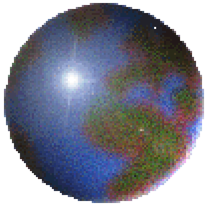


Gravity measurements supporting Kibble balances

Vojtech Pálinkáš

Research Institute of Geodesy, Topography and Cartography
Geodetic Observatory Pecný, Czech Republic

Outline

1) Definition of „gravity“ and „free-fall“ acceleration

2) What is needed for Kibble balance?

3 components and methods/techniquue for determination of:

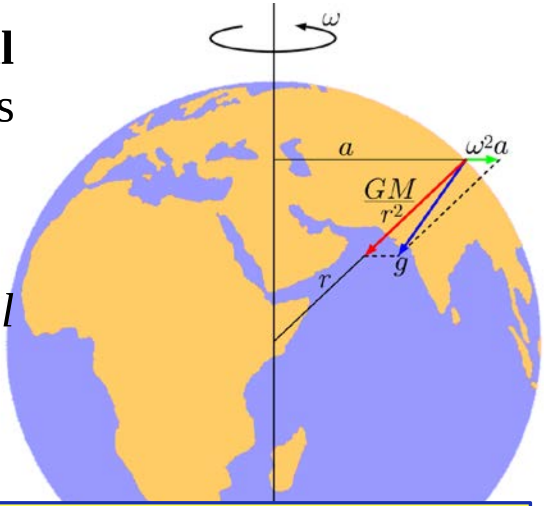
- **spatial „g“ variations,**
- **temporal „g“ variations,**
- **absolute „g“,**

3) Uncertainty of absolute „*g*“ measurements, key systematic effects

4) Conclusion: Uncertainty of different approaches of „*g*“ determination for Kibble balance

Definitions

In geodesy, the Earth’s Gravity Field is described by **non-inertial system** – geocentric terrestrial reference system (including oceans and atmosphere) :



gravity potential ***W*** *= gravitational. p.* ***U*** *+* ***Q*** *centrifugal p.*

gravity acceleration **grad W = g =** *gravitation* **b + z** *centrifugal acceleration*

$g=|\mathbf{g}|$ (magnitude of gravity acceleration)

International Earth Rotation and Reference Systems Service (IERS)
Service International de la Rotation Terrestre et des Systèmes de Référence

IERS Technical Note No. 36

IERS Conventions (2010)

Gérard Petit[1] and Brian Luzum[2] (eds.)

IERS Conventions Centre

[1] Bureau International des Poids et Mesures (BIPM)
[2] US Naval Observatory (USNO)

Not only gravitational force is relevant ⇒ the usual term of „gravitational“ acceleration is incorrect in this case

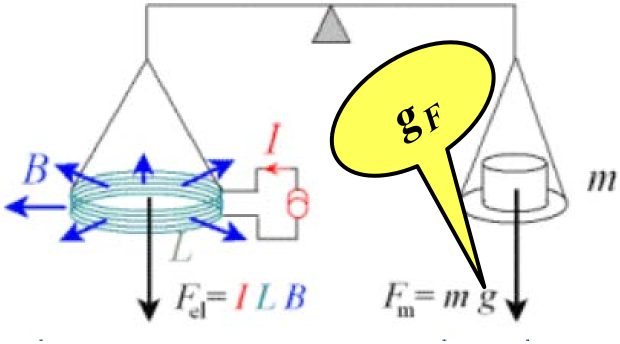
$$g = g_F + \Delta g_T + \Delta g_{Atm} + \Delta g_{Pol}$$

consensus quantity

gravity acceleration = **<u>free-fall acceleration</u>** **<u>measured by absolute gravimeters</u>** corrected by set of conventional corrections for tides, polar motion and atmosphere variation

needed for Kibble balance

Required uncertainty



Target ***u*= 10 μg** $\Rightarrow$ relative standard uncertainty of **1E-8**

g_F contribution should be **<5E-9** $\Rightarrow$ 5E-8 m·s^{-2} (50 nm·s^{-2})

5 μGal

at the centre of mass and time of the KB experiment

BASIC CONCEPT

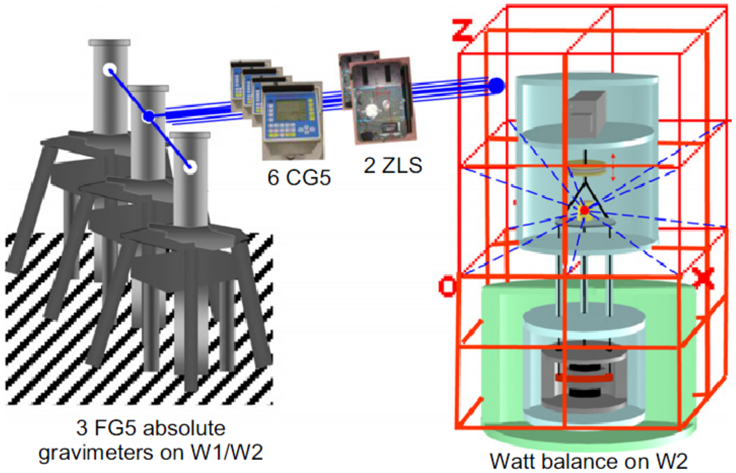


Figure 2. Gravity measurements in the WB laboratory. The 3 AGs occupied W1 and W2 and the 8 RGs measured the ties between W1 and W2 at the five levels and the 3D grid around the WB.

Jiang et al. (2013) Metrologia 50

1) Measure **absolute *g***

2) Measure/model temporal **gravity variations** (tides, polar motion, atmosphere, hydrosphere, geodynamics**)**

***obtain g_F** (t)*

3) Transfer ***g_F*** (t) to the centre of mass

Transferirng „g“ to the centre of mass


IOP Publishing
Metrologia **45** (2008) 265–274

Micro-gravity inve
LNE watt balance

Sébastien Merlet[1], Alexander Kopaev[2], Mi
Gérard Geneves[4], Arnaud Landragin[1] and

IOP Publishing | Bureau International des Poids et Mesures
Metrologia **51** (2014) S32–S41

Gravimetry for wat

J O Liard[1], C A Sanchez[1], B M Wood[1], A

[1] National Research Council Canada, 1200 Montreal Road
[2] Natural Resources Canada, 615 Booth St, Ottawa, ON, K

IOP Publishing | Bureau International des Poids et Mesures
Metrologia
Metrologia **54** (2017) 445–453
https://doi.org/10.1088/1681-7575/aa71e1

Self-attraction mapping and an update on local gravitational acceleration measurement in BIPM Kibble balance

Shisong Li, Franck Bielsa, Adrien Kiss and Hao Fang

Carefull **3D gravity mapping** is needed (↑3 mm ≈ -1 µGal) including determination of the **self- attraction** effect ⇒ standard uncertainty of **2.0 µGal**

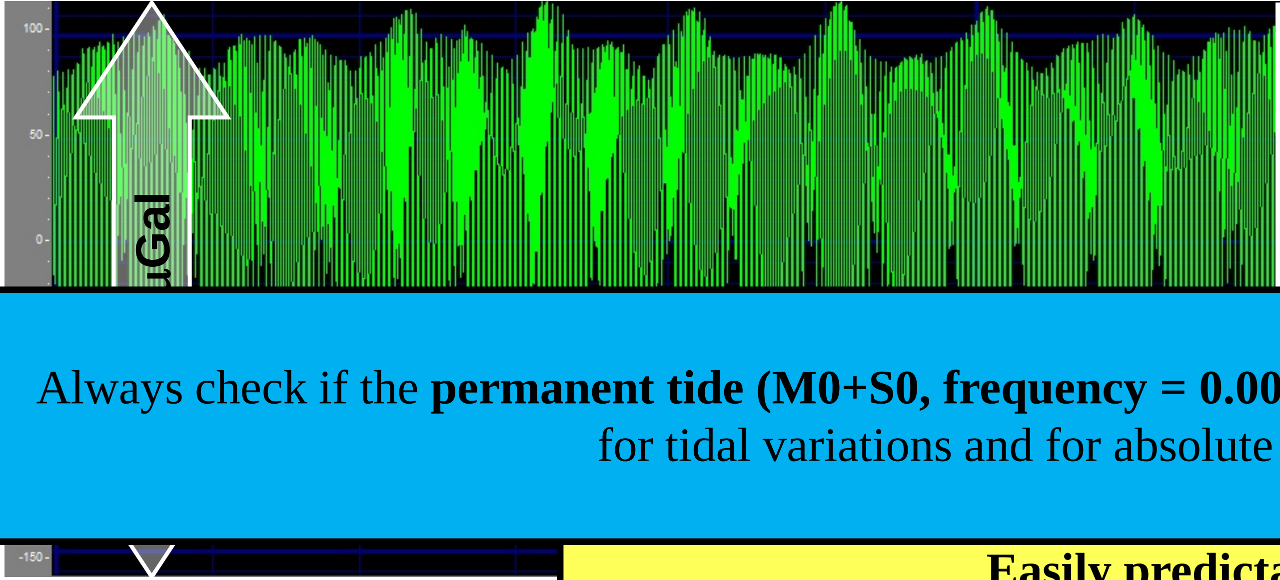
Well calibrated relative gravimeters are needed avoiding those having significant **magnetic sensitivity**.

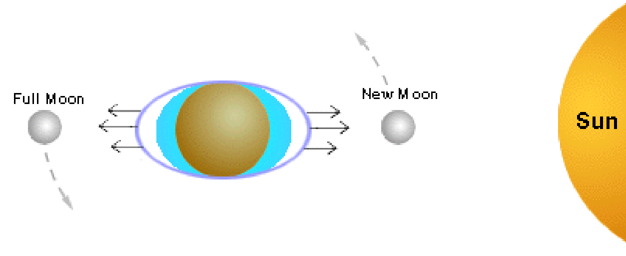
The gravity difference should be computed from the position of **AG** where *g* is invariant of the gradient - **effective position of the free-fall** (1.22 m / 1.27 m for FG5/FG5X)

$(5^2 – 2^2)^{0.5}$ = **4.6 µGal**

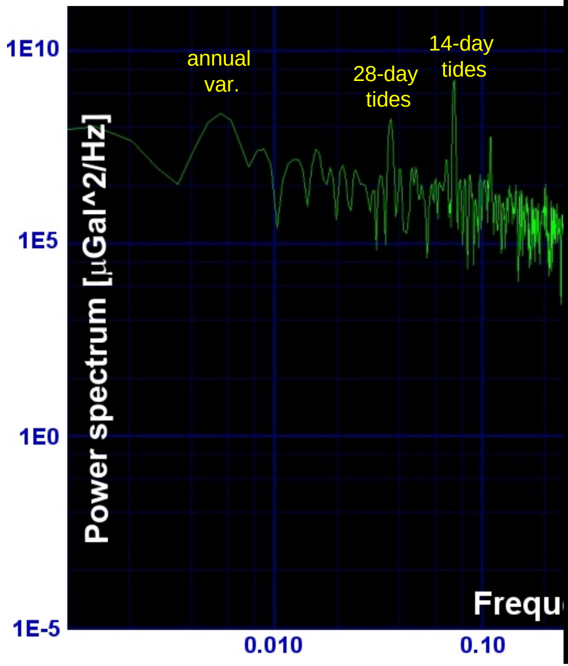
contribution from determination of absolute g_F (including temporal variations)

Temporal variations - Tides





Always check if the **permanent tide (M0+S0, frequency = 0.000)** are treated by the same way for tidal variations and for absolute *g*



Easily predictable effect

Possibility to determine:

1) **modelling** only (solid tides + ocean loading) **±1 μGal**

2) **6-months observations** with well calibrated relative gravimeter **±0.2 μGal**. Tidal parameters for main tidal waves are determined (**ocean tides** are included).
No need to measure exactly on the site of Kibble balance experiment. Tidal parameters have **large spacial validity** if the ocean tides are small – e.g. differences between using parameters from Prague or Vienna reach below 0.1 μGal

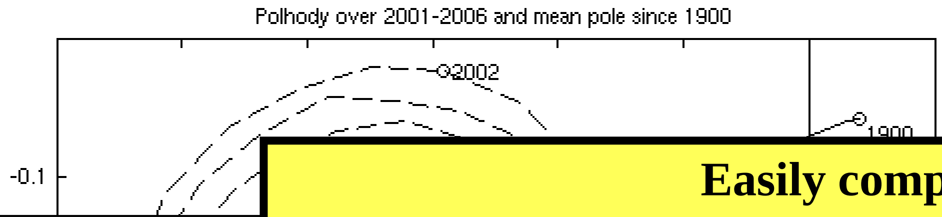
Temporal variations - polar motion

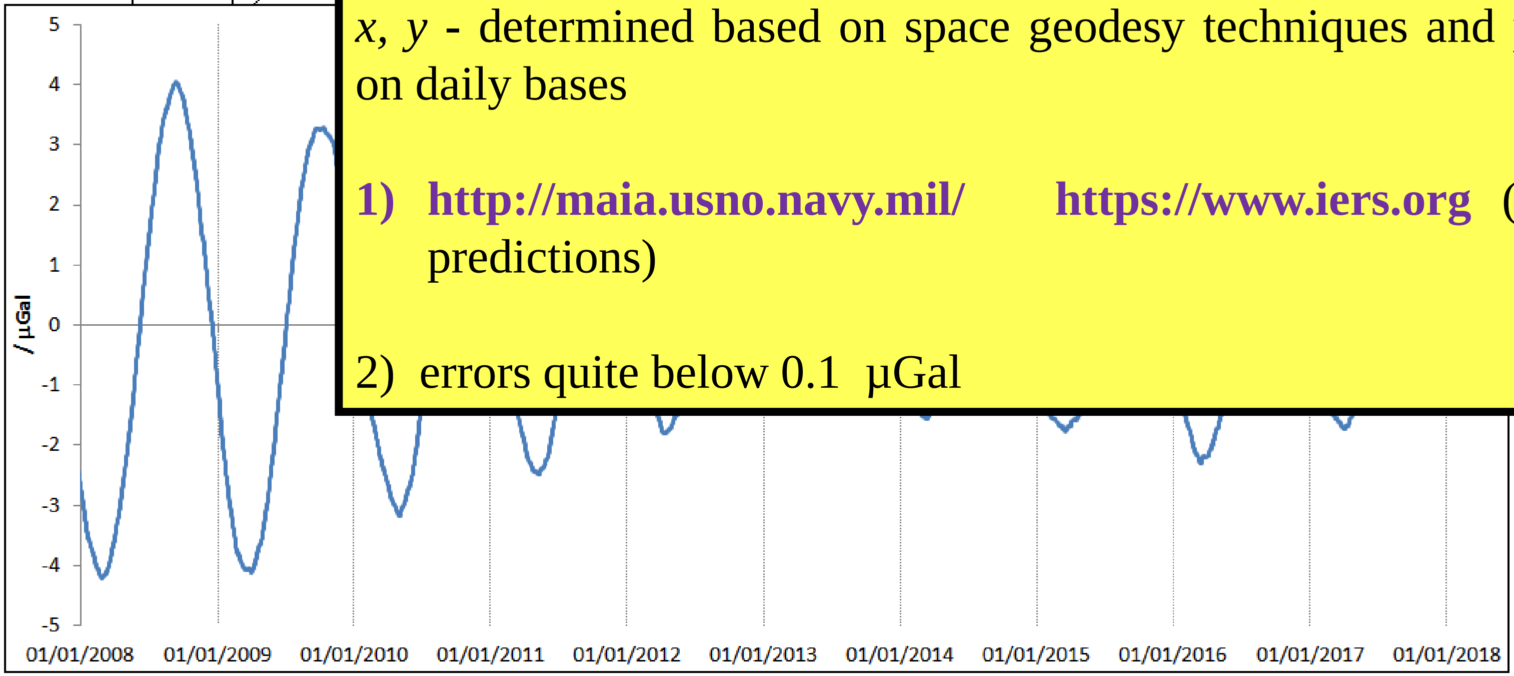
The motion of the rotation axis of the earth **relative to the crust**. Main components: a free oscillation with period about **435 days** (Chandler wobble) and an **annual oscillation**.

The momentary latitude is changing ⇒ **change of the centrifugal acceleration**

$$\Delta g_{Pol} = \frac{19.074}{\sqrt{1-0.0066944\sin^2\varphi}}\sin 2\varphi\,(x_{mas}\cos\lambda - y_{mas}\sin\lambda)$$

/ μGal





Easily computable effect

x, *y* - determined based on space geodesy techniques and published on daily bases

1) **http://maia.usno.navy.mil/** **https://www.iers.org** (including predictions)

2) errors quite below 0.1 μGal

Temporal variations – Atmosphere

DIN 5450 (ISO 2533:1975) Standard Atmosphere ⇒ Nomal pressure (depends on elevation H:

$$p_n = 1013{,}25\,(1 - 0{,}0065\,H/288{,}15)^{5{,}2559} \quad \text{hPa}$$

Simply model/correction using single admittance: $\Delta g_P = 0{,}3\,(p - p_n)$ µGal

Pressure variations up to 60 hPa at given site ⇒ variations of about **18 µGal**

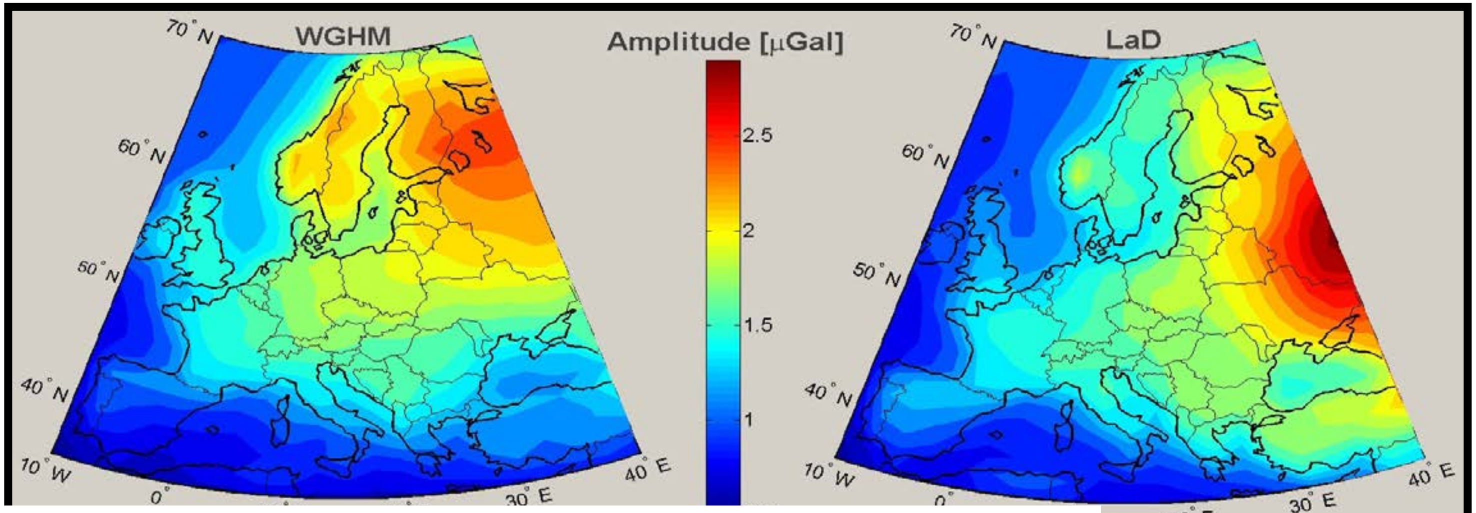
- the simple model works well in most of cases (errors up to 2 µGal with respect to 3D model) ***u = ± 1.0 µGal***

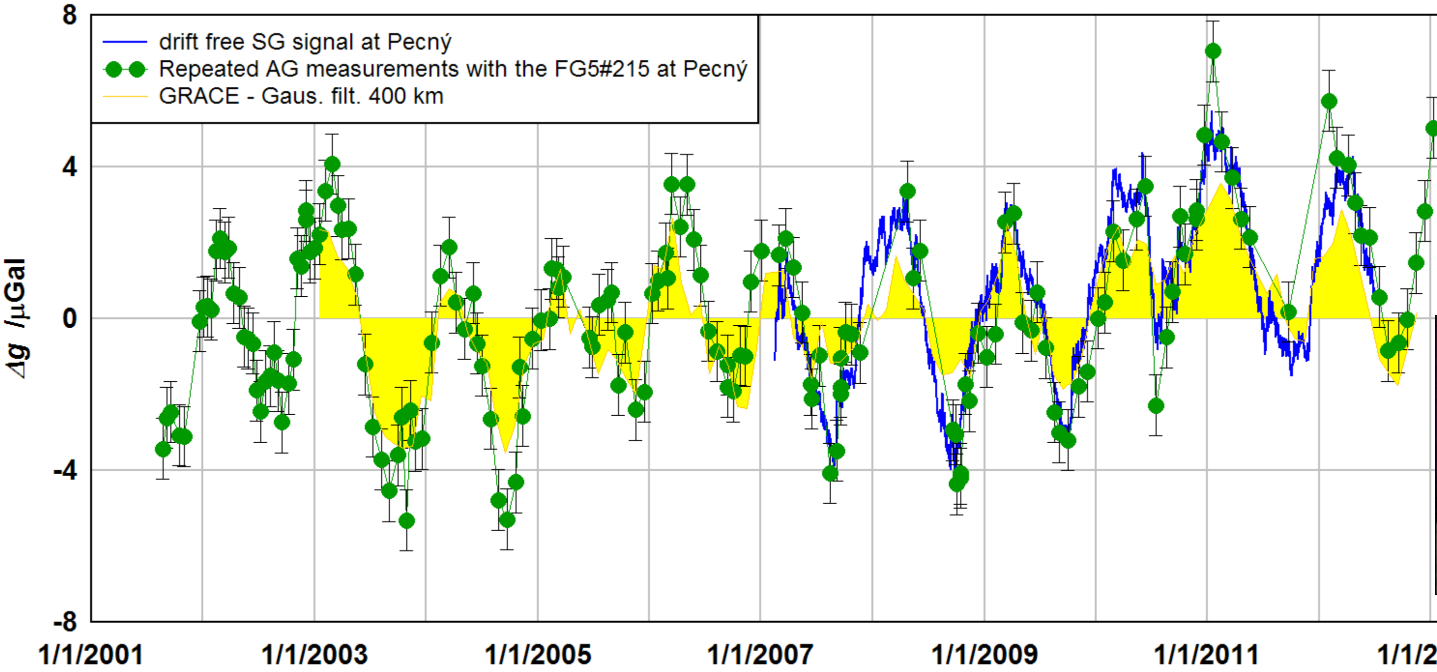
- especially at high elevations, the single admittance should be verified
- atmospheric pressure has to be measured simultaneosly with KB experiment (daily variations up to 20 hPa)

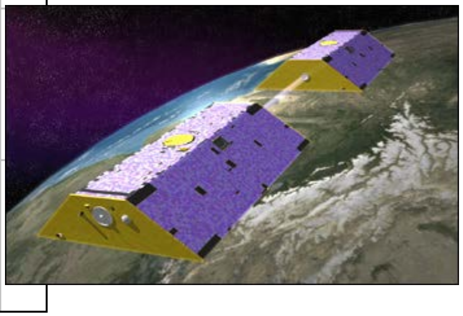
Temporal variations – Global hydrology

Continental water storage variations (hydrological models WGHM, LaD, etc.) – validateted by **GRACE**

”g” variations up to 6 µGal in Europe
Maximum *g* in autumn







Temporal variations – Local hydrology

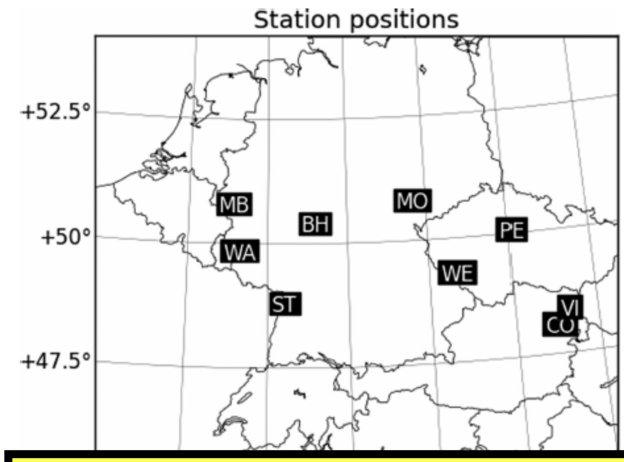
Geophysical Journal International

Geophys. J. Int. (2014) doi: 10.1093/gji/ggt524

Geophysical Journal International Advance Access published January 29, 2014

The quest for a consistent signal in ground and GRACE gravity time-series

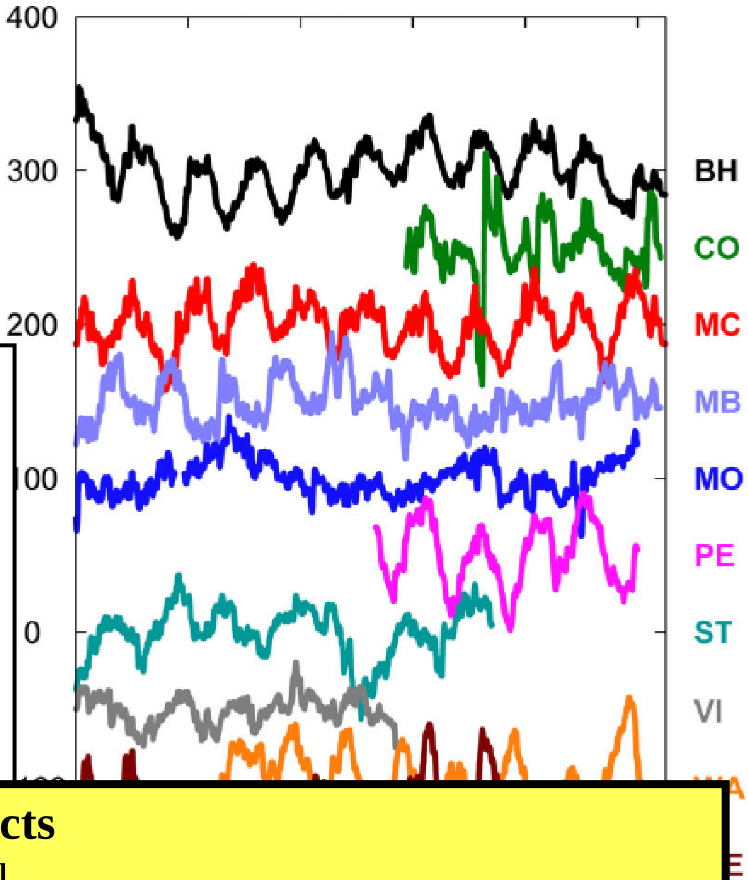
Michel Van Camp,[1] Olivier de Viron,[2] Laurent Métivier,[3] Bruno Meurers[4] and Olivier Francis[5]



The signal is variable at stations due to LOCAL hydrology (100 m radius is critical)

Seasonal effect is dominant.

Amplitudes depends on the hydrology and localization of the station



Hardly modeled effects

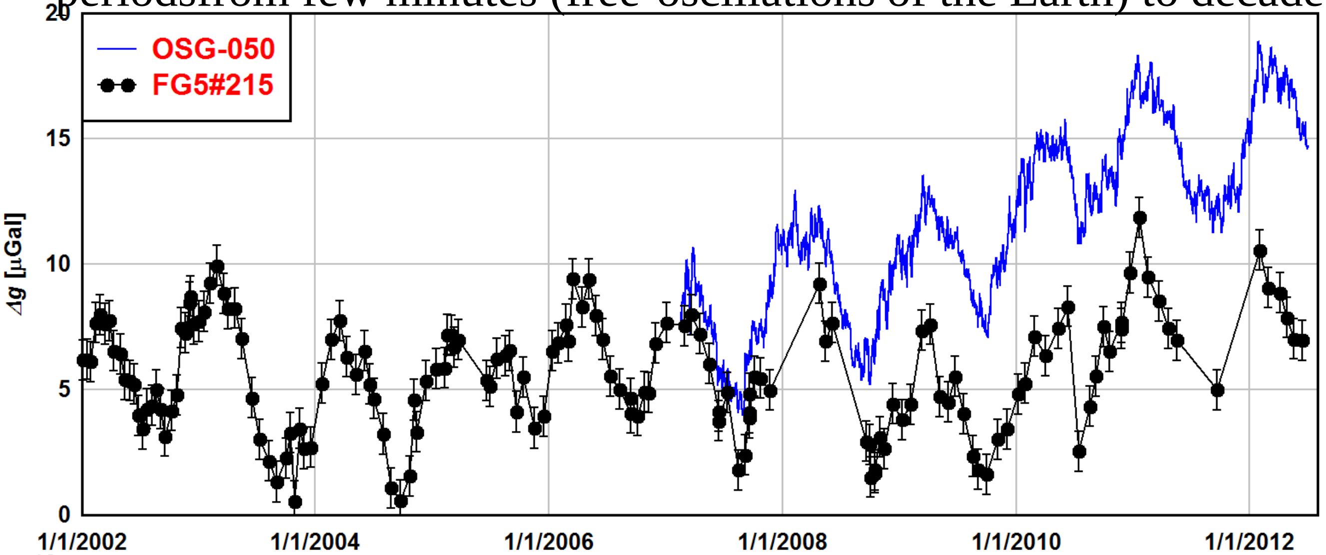
Seasonal variations approaching 10 µGal are quite usual

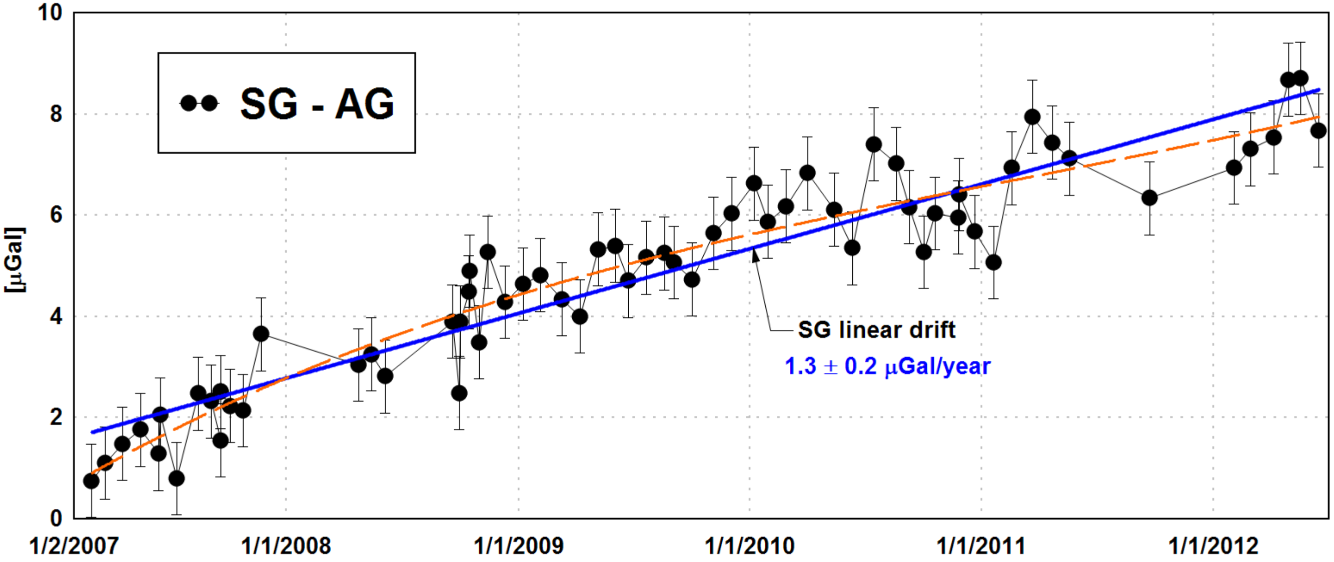
The efect should be **verified by measurements** (carried out for different periods of the year) – at least to determine the “hydrological sensitivity” of the station

Localization of the **Kibble balance and “*g*”** measurements should be **close to each other**

Combination of AG and SG

In situ geodynamic stations: used for the gravity variations in periodsfrom few minutes (free-oscillations of the Earth) to decades





GWR INSTRUMENTS, INC.

- relative values
- precision < 0.1 µGal
- continuous registration
- high temporal resolution

Worldwide SG stations

International Geodynamics and Earth Tides Service (IGETS)

Continuous monitoring of temporal gravity variations with Superconducting Gravimeters

Selection from active 21 (+2) stations:

Europe (13+1):
BG, BH, CO, MC, MB, ME, MO, NY, OS, PE, ST, WE, YS, [BF]

North America (3): AP, BO, CA

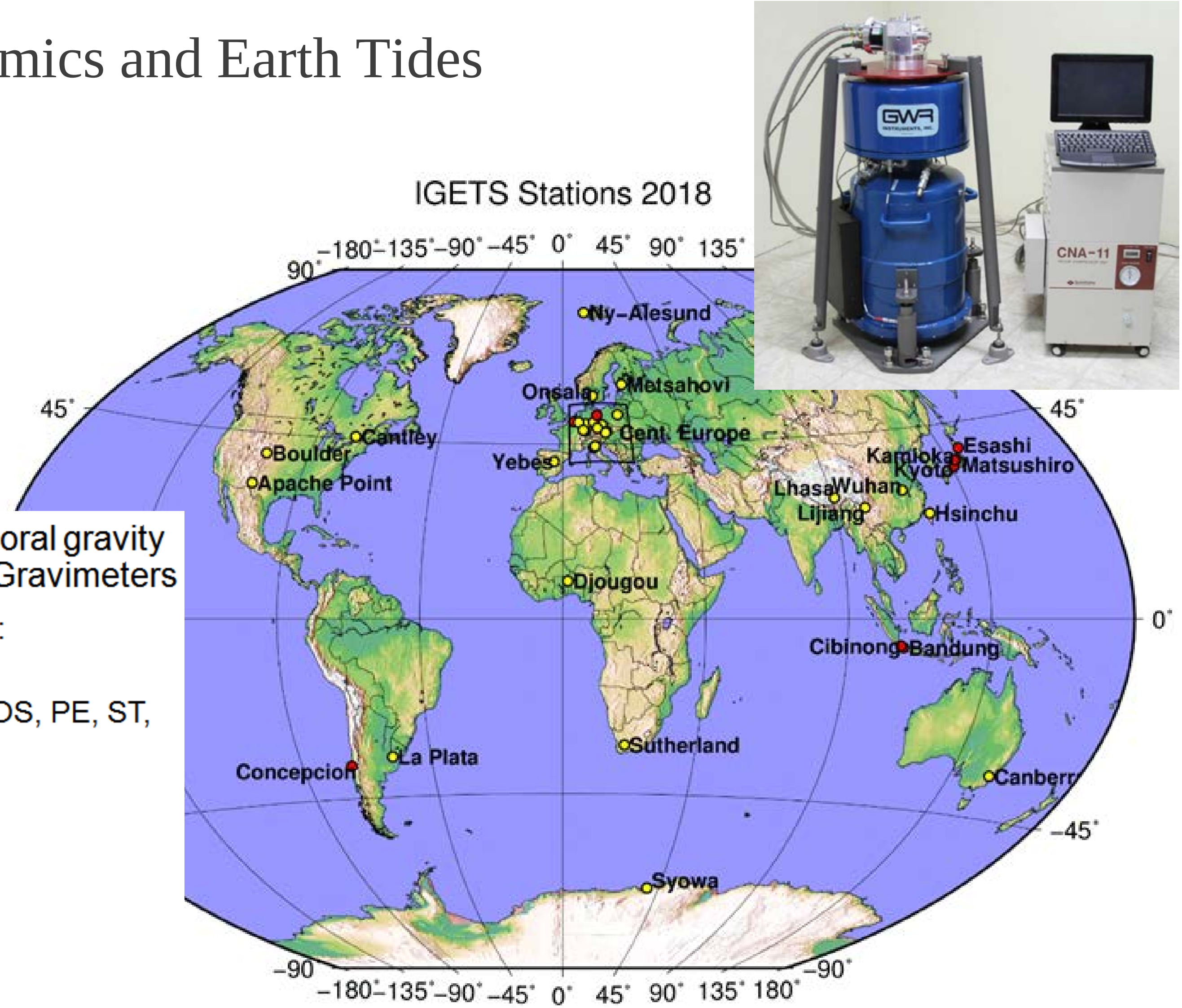
South America (1): LP

Asia:

- China (3): LH, LI, WU
- Japan (1+2): MA, [MI, IS]

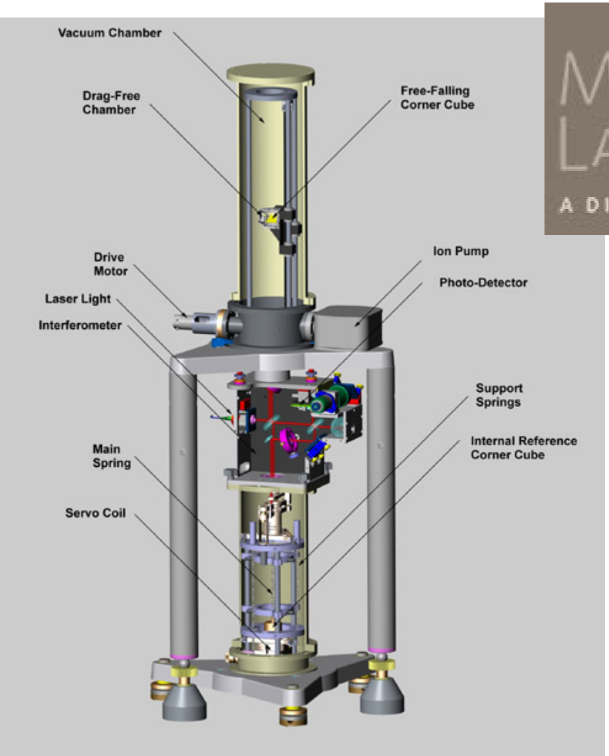
Africa (1+1): SU, [DJ]

Antarctica (1): SY



http://isdc.gfz-potsdam.de/igets-data-base

FG5/X absolute gravimeters



MICRO g LACOSTE
A DIVISION OF LRS

Most accurate and commercial available absolute gravimeters

Corner-cube absolute gravimeter
The freely falling test mass is tracked by laser interferometer

$$z_i = z_0 + v_0\,(t_i + \frac{W_{zz}}{6}\,t_i^3) + \frac{1}{2}\,g_0(t_i^2 + \frac{W_{zz}}{12}\,t_i^4)\,,$$

Absolute gravimeter:

- based upon physical standards
- no drift
- uncertainty: ± 2.5 µGal
- Long-term reproducibility : ± 1.5 µGal
- observation epochs

Top of the drop

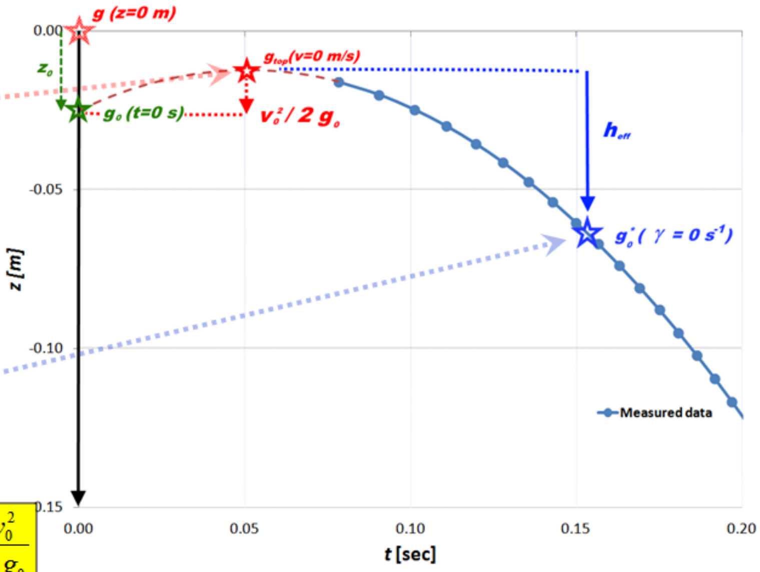
$$\boxed{v = \frac{dz}{dt} = 0} \qquad z = z_0 - \frac{v_0^2}{2g_0}$$

$$\boxed{g_{top} = g_0 - \gamma\,\frac{v_0^2}{2\,g_0}}$$

Effective position

$$z_i = z_0^* + v_0^*\,t_i + g_0^*\,\frac{t_i^2}{2}$$

$$\boxed{h_{eff} = \frac{g_0(\gamma_A) - g_0^*(\gamma = 0)}{\gamma_A} + \frac{v_0^2}{2\,g_0}}$$



CMCs

Austria, BEV (Bundesamt für Eich- und Vermessungswesen)

Complete CMCs in Mass and related quantities for Austria (.PDF file)

Gravitational acceleration. On (stable) site, **9.75 m/s^2 to 9.85 m/s^2**
Absolute expanded uncertainty (k = 2, level of confidence 95 %) in m/s^2: **1.0E-07**
Absolute measurement
Ambient temperature: (21 ± 5) °C

Finland, FGI (Finnish Geospatial Research Institute)

Complete CMCs in Mass and related quantities for Finland (.PDF file)

Gravitational acceleration. On (stable) site, **9.75 m/s^2 to 9.85 m/s^2**
Absolute expanded uncertainty (k = 2, level of confidence 95 %) in m/s^2: **8.0E-08**
Free fall experiment
Ambient temperature: 21 °C ± 8 °C
Approved on 03 January 2007

Italy, INRIM (Istituto Nazionale di Ricerca Metrologica)

Complete CMCs in Mass and related quantities for Italy (.PDF file)

Gravitational acceleration. On (stable) site, **9.75 m/s^2 to 9.85 m/s^2**
Absolute expanded uncertainty (k = 2, level of confidence 95 %) in m/s^2: **1.5E-07**
Absolute measurement
Ambient temperature: 23 °C ± 10 °C
Approved on 03 January 2007

Switzerland, METAS (Federal Institute of Metrology)

Complete CMCs in Mass and related quantities for Switzerland (.PDF file)

Gravitational acceleration. On (stable) site, **9.75 m/s^2 to 9.85 m/s^2**
Absolute expanded uncertainty (k = 2, level of confidence 95 %) in m/s^2: **8.0E-08**
Absolute measurement
Ambient temperature: (20 ± 5) °C
Approved on 02 July 2008

Ukraine, NSC "Institute of Metrology" (National Scientific Centre "Institute of Metrology")

Complete CMCs in Mass and related quantities for Ukraine (.PDF file)

Gravitational acceleration. On (stable) site, **9.77 m/s^2 to 9.85 m/s^2**
Absolute expanded uncertainty (k = 2, level of confidence 95 %) in m/s^2: **2.08E-07**
Absolute measurement
Ambient temperature: 14 °C to 34 °C
Approved on 21 June 2017
Internal NMI service identifier: NSCIM-M-8.1.1/7

u* = 4.0 *μGal

How accurate are the absolute measurements?

Comparisons of absolute gravimeters

ICAGs – at BIPM from 1981 to 2009

1981-82 7 AG	1985 6 AG	1989 10 AG	1994 11 AG	1997 17 AG	2001 15 AG	2005 19 AG	2009 22 AG
IMGC, Italy	IMGC	IMGC	IMGC	IMGC	IMGC	IMGC-2	IMGC-2*
GABL, USSR	GABL	GABL		GABL	A10-b02*	GABL	CAG-01
JAEGER, BIPM	JAEGER, BIPM		JAEGER, Japan	NIM-2A	A10-03	A10-8	NIM-02
NIM, China	NIM-1	NIM-1		NIM-2B*		FGC-01	MPG-2
Hammond, USA	Zumberge, USA	NAO, Japan		ZZB		GABL-01	A10-05
Sakuma, static array		Sakuma, static array*				TBG-01	A10-14
Faller, USA	JILAg-1	JILAg-4					A10-20
		JILAg-2	JILAg-2	JILAg-2	JILAg-2	JILAg-2	
		JILAg-3	JILAg-3	JILAg-3			
		JILAg-5	JILAg-5	JILAg-5	JILAg-5		
		JILAg-6	JILAg-6	JILAg-6	JILAg-6	JILAg-6	JILAg-6
							FGL-103
			FG5-101	FG5-101	FG5-101	FG5-101	FG5-101
			FG5-102	FG5-103	FG5-103	FG5-108	FG5-102
			FG5-104	FG5-105	FG5-105	FG5-202	FG5-105
			FG5-107	FG5-107	FG5-108	FG5-206	FG5-209
			FG5-108	FG5-108	FG5-202	FG5-209	FG5-213
				FG5-202	FG5-204	FG5-211	FG5-215
				FG5-206	FG5-206	FG5-213	FG5-220
					FG5-209	FG5-215	FG5-221
					FG5-211	FG5-216	FG5-224
					FG5-213	FG5-221	FG5-228
					FG5-301*	FG5-224	FG5-230
						FG5-228	FG5-233
							FG5-238
	1st JILAg	**5 JILAg**	**5 FG5**	**7 FG5**	**11 FG5**	**12 FG5**	**13 FG5**

2003, 2007 – Walferdange in Luxembourg

3 CIPM_KC: 2009 (11 KC + 10 PS), 2013 Walferdange (10 KC + 15 PS), 2017 Beijing (13 KC + 17 PS);

3 EURAMET_KC + PS: 2011, 2015, 2018

Comparisons of absolute gravimeters

The KCRV is defined by KC gravimeters only using the weighted constraint

$$\Sigma w_k \delta_k = 0$$

CCM-KC 2009: 7 from 11 AGs FG5/X

CCM-KC 2013: 8 from 10 AGs FG5/X

CCM-KC 2017: 12 from 13 AGs FG5/X

COMPARISONS – more than **90% of weights** are given by **FG5/X** gravimeters **declaring standard uncertainties of 2-3 µGal (confirmed at comparisons)**

However, KCRVs are **strongly "FG5/X dependent" !!!**

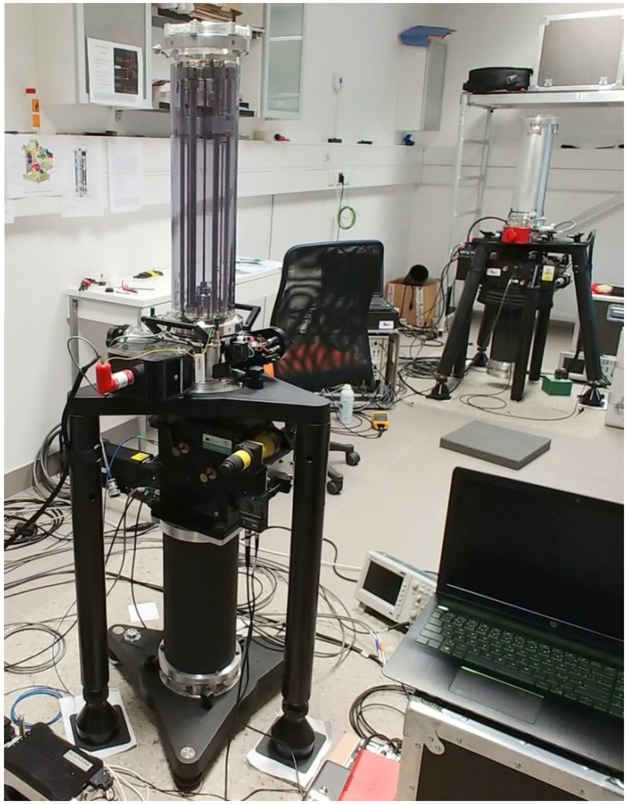
Possible **Systematic effects** have to be **captured!!!**

Another technique have to be used for verification !!!

Systematic errors of FG5/X

CZECH
METROLOGY
INSTITUTE

Research Institute of
Geodesy, Topography and
Cartography



Experiments on FG5-215 and FG5X-251:

- validation of measurement results
- determination of particular systematic errors
- improvement of the original measurement technology
- developping new measurement systems, methods, software, analysis

Petr Křen et al.

Metrologia 53 (2016) 27-40

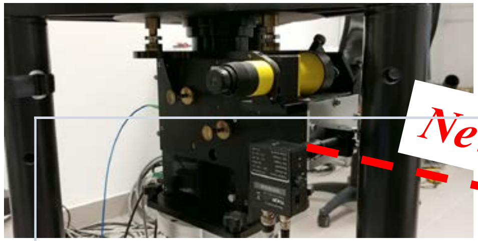
Journal of Geodesy 93 (2019) 27-40

Metrologia 55 (2018) 451-459

Metrologia 54 (2017) 161-170

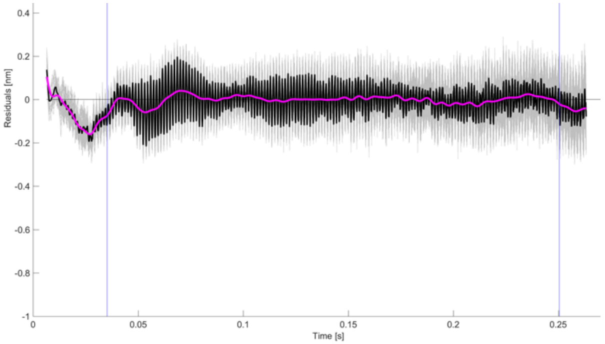
Metrology and Measurement Systems 25 (2018) 701-713

FG5(X) uncertainty

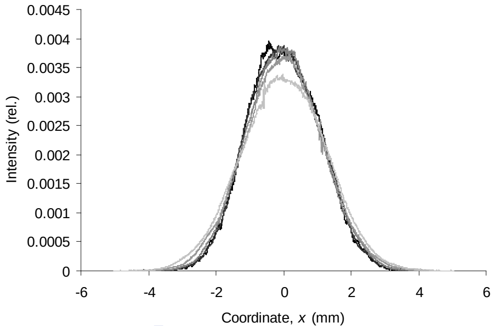


New system

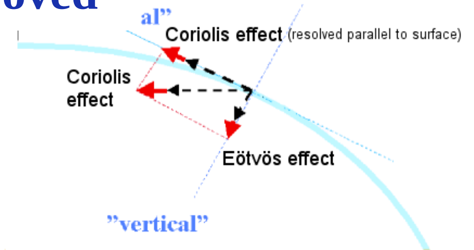
	FG5/X with HS5 system + additonal corrections	FG5/X - original system + standard corrections
Influence parameters x_i	**Contribution to the uncert. $/\mu Gal$**	**Contribution to the uncert. $/\mu Gal$**
Laser frequency	0.02	0.02
Rb-oscillator frequency	0.02	0.02
Test mass rotation, mechanical effects	**0.70**	**0.70**
Air gap modulation, floor recoil, fringe interval	**1.15**	**1.15**
Vacuum pressure	0.15	0.15
Self attraction correction	0.20	0.20
Electrostatic effect	0.12	0.12
Magnetic gradient field	0.23	0.23
Temperature sensitivity	0.50	0.50
Determination of the reference instr. height	0.35	0.35
Perturbation due to non-constant gravity gradient	0.20	0.20
Electronic phase shift and timing electronics	**0.40**	**2.00**
Impedance mismatch	**0.05**	**0.70**
Coriolis effect	**0.10**	**0.60**
Verticality of the test beam	**0.05**	**0.60**
Diffraction correction	**0.45**	**1.00**
Dispersion in cable	**0.02**	**0.50**
Setup-error, interferometer alignment	**0.90**	**0.90**
	1.88	**3.10**



Investigated



Improved



FG5(X) Original system bias

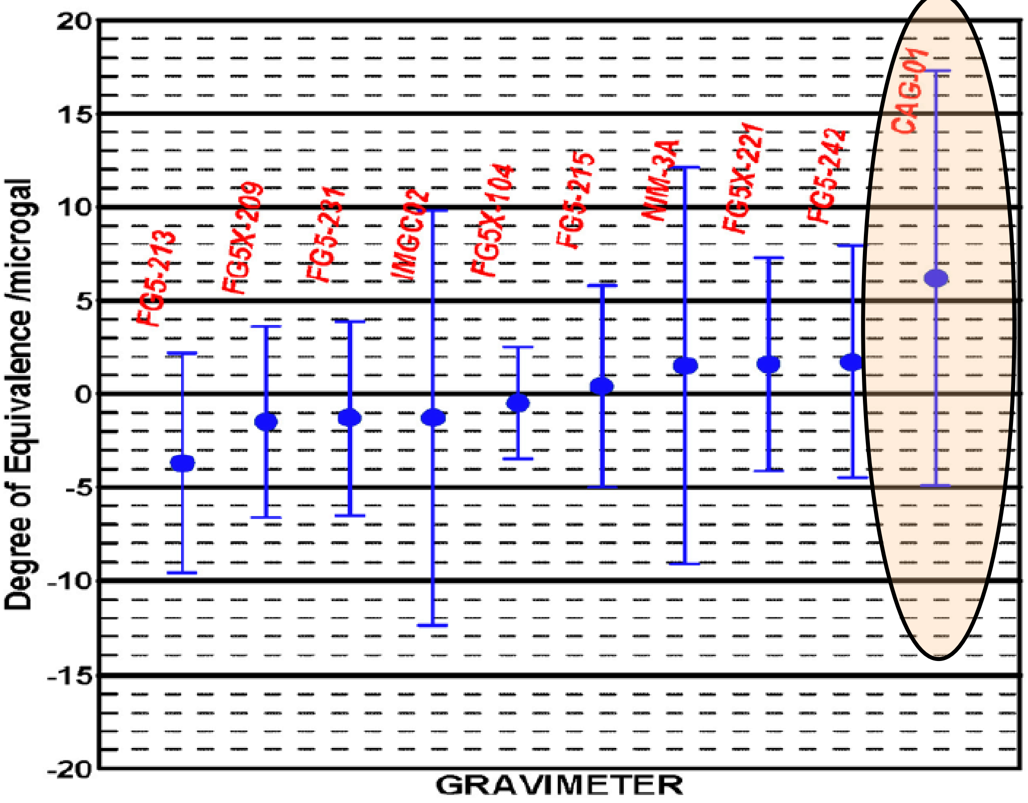
Corrections / µGal	**Standardly applied**	**New corrections**	**Difference**
Speed-of-light	$\tilde{t}_i = t_i + \frac{(z_i - z_0)}{c}$	$\tilde{t}_i = t_i + \frac{(z_i - z_0)}{c}$	0.0
Self-attraction	-1.2	-1.2	0.0
Diffraction	+1.2	+2.4	+1.2
Distortion (350 mV fringes)	N/A	-2.2	-2.2
Cable length (5 m)	N/A	-1.0	-1.0
Impedance mismatch	N/A	-1.4 / +1.4	?
Verticality	N/A	+0.2 / +1.0	+0.6
Eötvös/Coriolis	N/A	-1.0 / +1.0	?
Air-gap modulation etc.	?	?	?
Sum			**-1.4**

According to present estimates, the g-values of FG5/X should be too high at present and decreased by -1.4 µGal in average. However few next effects have to be estimated.

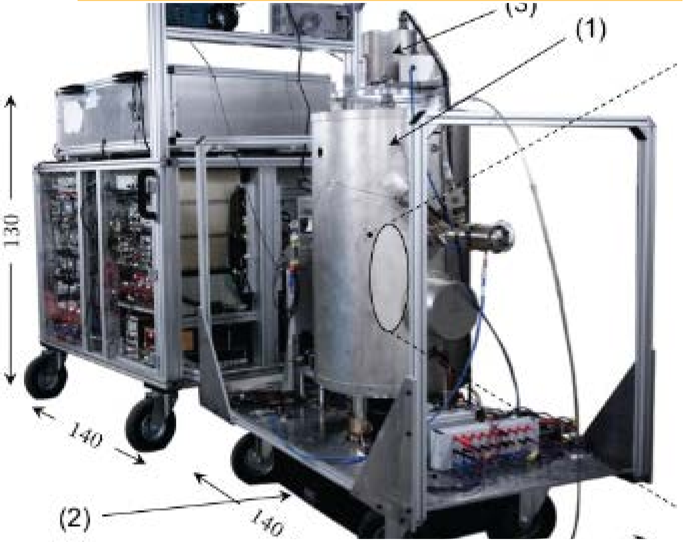
Generaly, the present bias of KCRV is expected to be up to 2 µGal.

Comparison with cold atom gravimeters

CAG-01 (LNE-SYRTE) – till 2015 the only CAG at International Comparisons

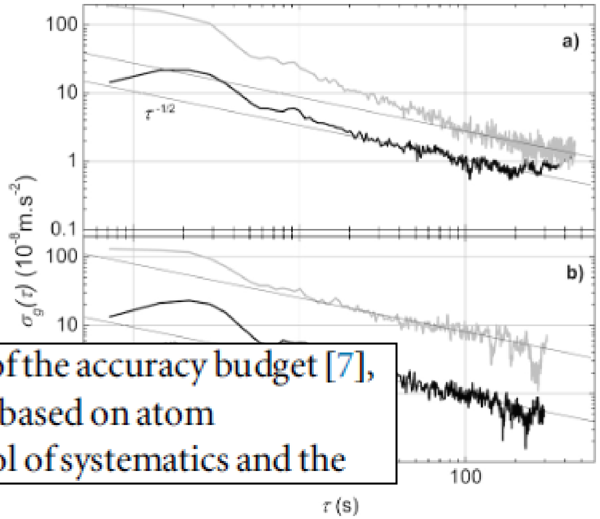


$$\Delta\Phi_{\text{int}} = -\vec{k}_{\mathit{eff}}\,\vec{g}\,T^2 + \delta\Phi_{\mathit{noise}} + \delta\Phi_{\mathit{sys}}$$



Karcher et al. (2018) New Journal of Physics 20

+5.6 ± 1.3 µGal bias explained by wavefront aberation

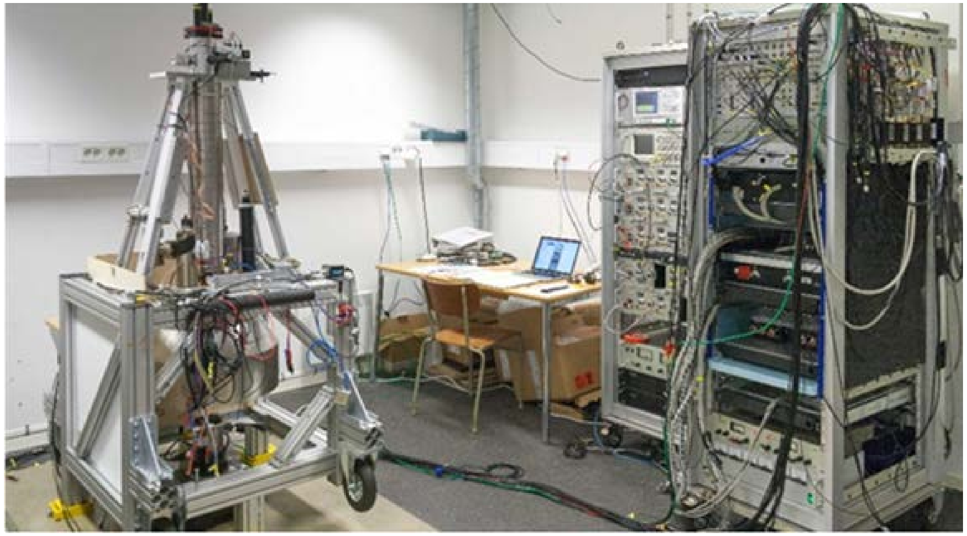


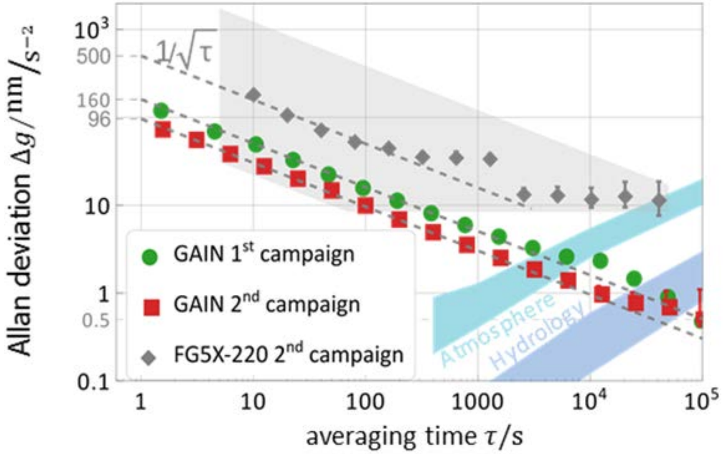
> With the above-mentioned improvements, and after a careful re-examination of the accuracy budget [7], accuracies better than 10 nm s^{-2} are within reach. This will make quantum sensors based on atom interferometry the best standards in gravimetry. Furthermore, the improved control of systematics and the

Comparison with cold atom gravimeters

GAIN

Freier et al. 2016, Journal of Physics: Conference Series 723 012050





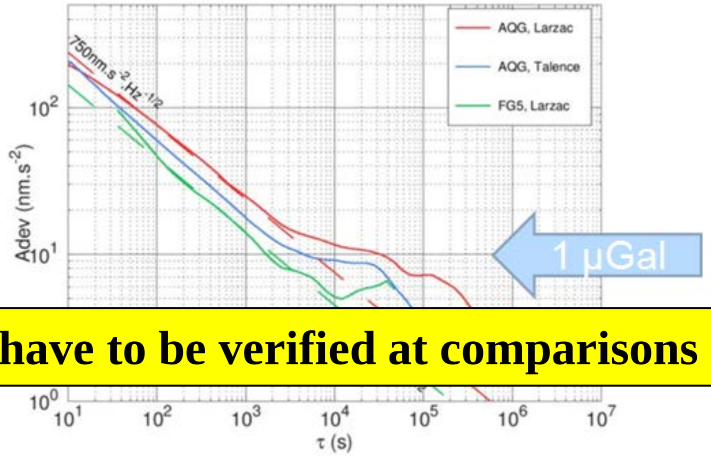


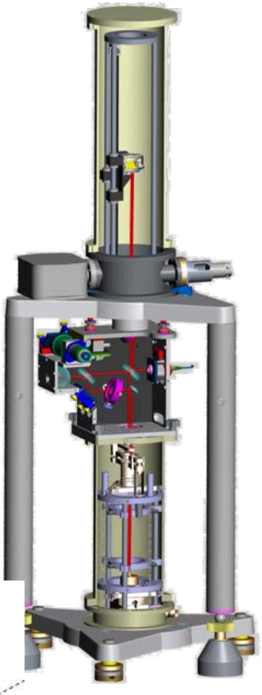
Figure 5. Allan deviation (left) and power spectral density (right) of the gravity measurements with AQG-A01 in Larzac (solid red) and Talence (blue), and with FG5#228 in Larzac (solid green). The effective sampling interval of the FG5 was taken as 36 s. The red (resp. green) dashed line in the Allan plot indicates a sensitivity of 750 (resp. 450) nm.s^{-2}.Hz$^{-1/2}$.

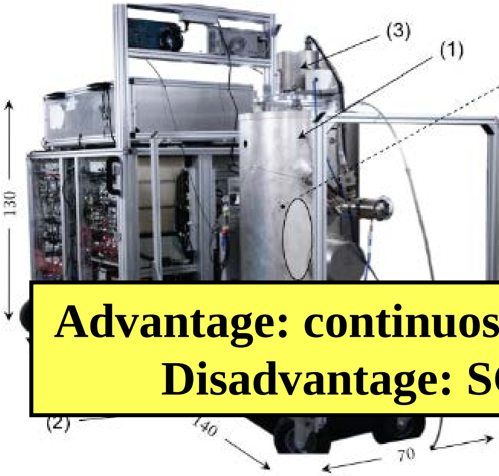
Scientific REPORTS | *(2018) 8:12300* | DOI:10.1038/s41598-018-30608-1

Nice stabilities are showed but uncertainties have to be verified at comparisons

Conclusion: Accuracy of g_F 1/3

The most accurate method: Combination of a continuous Superconducting Gravimeter (SG) and Absolute measurements (e.g. once per year)





Parameter	**Standard uncertainty / µGal**
Absolute *g: FG5X-HS5 (CAG-01) / FG5X*	1.9 / 3.1
Time variability	0.1
3D mapping and self-attraction	2.0
Combined	**2.8 / 3.7**

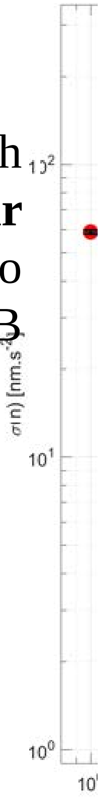
relative standard uncertainty **2.8E-9 / 3.7E-9**

Advantage: continuos g/g_F time series, possibility to invite more AGs, compare them etc.
Disadvantage: SG needs ice-cleaning once/year, cold-head repair each 3-years

Conclusion: Accuracy of g_F 2/3

Absolute method, based on absolute measuments. Monthly measurements are able to clearly detect the seasonal signal. Th **minutes also by an FG5X**

To determine„g_F“combination with **models of tides, polar motion, air pressure is efficient**– no need to operate AG exactly at the time of KB experiment



Parameter	**Standard uncertainty / µGal**
Absolute *g: FG5X-HS5 (CAG-01) / FG5X*	1.9 / 3.1
Tides	0.2
Polar motion	0.05
Atmosphere	1.0
Seasonal signal	≈1.0
3D mapping and self-attraction	2.0
Combined	**3.1 / 4.0**

Disadvantage: AG offset variations are not controlled, regular validation is needed

Conclusion: Accuracy of g_F 3/3

Minimalistic method, based on rarely absolute measuments and modelling tides, atmosphere and polar motion effects.

“Seasonal” variations are dangerous:

- the measured absolute *g* (e.g. in spring) might be deviated from the “middle”
- KB experiment can be done in the opposite season (in autumn)

Maximal systematic error (peak to peak variability of seasonal signal)

Parameter	**Standard uncertainty / µGal**
Absolute *g: FG5X-HS5 (CAG-01) / FG5X*	1.9 / 3.1
Tides	0.2
Polar motion	0.05
Atmosphere	1.0
Seasonal signal	??????????
3D mapping and self-attraction	2.0
Combined	**>3.1 / >4.0**

Thank you for your attention!

New gravity lab at the Pecný station