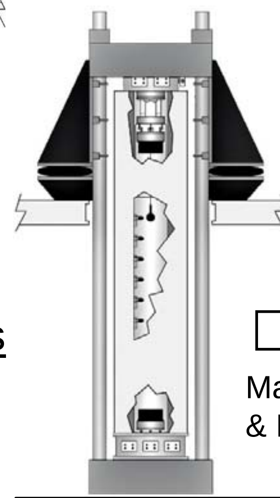
Magnitude & Phase

HP 3562
Signal Analyzer



Magnitude & Phase

<u>Secondary Reference Standards</u>
Standing Wave Tube
Traveling Wave Tubes



Magnitude & Phase

Custom Measurement System

<u>User Applications</u>
USRD Facility Standards
USRD Standards Leasing
Other Navy Labs
Gov’t Agencies
Non-Gov’t Orgs.
Academia
Industry