

Improving the clock frequency prediction in TAI algorithm

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Outline

- Prediction Algorithm
- Frequency drift in EAL
- EAL calculated without H-Masers
- New prediction algorithm
- EAL calculated by using the new prediction algorithm
- Conclusion and discussion

Prediction Algorithm on EAL

The prediction is useful to avoid or minimize the time and frequency jumps of the time scale when a clock is added or removed from the ensemble or when its weight changes.

The prediction term $h_i'(t)$ for clock H_i is the sum of two terms:

Term to avoid time steps

Term to avoid frequency steps

$$h_i'(t) = a_i + B_{ip}(t - t_i)$$

t_{i-1} t_i t_{i+1}

EAL- H_i ($t_i - t_{i-1}$) EAL- H_i ($t_{i+1} - t_i$)

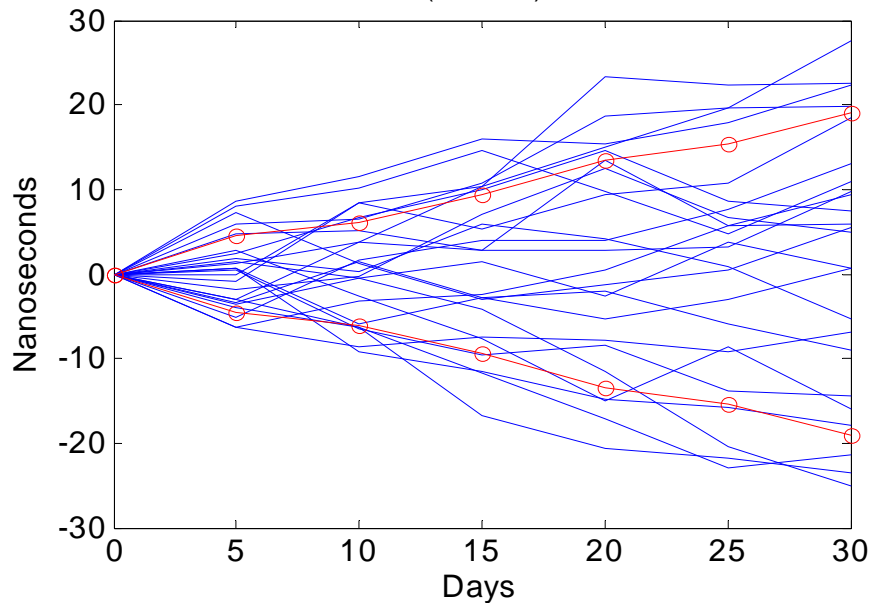
- a_i is the time correction relative to EAL of clock H_i at date t_i
- B_{ip} is the frequency of clock H_i , relative to EAL, predicted for the period $[t_i, t]$

Linear model: the frequency is considered constant during the month

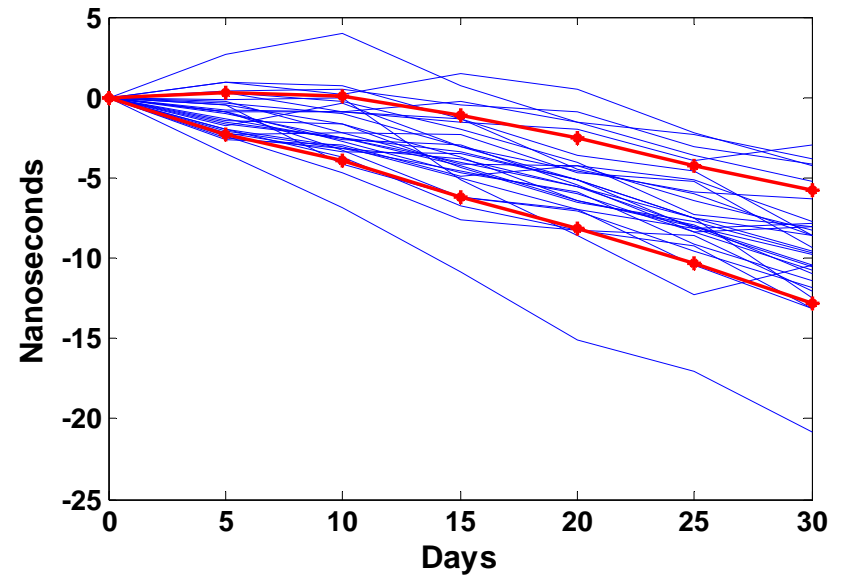
Prediction Analysis

3 years test period (2006-2008)

The difference (prediction-reality) of the EAL-CS Clock with standard deviation (red lines).



The difference (prediction-reality) of the EAL-Hm Clock with standard deviation (red lines).



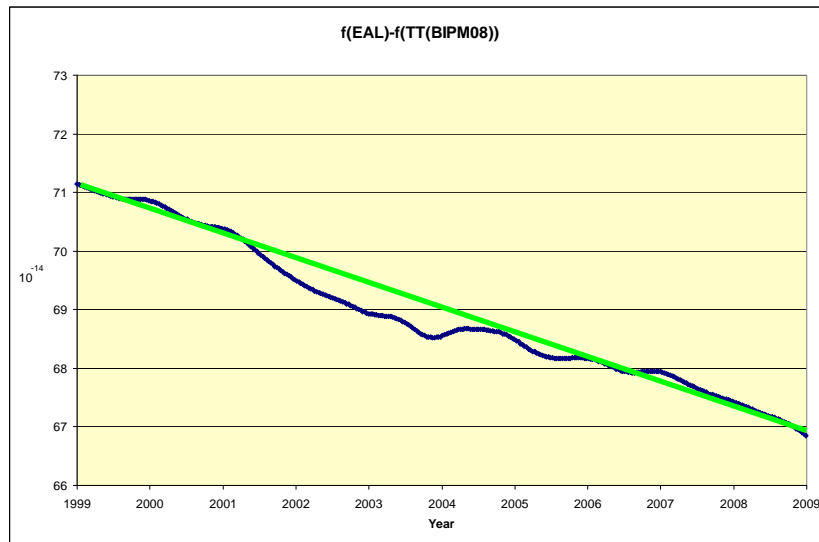
The mean value of the difference (prediction-reality) at 30 days

- for 100 EAL-CS Clock is about 0.2 ns
- for 20 EAL-Hm is about -30 ns

As expected, linear model does not take care of the H-maser frequency drift

TT(BIPM) as a frequency reference

TT(BIPM) is a time scale optimized for frequency accuracy. It uses primary frequency standard data provided to BIPM from laboratories all over the world



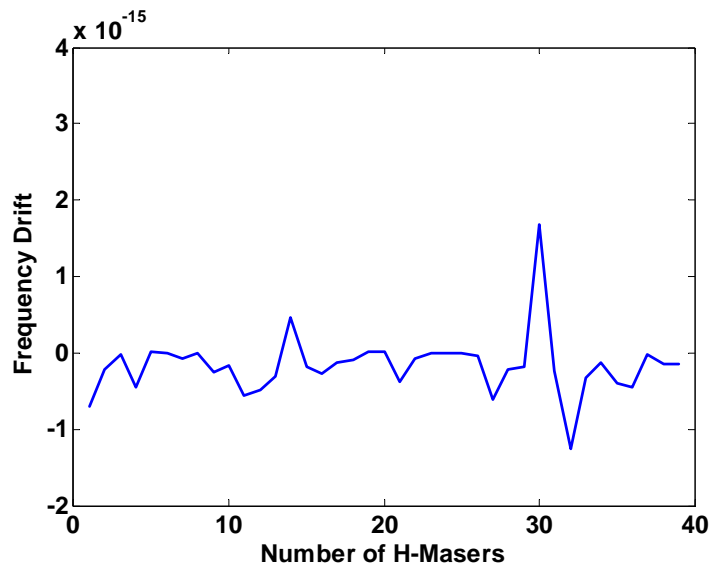
EAL shows a frequency drift w.r.t. TT

$$4 \times 10^{-16} / \text{month}$$

Drift evaluation

3 years test period (2006-2008)

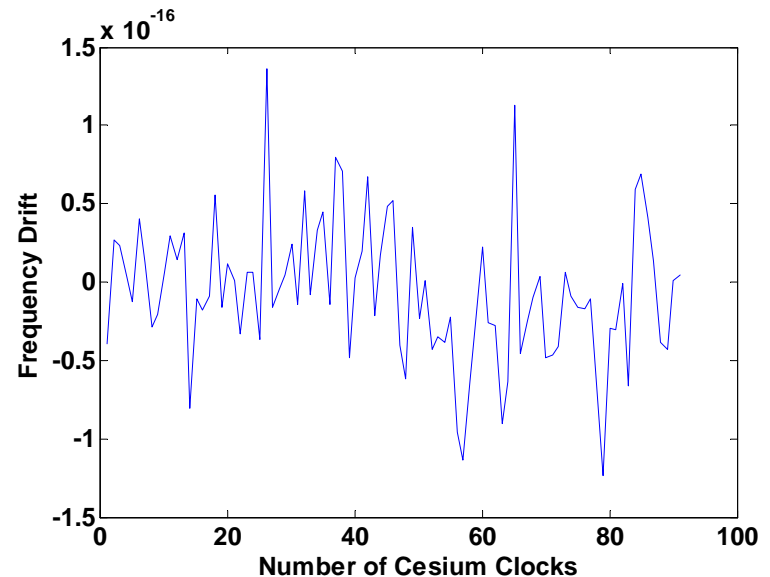
Drift of 40 H-Masers respect to TT



The mean on the frequency drift of 40 (H Masers - TT) is:

$$-4 \times 10^{-16} / \text{day}$$

Drift of 100 Cesium Clocks respect to TT



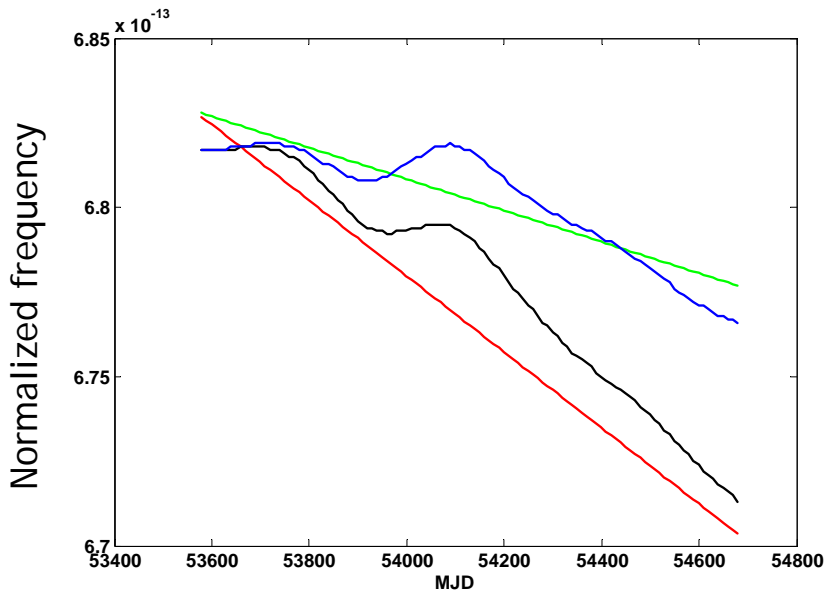
The mean of the frequency drift of 100 (Cesium clock - TT) is:

$$-1 \times 10^{-17} / \text{day}$$

EAL-TT

As H-Masers has a frequency drift, a test version of EAL has been calculated removing the H-Masers from the clock ensemble. To show the influence of H-Masers on EAL drift we consider TT as independent reference:

Test period: January 2006 - July 2008



— $f(\text{EAL}_{\text{without Hmaser}}) - f(\text{TT})$
— $f(\text{EAL}) - f(\text{TT})$

About 40% of EAL frequency drift is due to the H-masers

Frequency drift on $f(\text{EAL}) - f(\text{TT})$ is: $4 \times 10^{-16} / \text{month}$

Frequency drift on $f(\text{EAL}_{\text{without Hmaser}}) - f(\text{TT})$ is: $2.4 \times 10^{-16} / \text{month}$

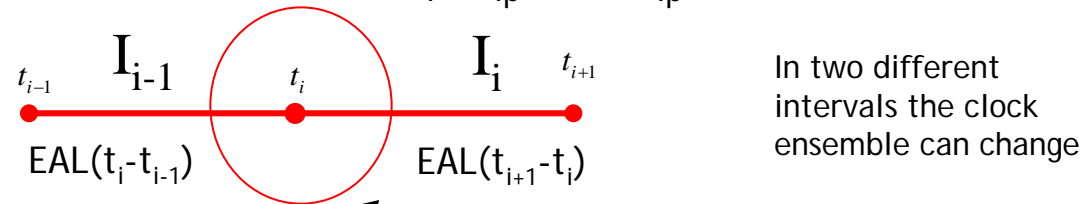
1. New prediction algorithm for the H-Masers

We consider a quadratic behaviour for the correction term $h_i'(t)$

$$h_i'(t) = a_{i,I_i} + B_{ip,I_i} (t - t_i) + \frac{1}{2} C_{ip,I_i} (t - t_i)^2$$

The frequency drift is considered constant during the month

How to estimate the parameters a_i , B_{ip} and C_{ip} related to the interval I_i ?



We impose the following conditions on h_i' at time t_i :

- 1- No time steps → continuity
- 2- No frequency steps → continuity of first derivative
- 3- No changing in frequency drift → continuity of second derivative

2. New prediction algorithm for the H-Masers

$$h_i'(t) = a_{i,I_i} + \underbrace{B_{ip,I_i}(t - t_i) + \frac{1}{2} C_{ip,I_{i-1}}(t_i - t_{i-1})(t - t_i)} + \frac{1}{2} C_{ip,I_i}(t - t_i)^2$$



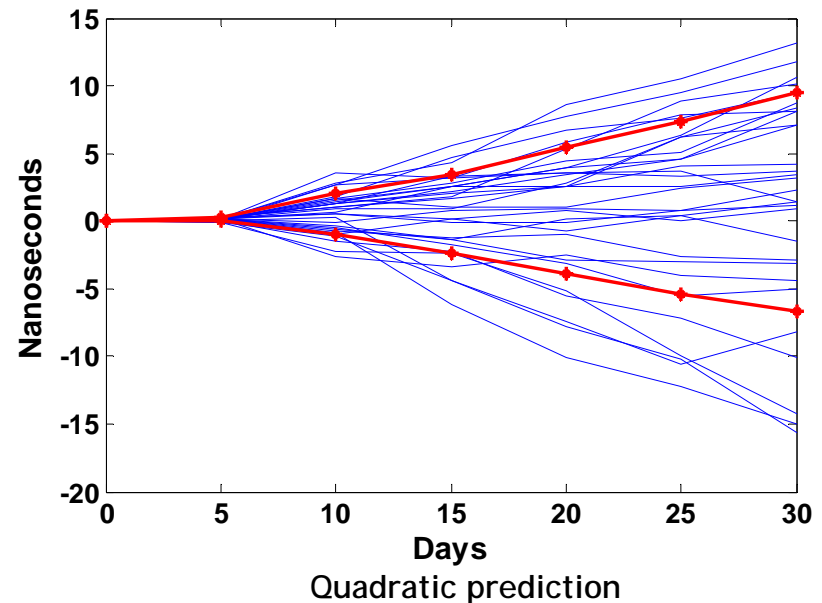
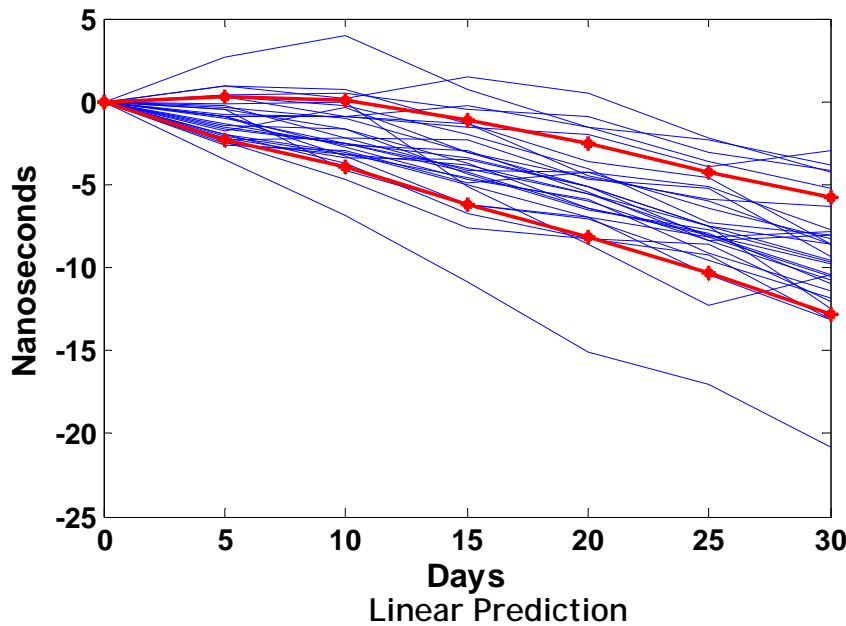
Now the frequency is not constant on the interval!

- a_{i,I_i} is the time correction relative to EAL of clock H_i at date t_i
- B_{ip,I_i} is the frequency of clock H_i , relative to EAL, predicted for the period $[t_i, t]$
- C_{ip,I_i} is the frequency drift of the clock H_i , relative to EAL, predicted for the period $[t_i, t]$
- $C_{ip,I_{i-1}}$ is the frequency drift of the clock H_i , relative to EAL, predicted for the period $[t_{i-1}, t_i]$

1. Effect of the new prediction algorithm

We calculate EAL for 3 years by using linear prediction for the cesium clocks and quadratic prediction for the H-masers. The frequency drift for the H-masers was evaluated respect to EAL on one month past period.

The difference (prediction-reality) of the EAL- H Maser using two different prediction techniques

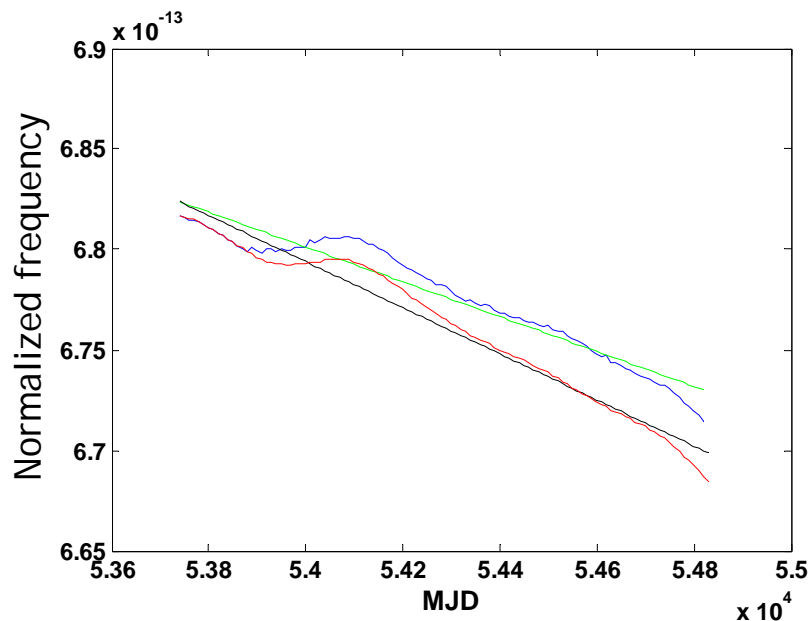


Considering 20 (EAL-H Masers): The mean value of the difference (prediction-reality) after 30 days is about 2 ns

Test period: January 2006 - December 2008

2. Effect of the new prediction algorithm - TT

To show the influence of new prediction algorithm on EAL drift we use TT as independent frequency reference:



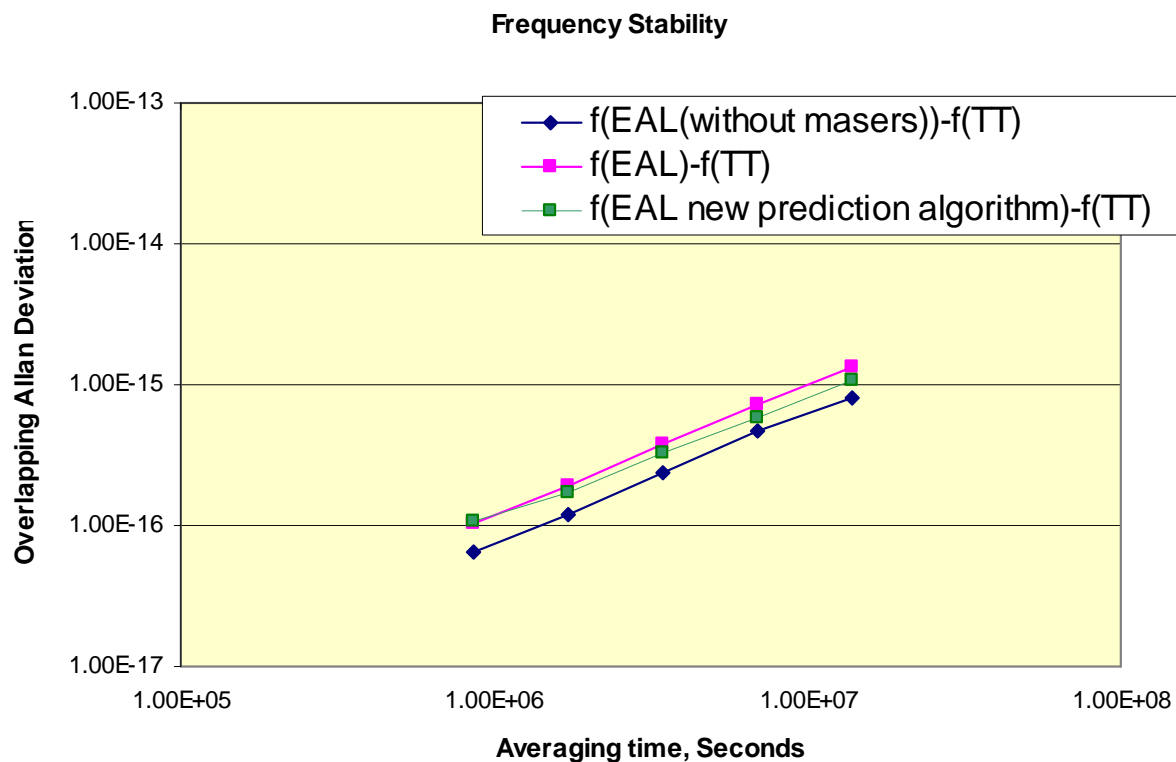
— $f(\text{EAL}_{\text{new prediction algorithm}}) - f(\text{TT})$
— $f(\text{EAL}) - f(\text{TT})$

About 20% of EAL frequency drift is due to the linear prediction used in ALGOS

Frequency drift on $f(\text{EAL}) - f(\text{TT})$ is: **4×10^{-16} / month**

Frequency drift on $f(\text{EAL}_{\text{new prediction algorithm}}) - f(\text{TT})$ is: **3.2×10^{-16} / month**

Stability Analysis



Allan deviation values

8.64E+05	6.56E-17	1.02E-16	1.06E-16
1.73E+06	1.20E-16	1.94E-16	1.72E-16
3.46E+06	2.38E-16	3.76E-16	3.23E-16
6.91E+06	4.63E-16	7.14E-16	5.93E-16
1.38E+07	8.16E-16	1.33E-15	1.08E-15

Discussion and Conclusions

A test for the 3 years period was done applying the linear prediction to the cesium clocks and the quadratic prediction to the H-masers.

One month of past data have been used to evaluate frequency drift, a longer period could be tested.

EAL still shows a relevant drift, further work needs to be done on EAL weighting algorithm.