The prediction uncertainty

Participating time laboratories provide the evaluation for the prediction uncertainty of [UTC-UTC(*k*)] within the framework of Calibration and Measurement Capabilities (CMCs) and CCTF key comparisons for the key comparison database (KCDB). The time interval for declaring the prediction uncertainty is 20 days and the declared values show a large variation; from 20 ns to 200 ns. A study was initiated as a result of this large variation among the declared values to determine reasonable values for the prediction uncertainty of [UTC-UTC(*k*)] depending on the clock used and the time transfer method. The laboratories should exercise caution when evaluating the prediction uncertainty and to avoid misunderstandings due to the deviation between the values of [UTC-UTC(*k*)] over 20 days.

The prediction uncertainty depends on:

- The uncertainty on [UTC-UTC(*k*)] declared in Section 1 of *Circular T*
- The stability property of the atomic clock generating UTC(*k*).

The uncertainties reported in Section 1 of *Circular T* are linked to the link uncertainties reported in Section 6. There are different cases depending on the time transfer method used and the calibration status:

- TWSTFT - calibrated time transfer; global uncertainty between 1 ns to 5 ns
- GPSPPP, P3, MC, SC - calibrated time transfer; global uncertainty between 5 ns to 10 ns
- GPSPPP, P3, MC, SC - un-calibrated time transfer; global uncertainty of ~20 ns.

There are many papers dedicated to the study of the prediction [1-13] and the uncertainty linked to the prediction. The results reported in this note were obtained following the work presented in [3].

In this note we consider a time scale generated by using a free running caesium clock and H-masers with typical values for the Allan deviation to give consistent values to the uncertainty prediction.

We consider the caesium clock stability with a White Frequency Noise (WFN) from $\sigma_y(\tau) = 1\times10^{-14}/day$ to $\sigma_y(\tau) = 7\times10^{-14}/day$. In such a way we aim to consider two extreme cases. Depending on the stability of the clock used to generate the internal realization of UTC, the declared prediction uncertainty should be consistent with the values declared in this guideline.

In addition, for the hydrogen masers we consider two possible ranges of values for the WFN (from $\sigma_y(\tau) = 1\times10^{-15}/day$ to $\sigma_y(\tau) = 1\times10^{-14}/day$) and for the Random Walk Frequency Noise (RWFN) (from $\sigma_y(\tau) = 1\times10^{-16}/day$ to $\sigma_y(\tau) = 1\times10^{-15}/day$).

In figure 1 for example, the prediction uncertainty (2 sigma) at 20 days is reported in the case of a free running caesium clock with $\sigma_y(\tau) = 4\times10^{-14}/day$.

The results are related to:

- Calibrated TWSTFT, considered the best case with the uncertainty of about u = 1 ns (blue line)
- Calibrated GPS, the uncertainty of about u = 5 ns, 7 ns, and 10 ns (black, pink and green lines respectively)
- Un-calibrated GPS, considered the worst case with the uncertainty of about u = 20 ns (red line).

Table 1 shows the prediction uncertainties (2 sigma) at 20 days in the case of a time scale obtained by using a free running caesium clock with the stabilities equal to $\sigma_y(\tau) = 1 \times 10^{-14} / day$ and $\sigma_y(\tau) = 7 \times 10^{-14} / day$.

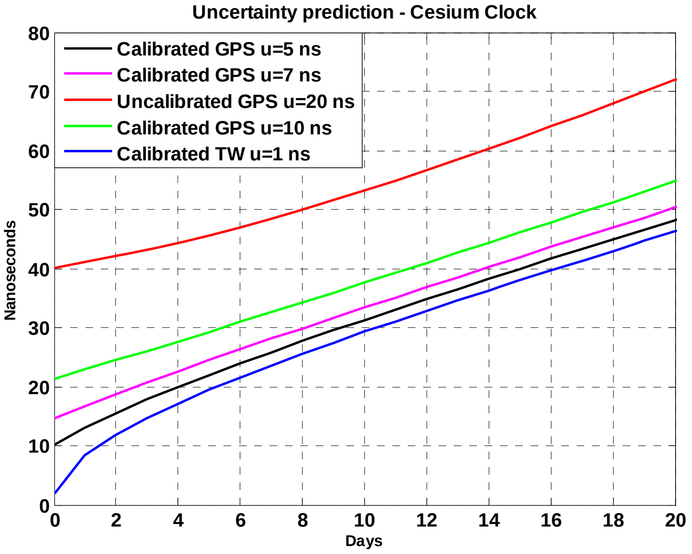

Figure1. Prediction uncertainty (2 sigma) in the case of a free running caesium clock with $\sigma_y(\tau) = 4 \times 10^{-14} / day$.

Measurement Uncertainty on UTC-UTC(*k*) / [ns]	Prediction uncertainty (2 sigma) at 20 days / [ns] $\sigma_y(\tau) = 1 \times 10^{-14} / day$	Prediction uncertainty (2 sigma) at 20 days / [ns] $\sigma_y(\tau) = 7 \times 10^{-14} / day$
1	12	81
5	19	83
7	24	84
10	32	87
20	57	98

Table 1. The prediction uncertainty (2 sigma) depending on measurement uncertainty at 20 days and at 45 days obtained by using a free running caesium clock with $\sigma_y(\tau) = 1 \times 10^{-14} / day$ and $\sigma_y(\tau) = 7 \times 10^{-14} / day$.

By analysing the values reported in this table we can observe the role of the clock generating the internal realization of UTC. Good clock stability has a predominant role in the uncertainty budget. When UTC(*k*) is realized by a clock with good stability the prediction uncertainty is dependent on the time transfer performance, as can be seen in the first column of table 1. The contribution to the uncertainty of the time transfer is almost negligible when the clock is characterized by a larger instability (second column in table 1).

Figure 2 shows the results for H-masers with WFN equal to $\sigma_y(\tau)=1\times10^{-15}/day$ and RWFN equal to $\sigma_y(\tau)=1\times10^{-16}/day$.

Table 2 gives the prediction uncertainties (2 sigma) at 20 days in the case of a time scale obtained by using a free running H-maser with the stabilities equal to WFN (from $\sigma_y(\tau)=1\times10^{-15}/day$ to $\sigma_y(\tau)=1\times10^{-14}/day$) and for the Random Walk Frequency Noise (RWFN) (from $\sigma_y(\tau)=1\times10^{-16}/day$ to $\sigma_y(\tau)=1\times10^{-15}/day$).

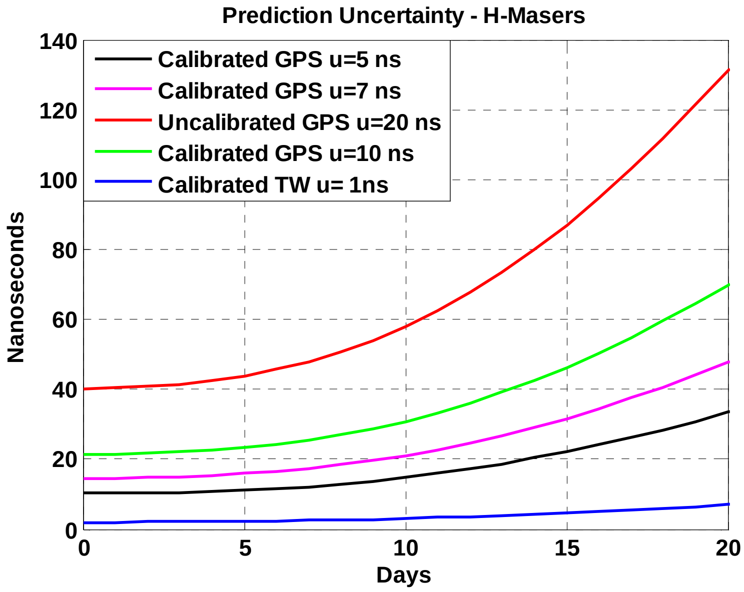

Figure2. Prediction uncertainty (2 sigma) of a free running H-maser with a WFN equal to $\sigma_y(\tau)=1\times10^{-15}/day$ and a Random Walk Frequency Noise (RWFN) equal to $\sigma_y(\tau)=1\times10^{-16}/day$.

Measurement Uncertainty on UTC-UTC(k) / [ns]	Prediction uncertainty (2 sigma) at 20 days / [ns] $\sigma_{WFN}(\tau)=1\times10^{-15}/day$ $\sigma_{RWFN}(\tau)=1\times10^{-16}/day$	Prediction uncertainty (2 sigma) at 20 days / [ns] $\sigma_{WFN}(\tau)=1\times10^{-14}/day$ $\sigma_{RWFN}(\tau)=1\times10^{-15}/day$
1	7	25
5	34	41
7	48	54
10	71	74
20	132	134

Table 2. The prediction uncertainties (2 sigma), depending on measurement uncertainty at 20 days by using an H-maser characterized by the stated frequency stability are reported.

By analysing the results reported in table 2 we can conclude that when a good quality time transfer technique is used, the noise affecting the atomic clocks has a significant impact on the uncertainty prediction but with an un-calibrated technique the results do not depend on the stability of the clock.

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