Sound field reconstruction and beamforming based on measurements of the acousto-optic effect

Guest lecture

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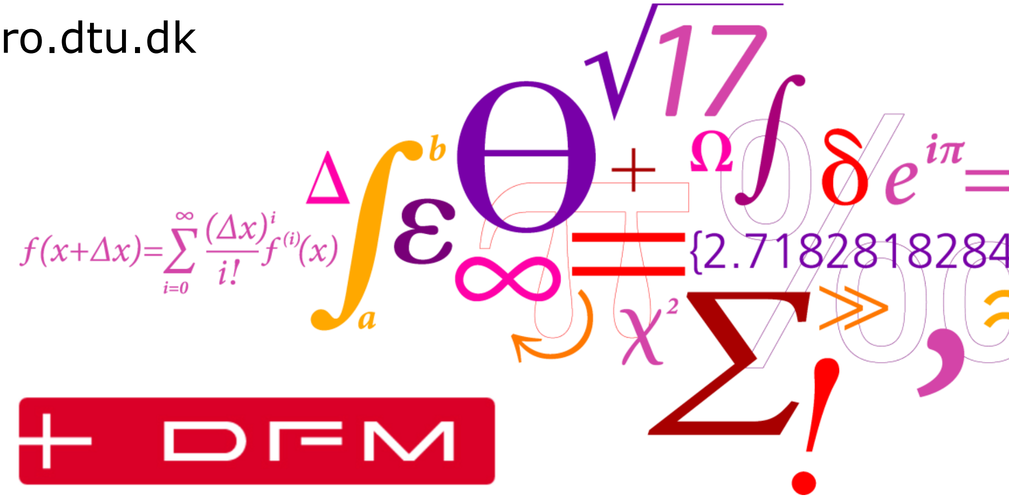
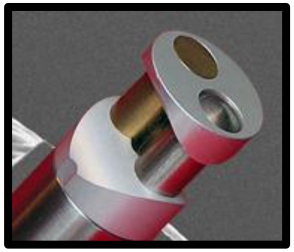


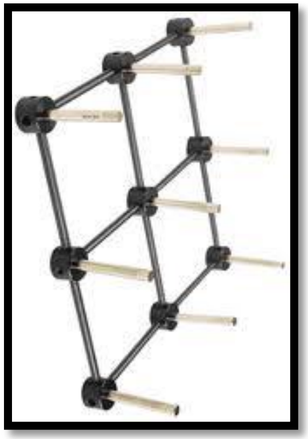
Acoustic Technology
Department of Electrical Engineering

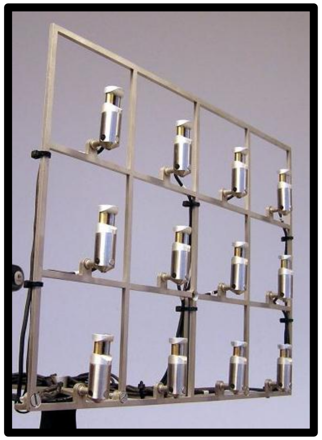
Outline

- ✓ Introduction
- ✓ The propagation of light
- ✓ The acousto-optic effect
- ✓ Application 1: Visualization of sound fields
  - Tomography
  - Acousto-optic tomography
- ✓ Application 2: Beamforming
  - Conventional beamforming
  - Spatial aliasing
  - Acousto-optic beamformer

Introduction







- The measurement of sound with conventional techniques requires the immersion of the transducers into the sound field.

The propagation of light

Electromagnetic wave equation:

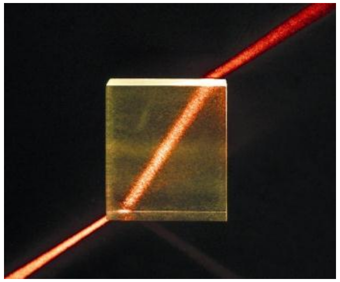
$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$\mathbf{E}$: Electric field

n : refractive index

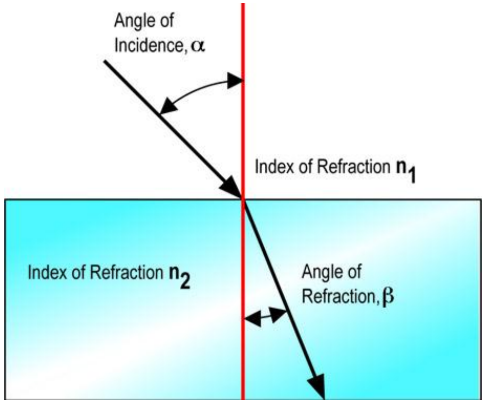
c_0 : speed of light in vacuum

$$c = \frac{c_0}{n}$$



OBS Light travels slower in dense media

	Vacuum	Air	Water
n	1	~1.0003	~1.3
ρ [kg/m³]	0	~1.2	~1000
$\frac{c_0 - c}{c_0}$ [%]	0	0.03	~23



- What happens if the properties of the medium are not constant?

The refractive index changes → The propagation of light is influenced

The propagation of light

- What does it influence the refractive index in air?

- Temperature
- Pressure
- Humidity
- …

The propagation of light

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  - Temperature
  - Pressure
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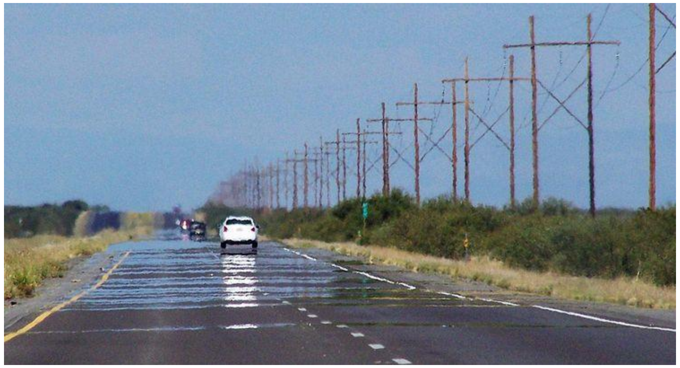
  - …

➢ Example: Road mirages



Large temperature gradients above the pavement bend the light rays coming from the sky.

The acousto-optic effect

Physical principle:

Propagation of sound

Pressure fluctuations (p)

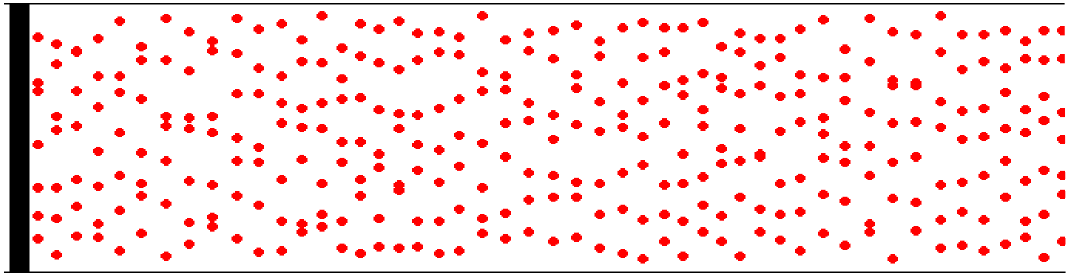
Density variations (ρ)

Refractive index (n) changes

$$n \cong n_0 + \boxed{\frac{n_0 - 1}{\gamma p_0}} p$$

Acousto-optic effect

Influence on the propagation of light



The acousto-optic effect

- In air and within the audible frequency range, the acousto-optic effect changes the phase of light (ϕ), but not its amplitude.

  - Can we still use the electromagnetic wave equation?

    $$\left|\frac{1}{n}\frac{\partial n}{\partial t}T\right| \ll 1$$

    (T: is the oscillation period of light)

  - Let us assume the following solution to the electromagnetic wave equation:

    $$\mathbf{E} = \mathbf{E}_0 e^{j(\omega t + \phi(x,y,t))}$$

(E.g. when light propagates along the x-direction)

$$j\frac{\partial^2\phi}{\partial x^2} - \left(\frac{\partial\phi}{\partial x}\right)^2 - \left(\frac{n}{c_0}\right)^2\left(j\frac{\partial^2\phi}{\partial t^2} - \left(\omega_o + \frac{\partial\phi}{\partial t}\right)^2\right) = 0 \quad \longleftarrow \quad \nabla^2\mathbf{E} - \left(\frac{n}{c_0}\right)^2\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$

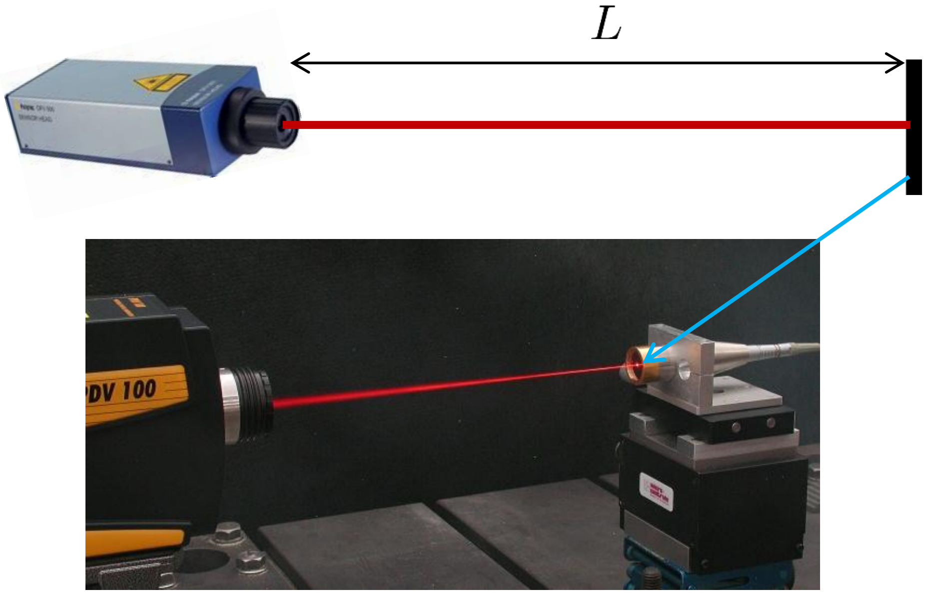
$$\phi(x,y,t) = k_0 n_0 L_0 + k_0\frac{n_0 - 1}{\gamma p_0}\int_L p(x,y,t)\mathrm{dl}$$

How can we measure the acousto-optic effect?

- We can measure the phase of a beam of light using a laser Doppler vibrometer (LDV)

Mechanical vibrations

$$v_{LDV}(t) = \frac{\mathrm{d}L}{\mathrm{d}t}$$



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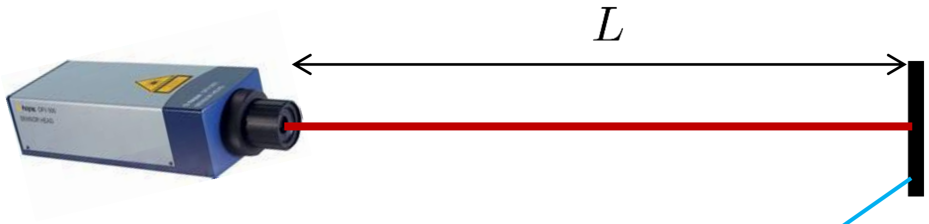
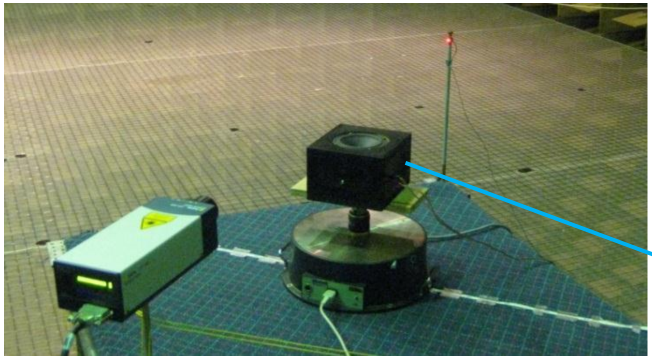
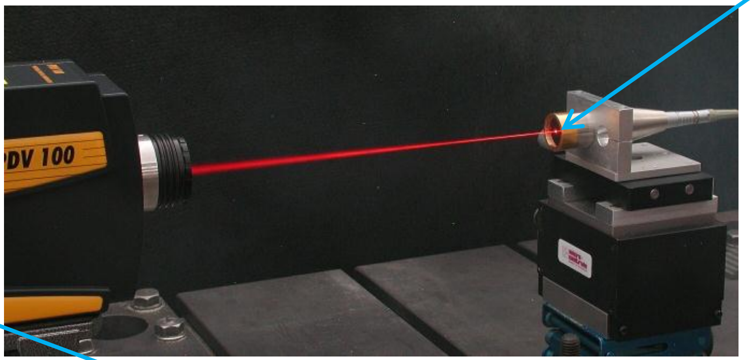
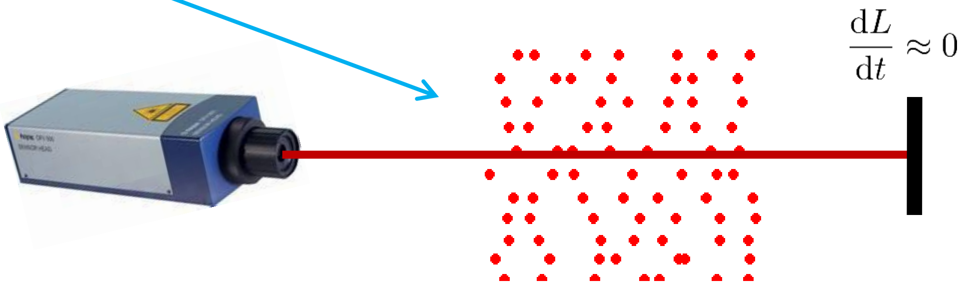






Acousto-optic effect

$$v_{LDV}(t) = \frac{n_0 - 1}{\gamma p_0 n_0} \frac{\mathrm{d}}{\mathrm{d}t} \left(\int_L p(x,y,t)\mathrm{d}l \right)$$



Application 1: Sound field visualization

- Can we use the apparent velocity measured with the LDV to visualize an acoustic field?

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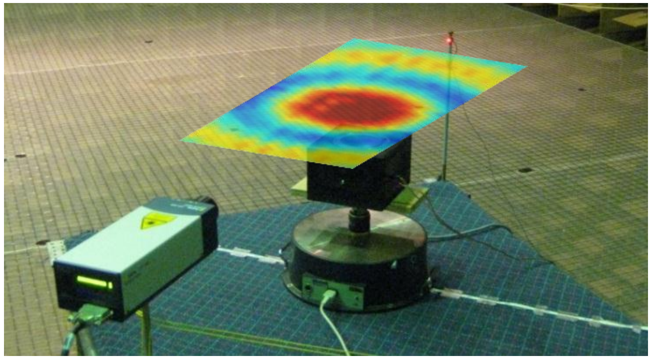
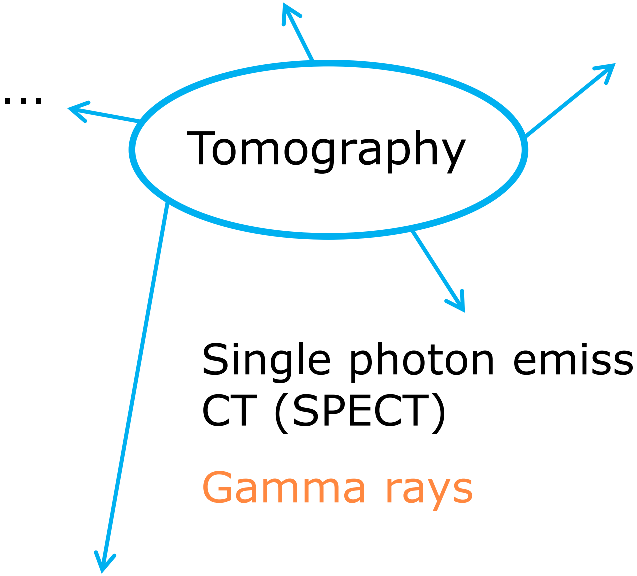


The sound pressure can be reconstructed using tomographic techniques

Tomography



Tomography

Ocean Acoustic Tomography

Sound waves

Computerized Tomography (CT)

X-rays

Single photon emission CT (SPECT)

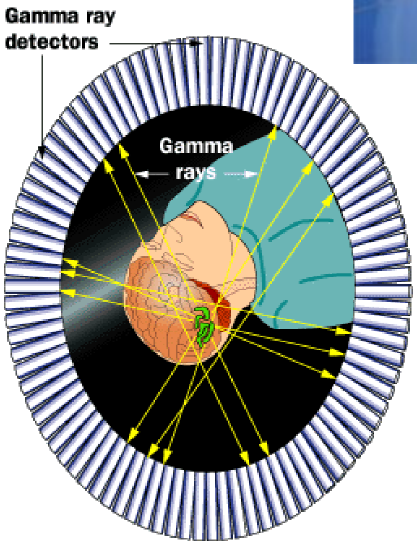
Gamma rays

Magnetic Resonance Imaging (MRI)

Radio-frequency waves

...

- “Non-invasive” techniques



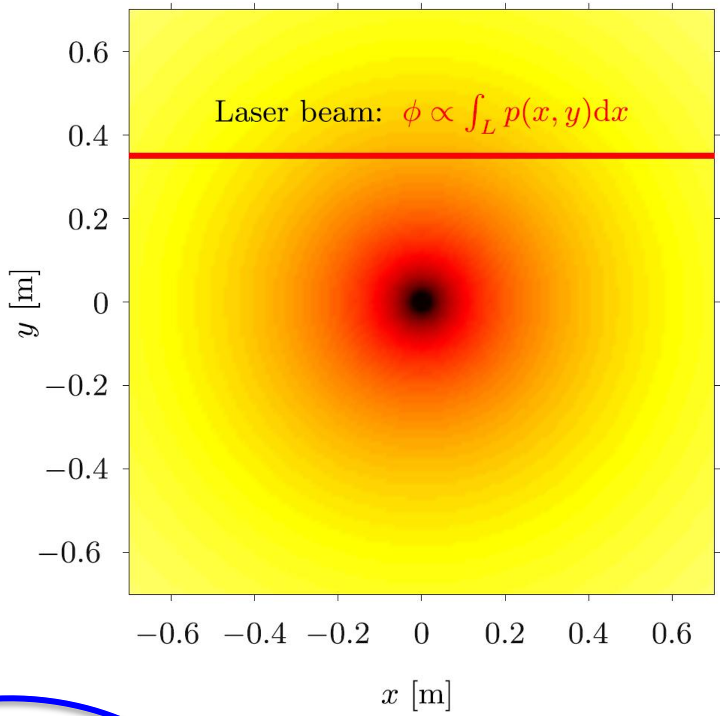
Acousto-optic tomography

The phase of the light integrates the acoustic field:

$$\phi = k_0 n_0 L + k_0 \frac{n_0 - 1}{\gamma p_0} \boxed{\int_0^L p(x,y)\mathrm{d}x}$$

Projection of the sound field

Radon Transform: $R_p(x', \theta) = \int_0^L p(x,y)\mathrm{d}x$



With a laser Doppler vibrometer:

$$v_{LDV}(t) = \frac{n_0 - 1}{\gamma p_0 n_0} \frac{\mathrm{d}}{\mathrm{d}t} \left(R_p(x', \theta)\right)$$

Acousto-optic tomography

- The acoustic field cannot directly be measured.
- The Radon transform of the acoustic field can be obtained with an LDV,

$$R_p(x',\theta) = \frac{\gamma p_0 n_0}{n_0 - 1} \int v_{LDV}(t) \mathrm{d}t$$

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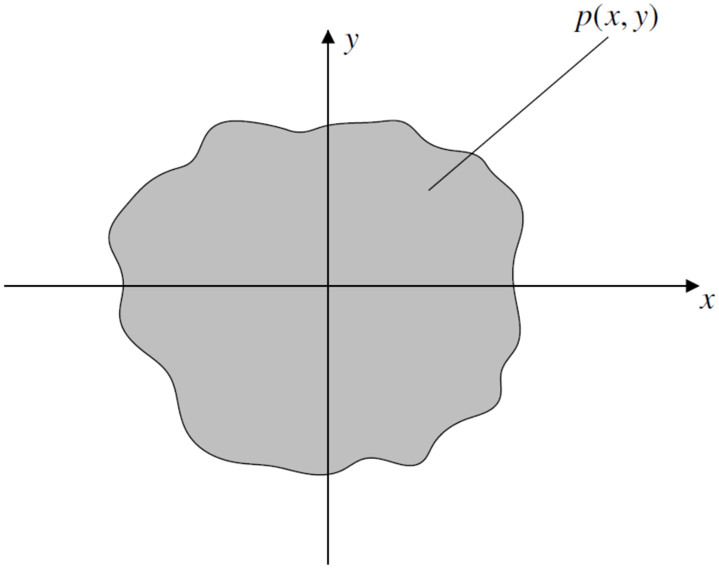

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- A single line scan is not enough.
- Complete reconstruction of an arbitrary sound field requires:
    - Parallel lines scanning over a plane.

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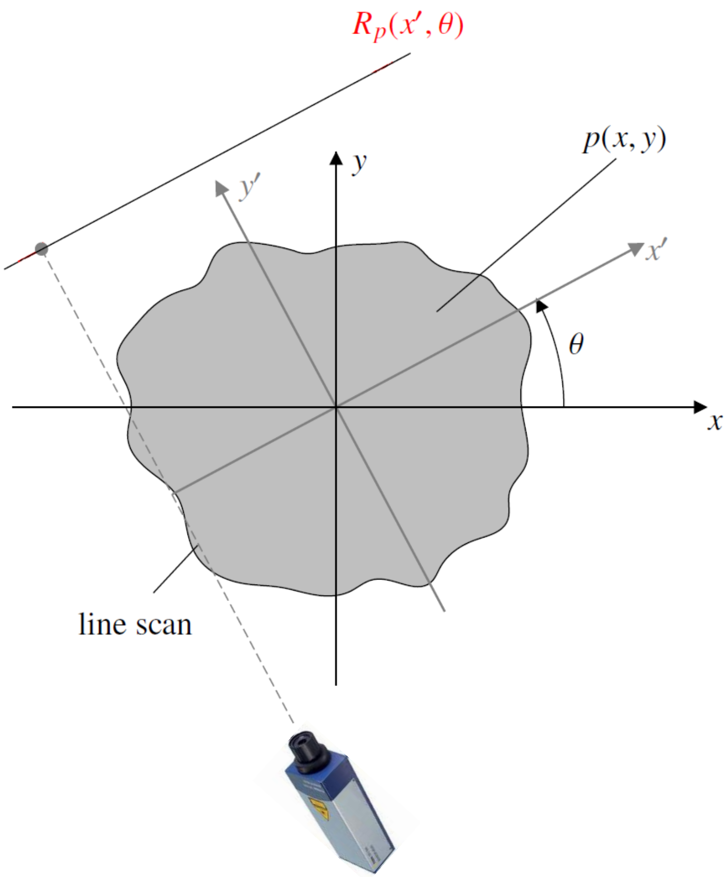

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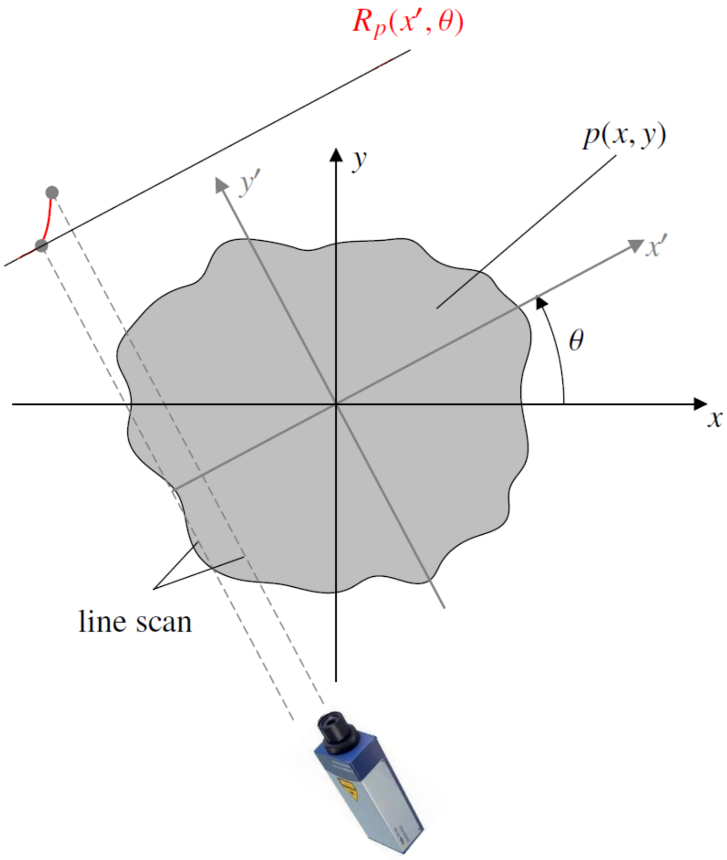

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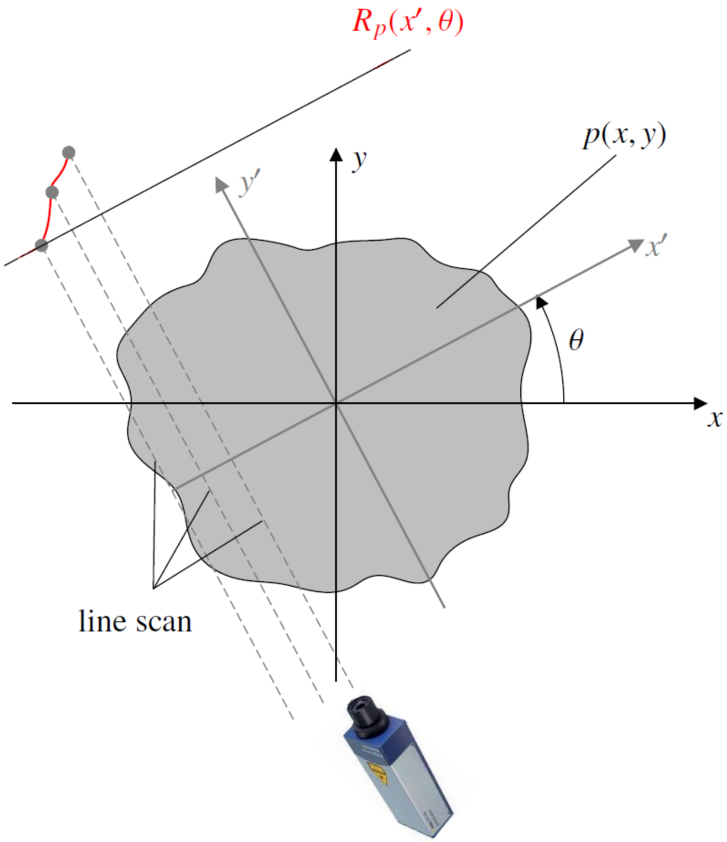

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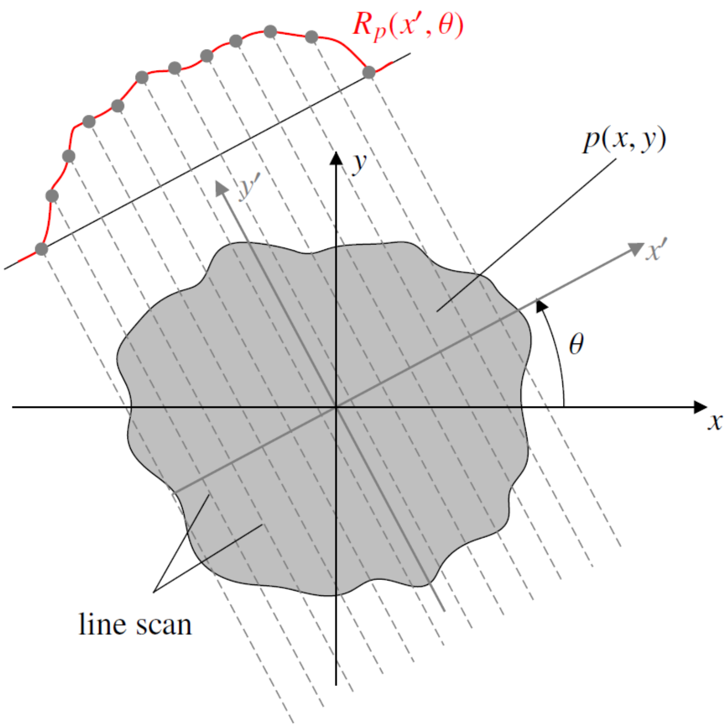

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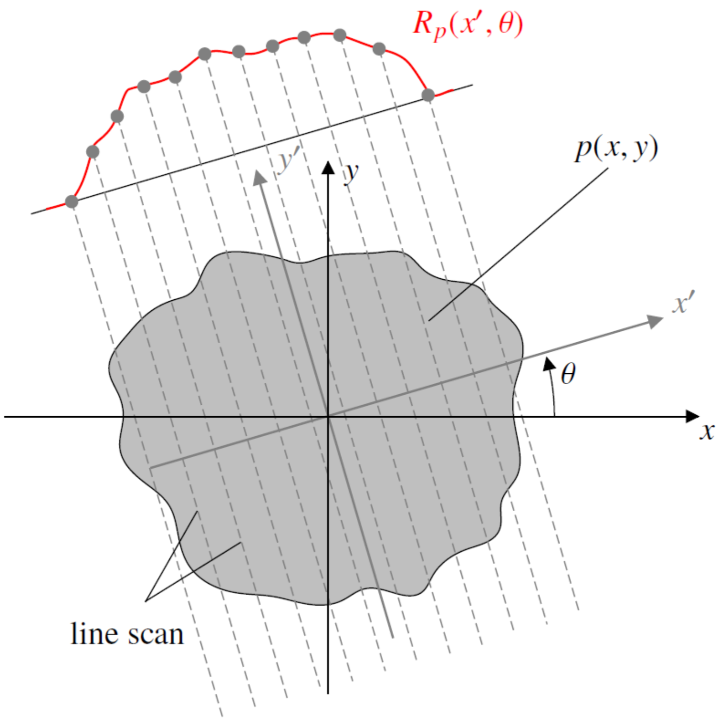

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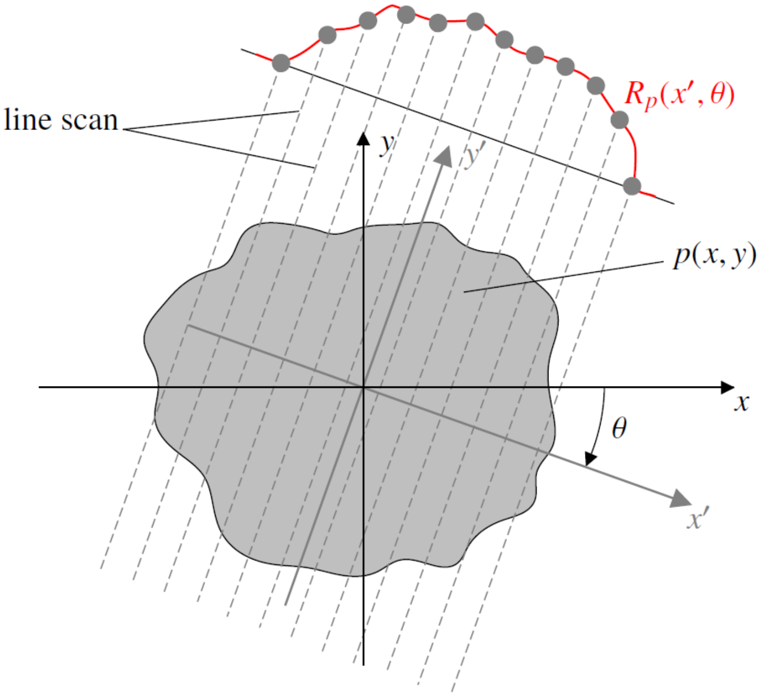
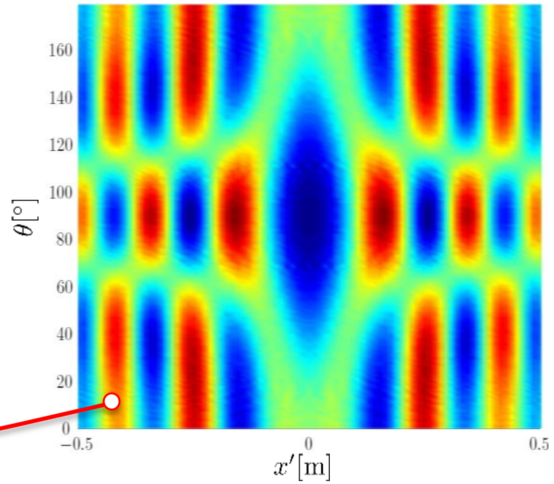


Tomographic reconstruction

- Projecting the acoustic field into different directions yields a very well defined inverse problem,

$$R_p(x',\theta) = \begin{bmatrix} R_p(x'_1,\theta_1) & R_p(x'_1,\theta_2) & \dots & R_p(x'_1,\theta_n) \\ R_p(x'_2,\theta_1) & \boxed{R_p(x'_2,\theta_2)} & \dots & R_p(x'_2,\theta_n) \\ \vdots & \vdots & \ddots & \vdots \\ R_p(x'_m,\theta_1) & R_p(x'_m,\theta_2) & \dots & R_p(x'_m,\theta_n) \end{bmatrix}$$



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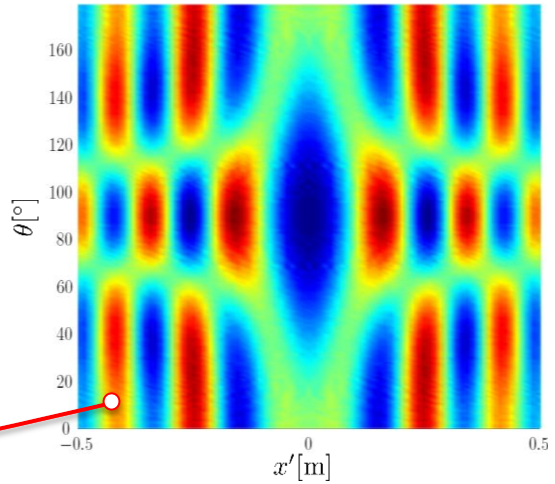
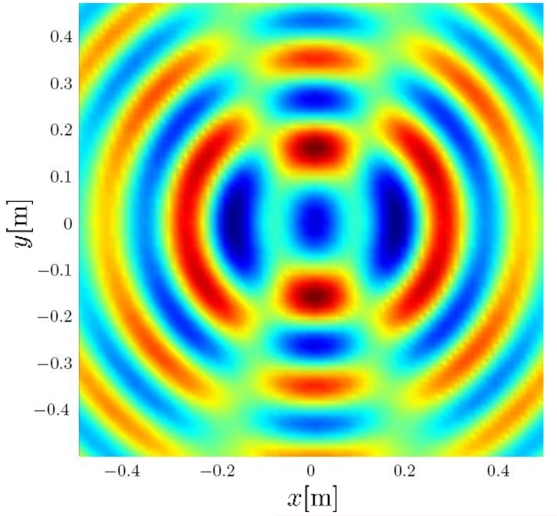
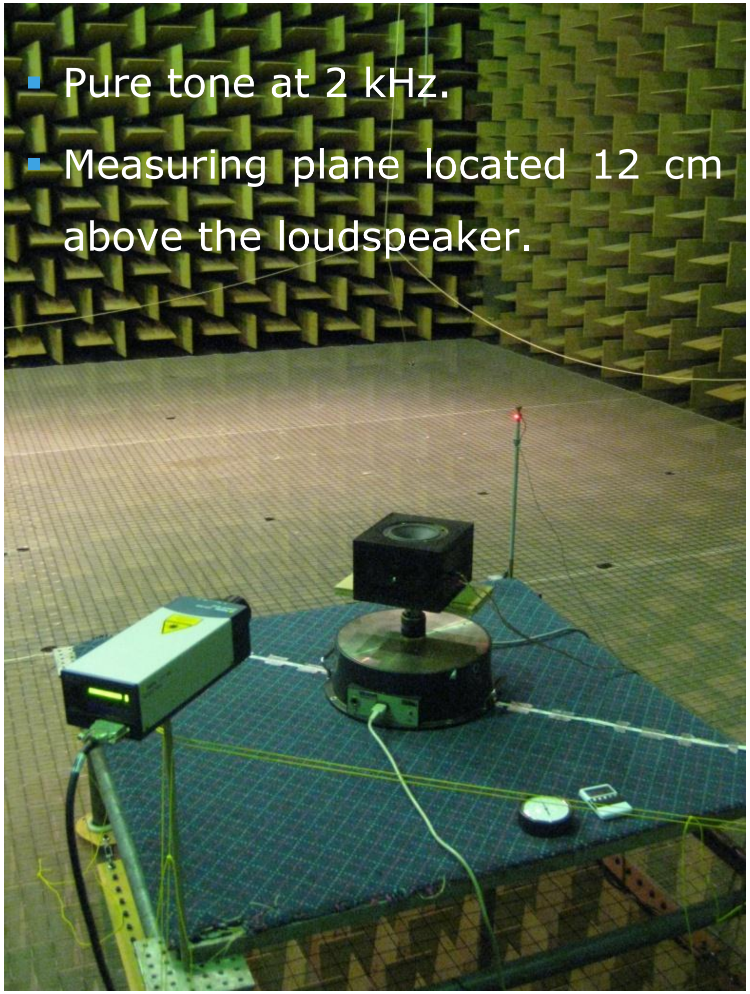


- The acoustic field is reconstructed by computing the **inverse Radon transform.**

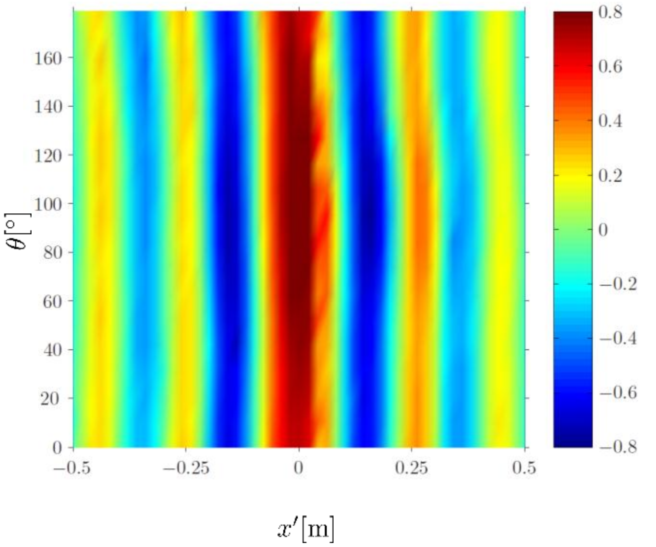


Sound field visualization



- Pure tone at 2 kHz.
- Measuring plane located 12 cm above the loudspeaker.

Measured
Radon transform



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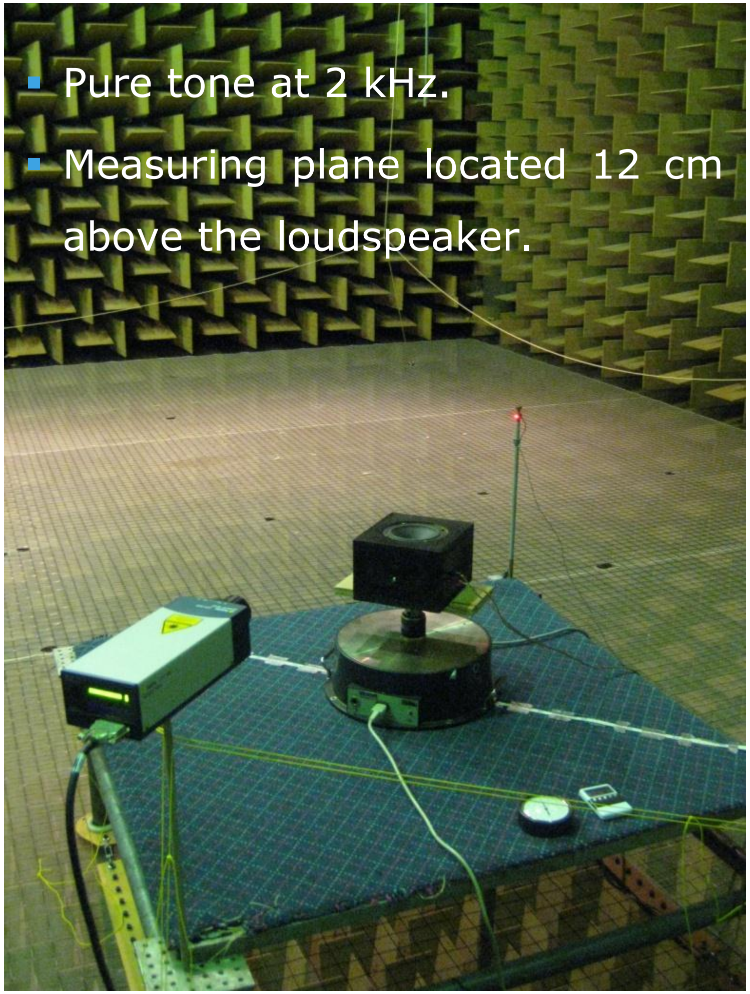

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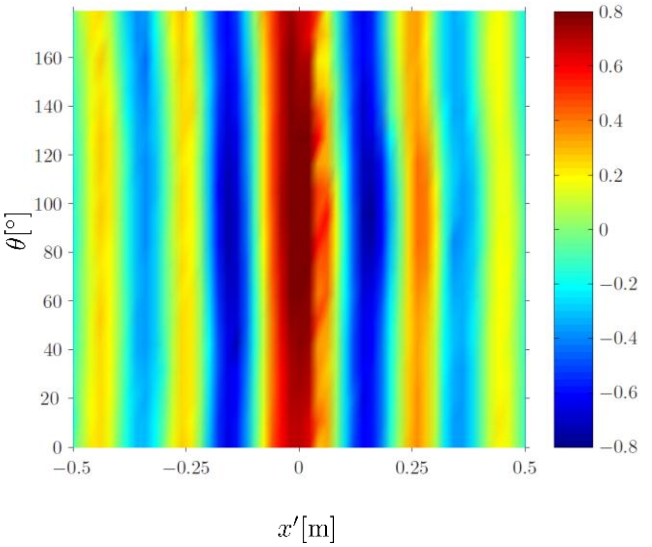
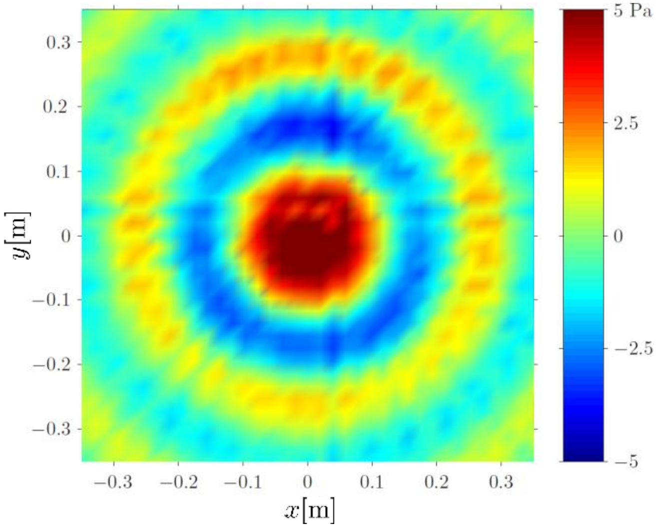
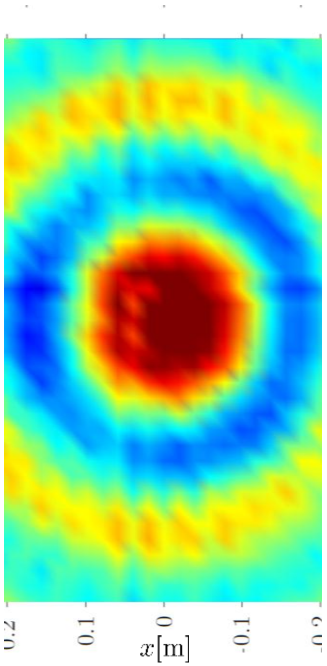




Reconstructed sound field

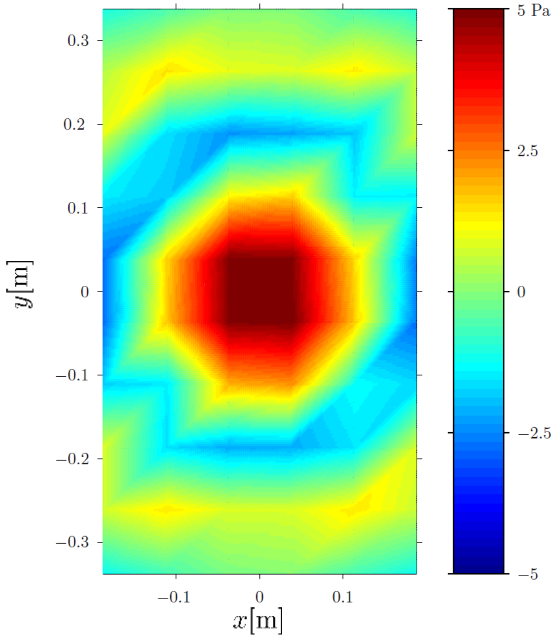
Tomography vs Microphone array

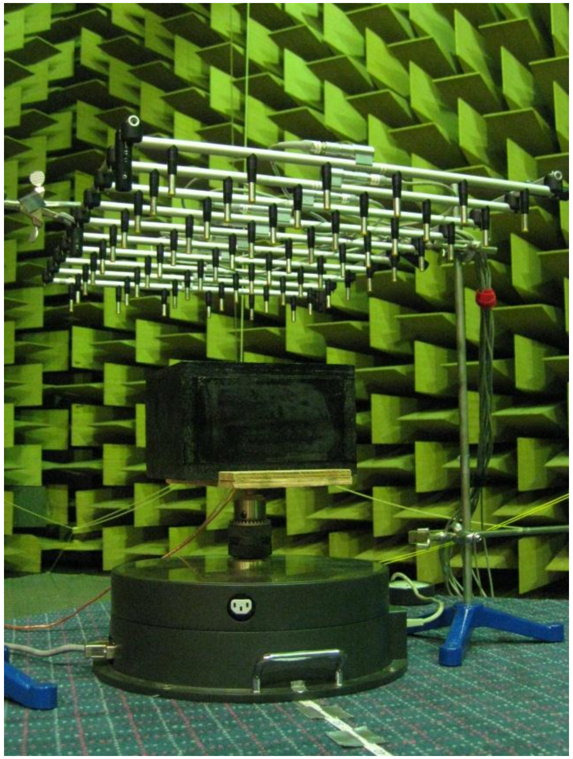
Tomographic reconstruction



- Spatial res.: 2 cm
- Angular res.: 10º

Microphone array

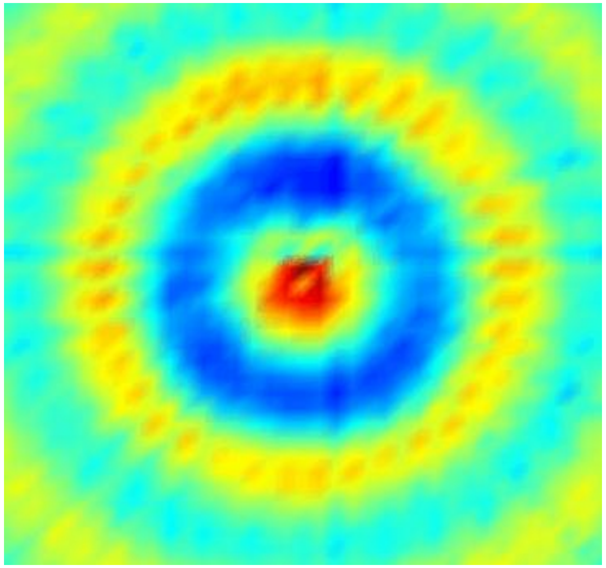




- Planar array of 60 microphones
- Spatial resolution: 7.5 cm

Tomography vs Microphone array

Tomographic reconstruction



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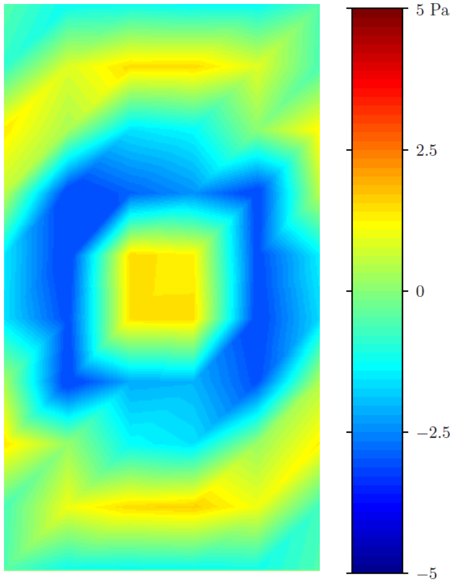
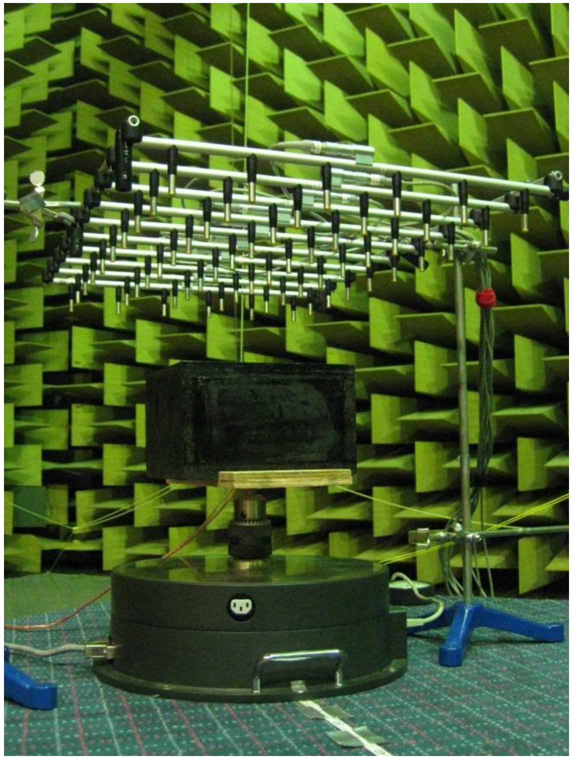
Microphone array





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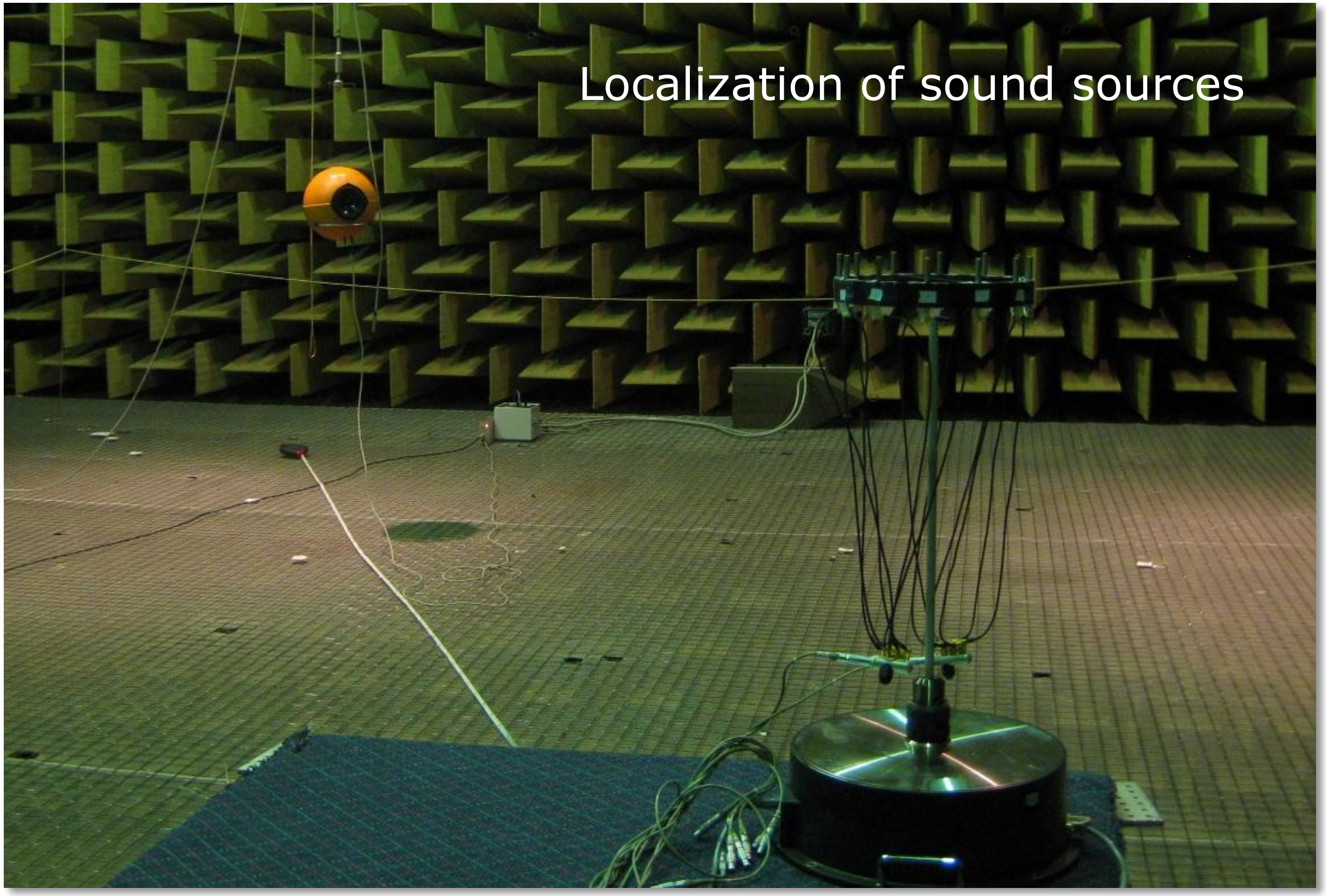
Application 2: Beamforming



Localization of sound sources

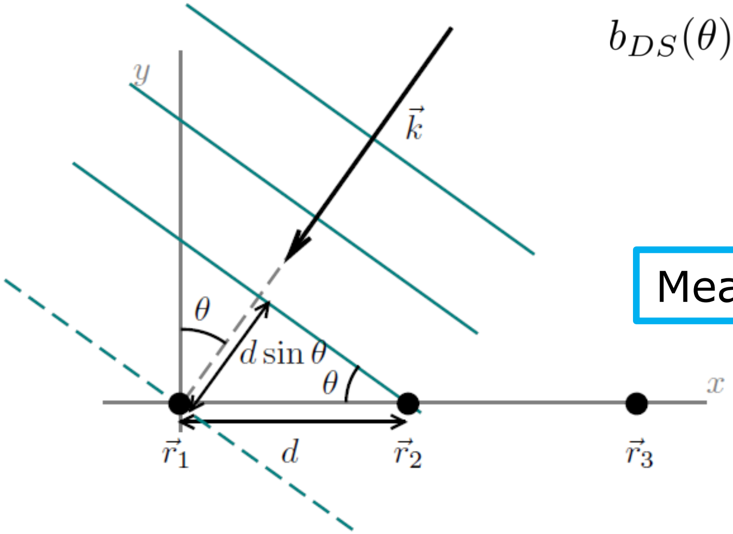
Conventional beamforming

Beamforming techniques	Region	Key feature
	Farfield	Phase
	Nearfield	Phase and amplitude

Conventional beamforming

Beamforming techniques	Region	Key feature
	Farfield	Phase
	Nearfield	Phase and amplitude

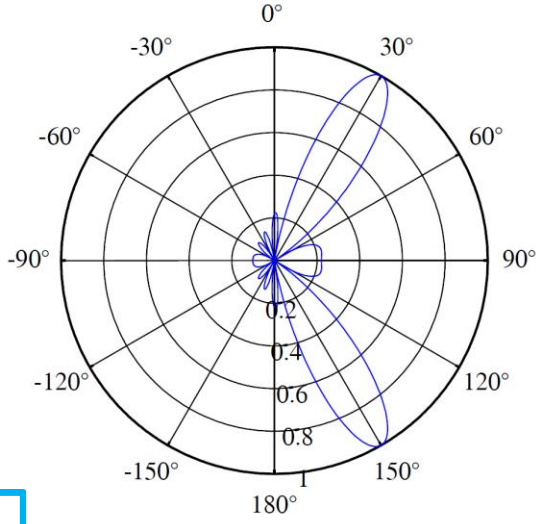
- Example: Delay and Sum Beamforming (DSB)



$$b_{DS}(\theta) = \left| \sum_{m=0}^{M-1} w_m \tilde{p}_m \boxed{e^{-jkmd\sin\theta}} \right|^2$$

Measured pressure

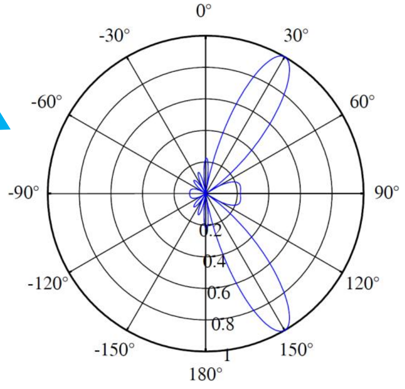
Phase shift

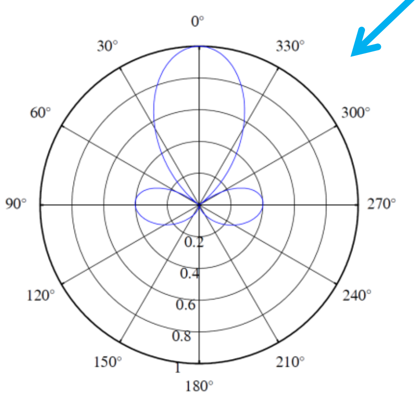


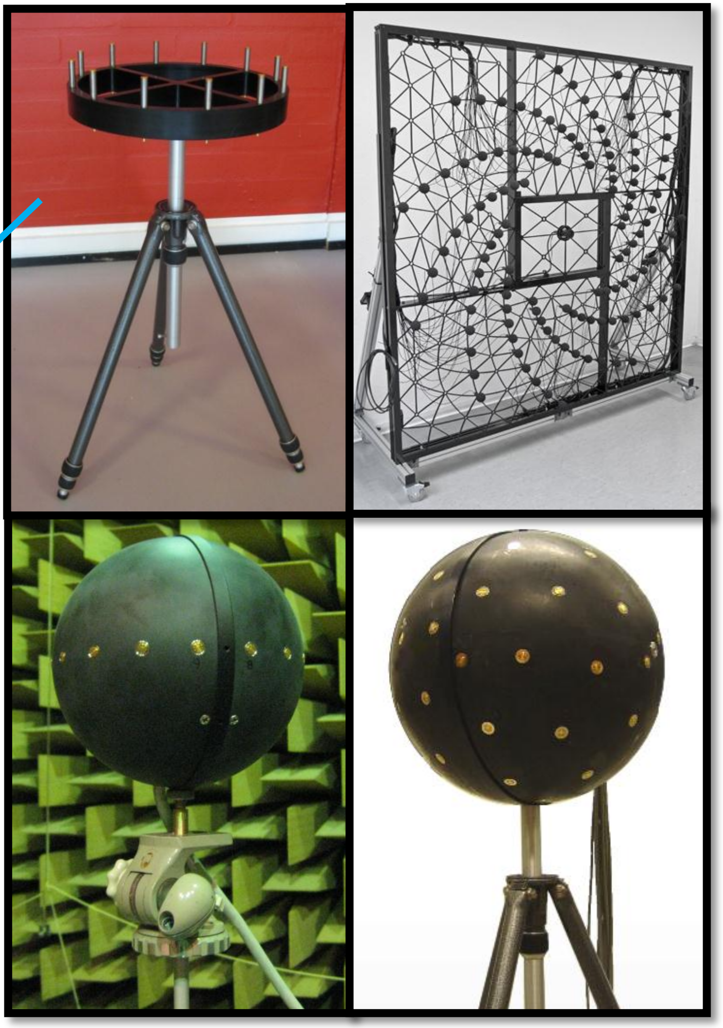
More advanced beamforming techniques

- To improve the beamforming output:
  - ✓ Design an optimum array layout/geometry
  - ✓ Optimize weighting coefficients (w_m)
  - ✓ Adaptive beamforming

Line array







- What do these techniques improve?
  - ✓ Resolution
  - ✓ Maximum sidelobe level (MSL)
  - ✓ Frequency range of analysis, etc.

Spatial aliasing

- Beamforming techniques are based on a finite number of input signals → This inevitably causes spatial aliasing!

➢ E.g. DSB when $\theta = 0$

$$b_{DS}(0) = \left| \frac{1}{M} \sum_{m=0}^{M-1} \tilde{p}_m e^{-jkmd\sin(0)} \right|^2$$

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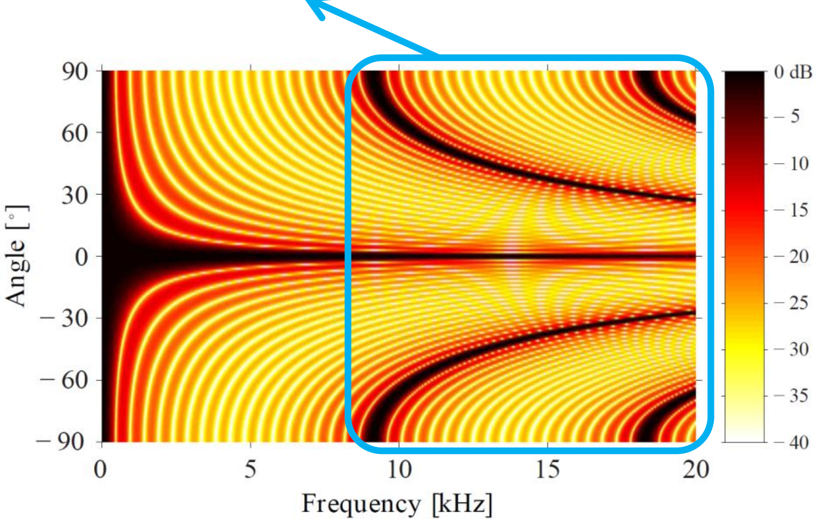

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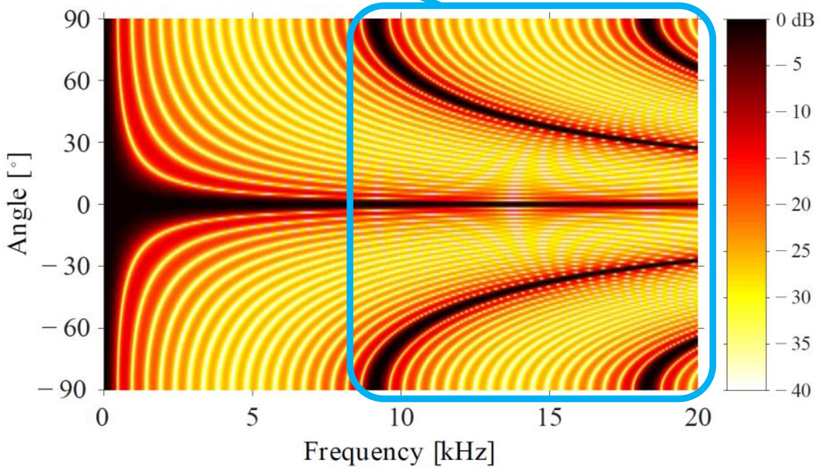

- Theoretical solution: Use an infinite number of transducers ($d \to 0$)

$$\lim_{M\to\infty} \sum_{m=0}^{M-1} \tilde{p}_m d = \int_0^{L_0} P(x,y,\omega)\mathrm{d}l \quad \overset{?}{\longleftrightarrow} \quad v_{LDV}(t) = \frac{n_0 - 1}{\gamma p_0 n_0} \frac{\mathrm{d}}{\mathrm{d}t} \left(\int_L p(x,y,t)\mathrm{d}l \right)$$

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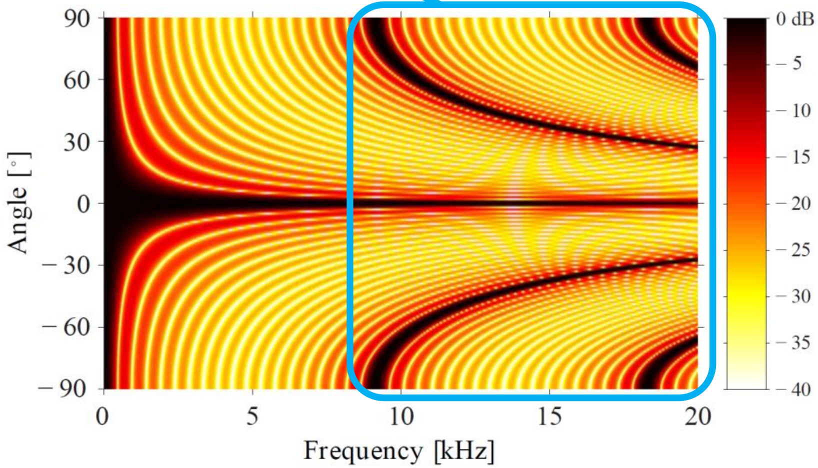

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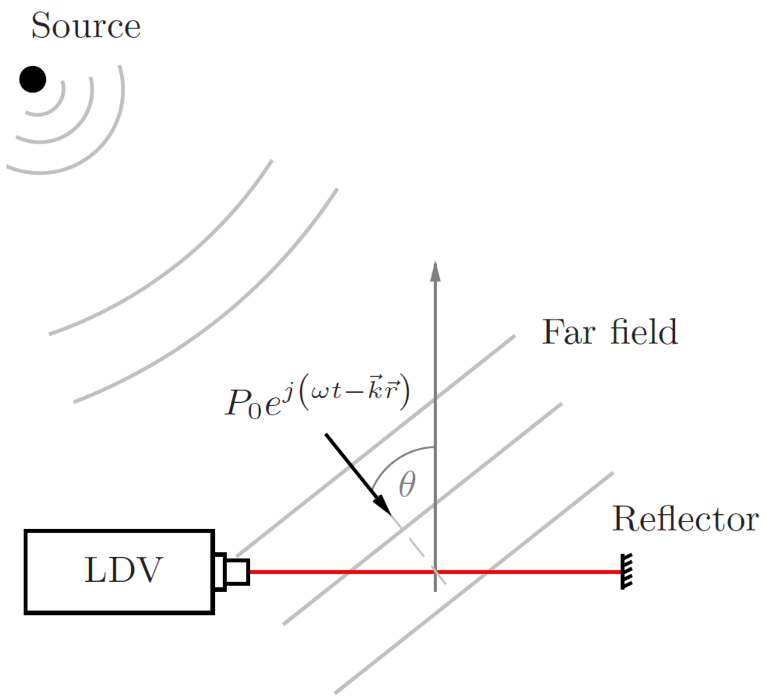
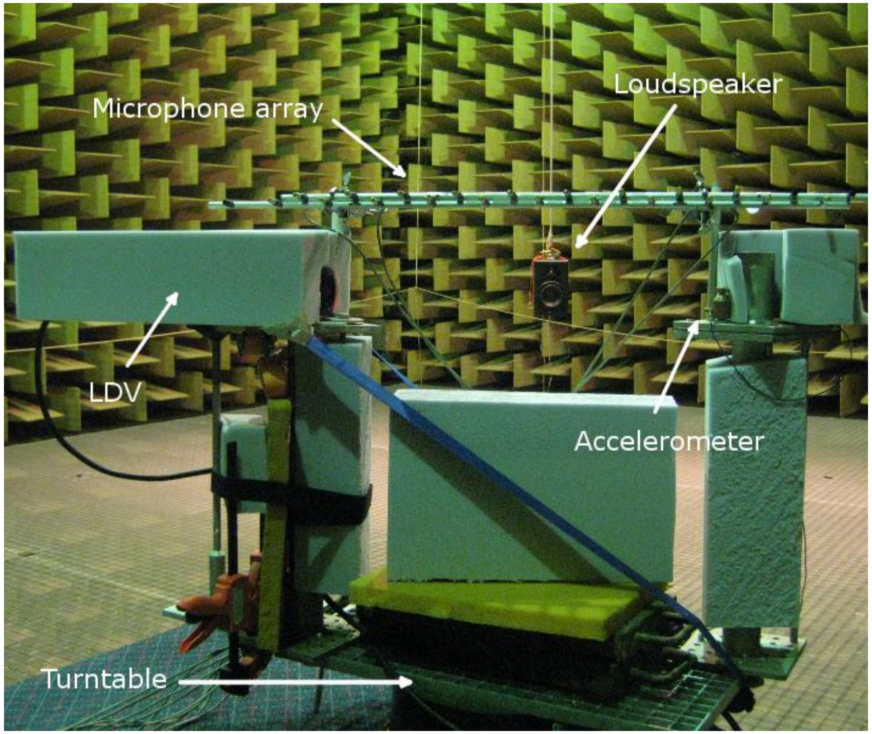
Acousto-optic beamformer



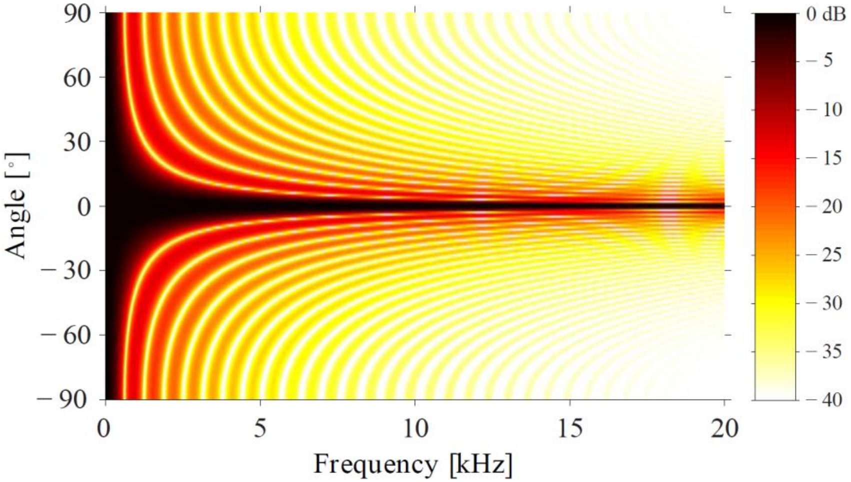


$$V_{LDV}(\omega) = j\omega \frac{n_0 - 1}{\gamma p_0 n_0} \left(\int_{\mathbf{L}} P(x,y,\omega)\mathrm{d}l \right) \rightarrow b_{AO} = \left| \frac{1}{L_0} \int_0^L P(x,y,\omega)\mathrm{d}l \right|^2$$

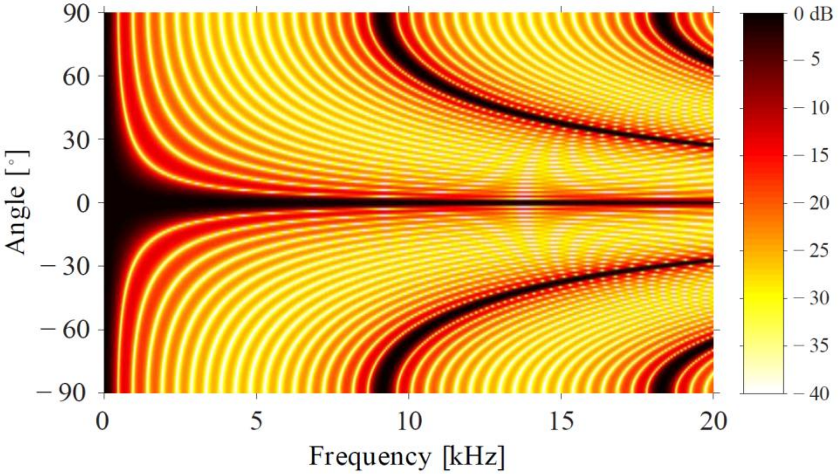
$$= \left| \frac{1}{L_0} \frac{\gamma p_0 n_0}{n_0 - 1} \frac{V_{LDV}(\omega)}{j\omega} \right|^2$$

Acousto-optic beamformer

- Acousto-optic beamformer



- DSB – Line array



Acousto-optic beamformer

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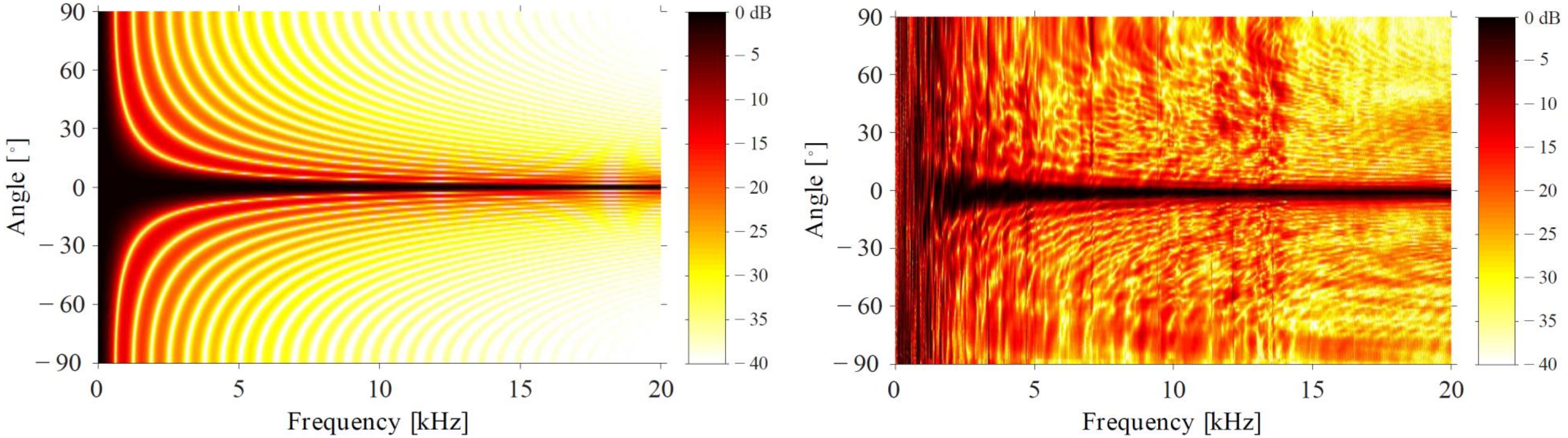

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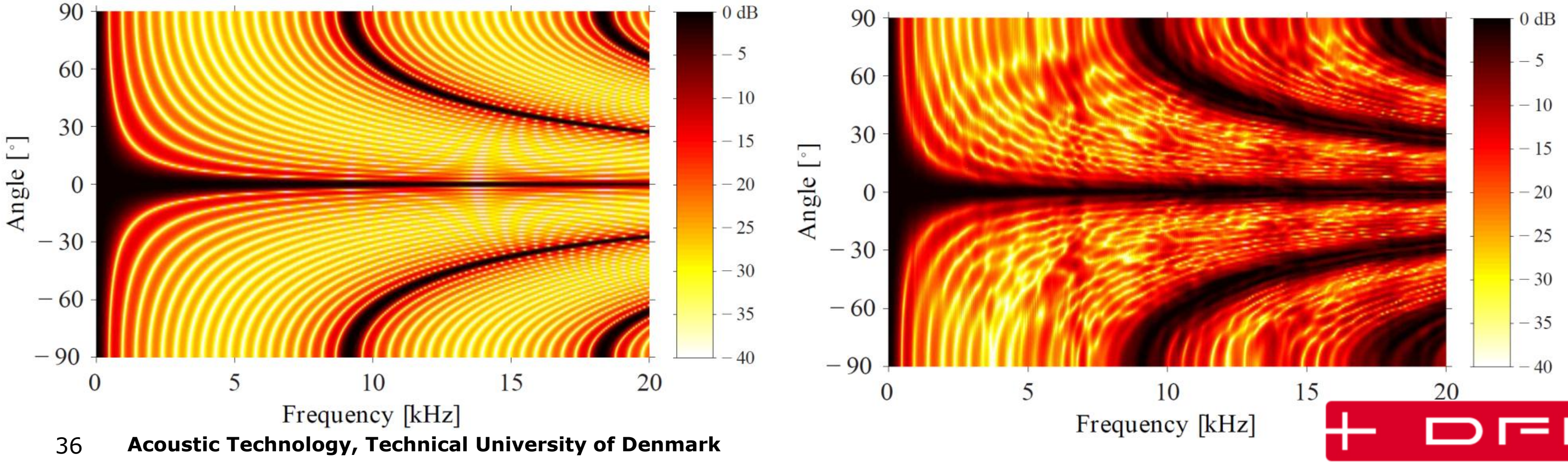

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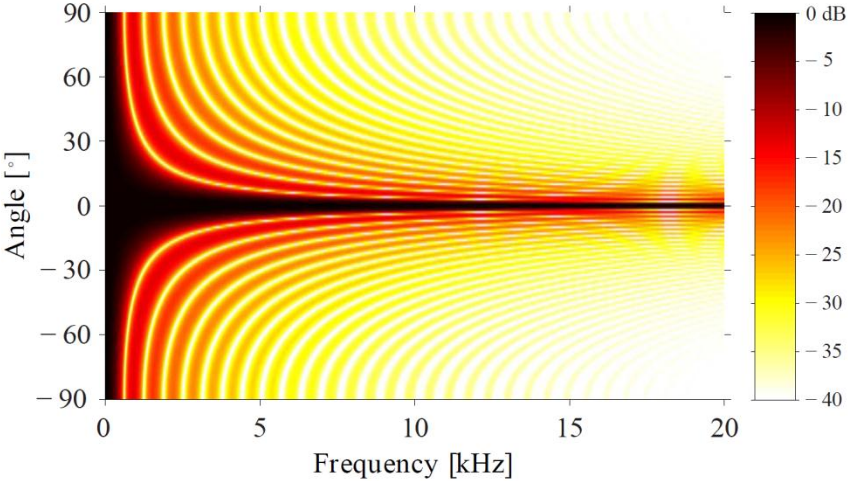

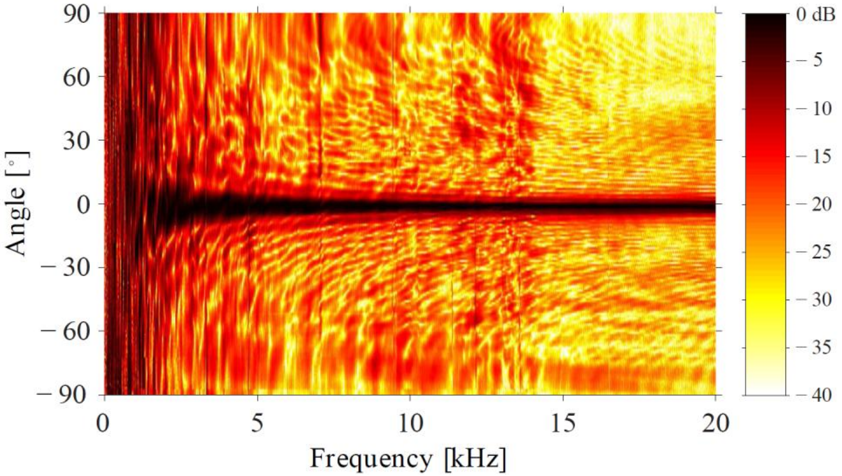

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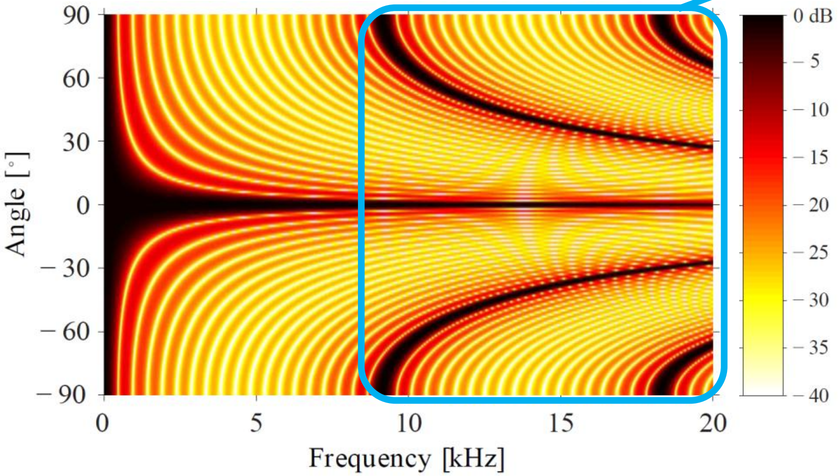
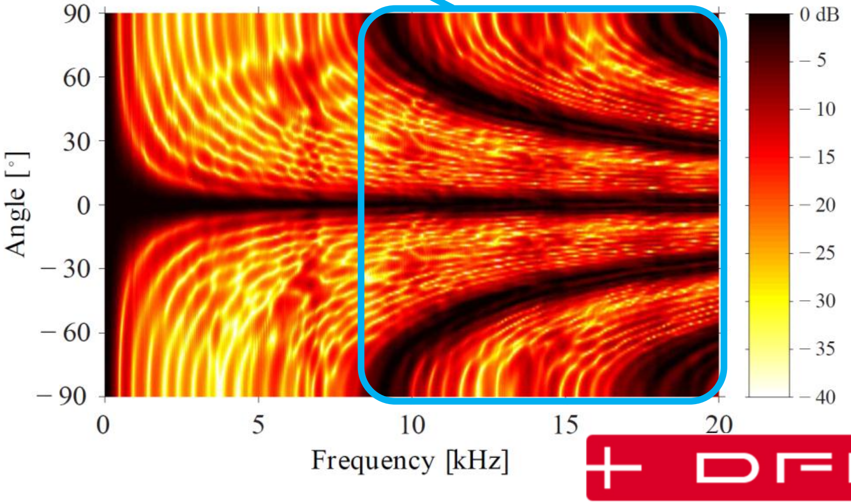
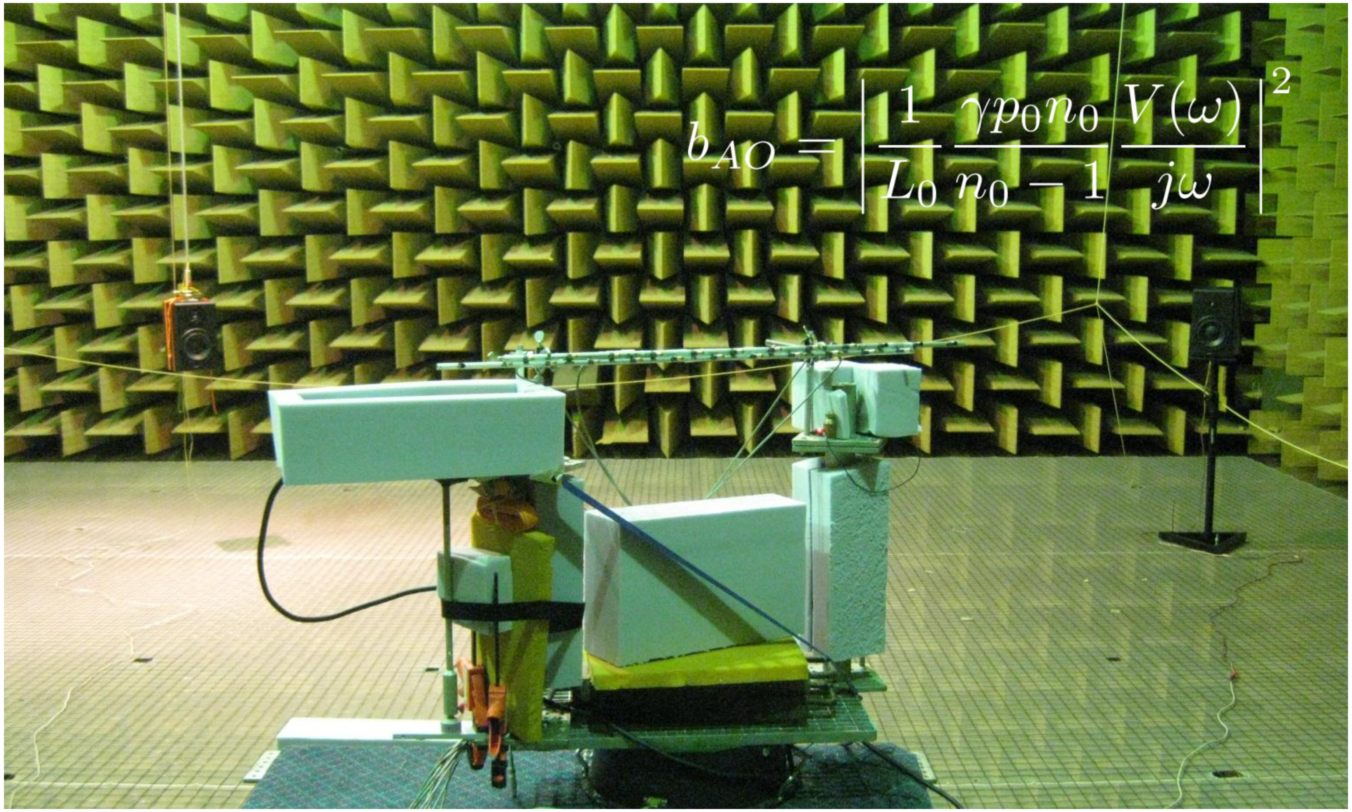


Spatial aliasing!



Acousto-optic beamformer



$$b_{AO} = \left| \frac{1}{L_0} \frac{\gamma p_0 n_0}{n_0 - 1} \frac{V(\omega)}{j\omega} \right|^2$$

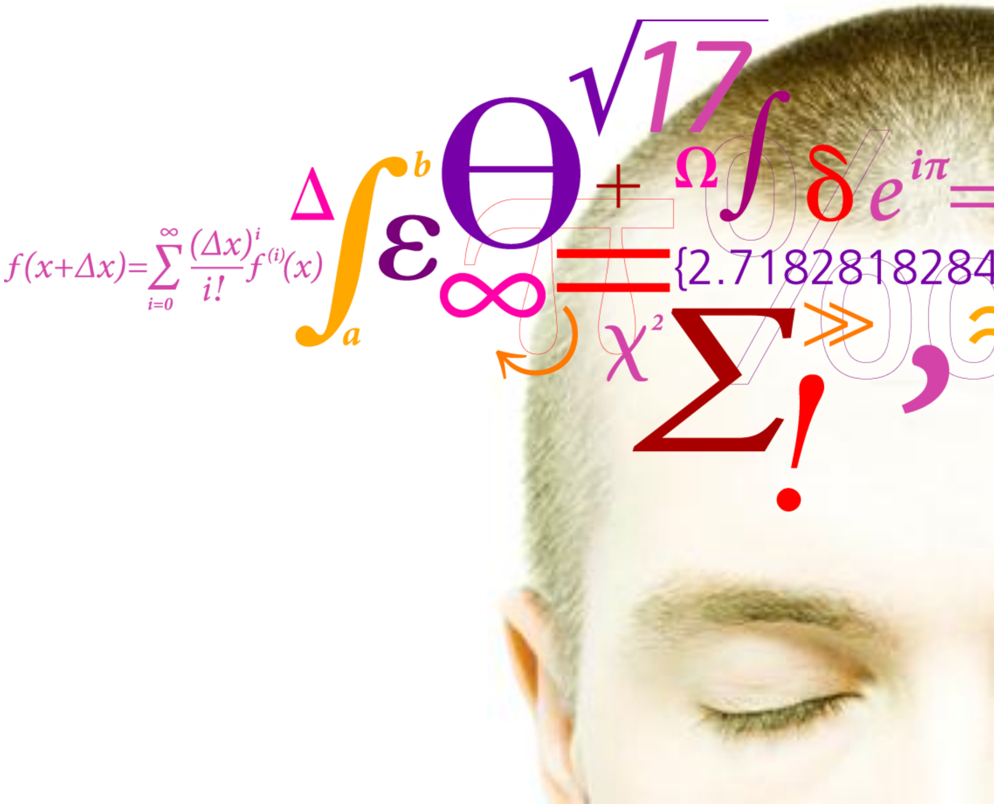
LDV $\theta = 90^\circ$ 🔉

Microphone 🔉

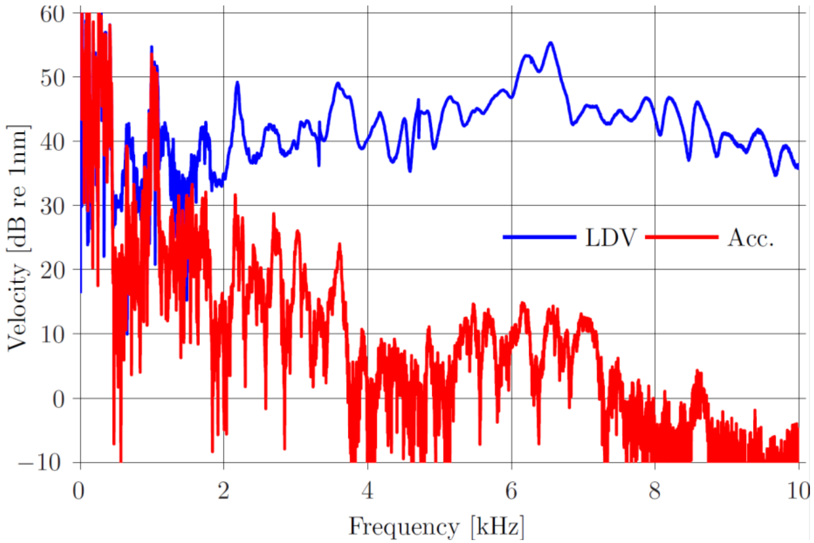
LDV $\theta = 30^\circ$ 🔉

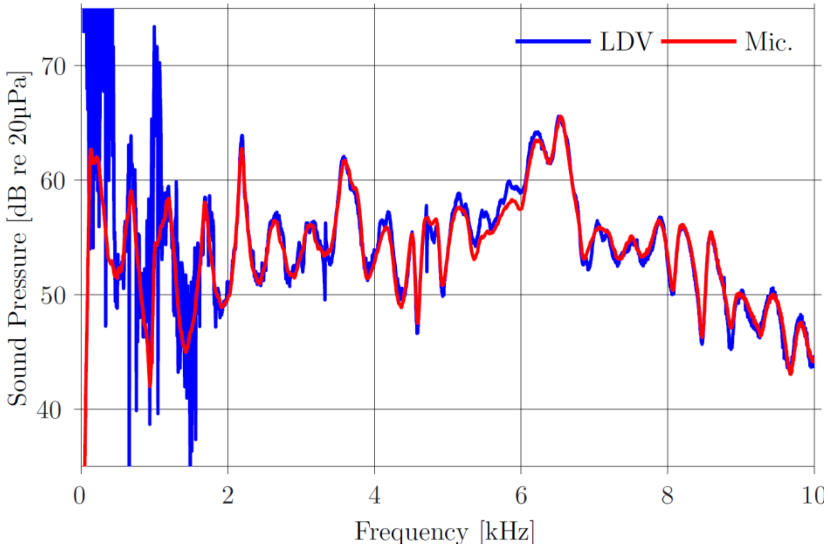
(High-pass filtered signals)

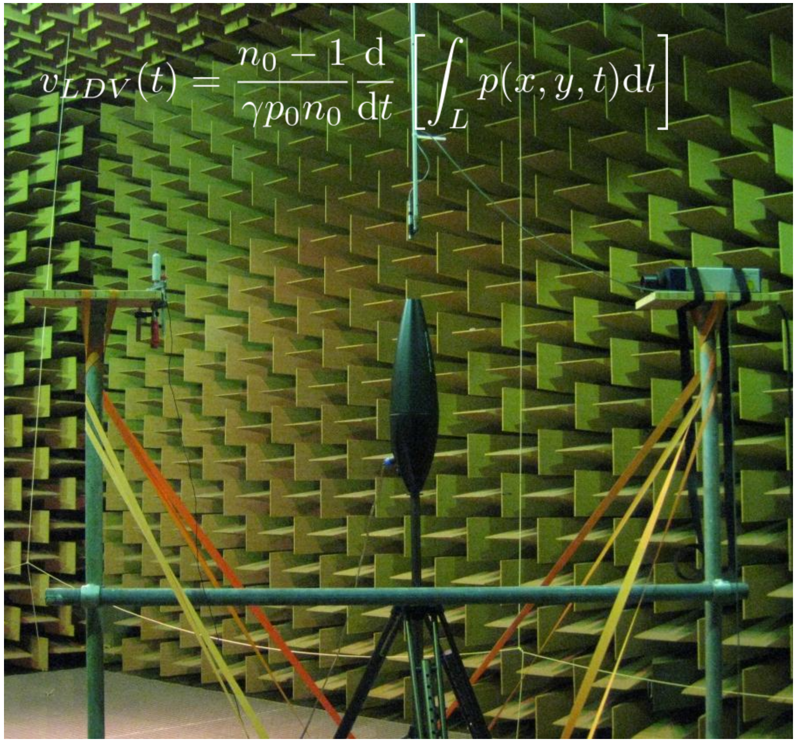
Thanks for your attention!



Measuring the acousto-optic effect







$$v_{LDV}(t) = \frac{n_0 - 1}{\gamma p_0 n_0}\frac{\mathrm{d}}{\mathrm{d}t}\left[\int_L p(x,y,t)\mathrm{d}l\right]$$

Microphone LDV

(High-pass filtered signals)