Studies and possible improvements on EAL algorithm

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Abstract— **A new algorithm for the clock frequency prediction used to calculate EAL is presented. The mathematical model adopted in the new prediction algorithm takes in account the effect of the frequency drift of the H-masers. We demonstrate that there is an improvement in long term stability of EAL by using the new prediction algorithm.**

I. INTRODUCTION

We focus our analysis on the algorithm used at the Bureau International des Poids et Mesures (BIPM) for the calculation of the free atomic scale EAL and we discuss possible improvements of its frequency stability. In the present version of the algorithm the clock frequency prediction is considered constant during a month for all type of clocks, process which is adequate for caesium standards. This simple approach does not take into account the frequency drift of the H-masers, which could be significant in a month interval. Comparing the frequencies of EAL and TT (Terrestrial Time, a time scale optimized for frequency accuracy) a frequency drift is clearly detected. We start by the analysis carried out to understand the reasons of this drift. In particular the effect of the H-masers on the frequency drift of EAL has been evaluated. In the second part a new mathematical model for the prediction algorithm is presented. This model takes in account the frequency drift of the H-masers. The results of tests on EAL using this new model are discussed in the last section of this paper.

II. THE CALCULATION OF UTC AND TAI

The time laboratories realize a stable local time scale using individual atomic clocks or a clock ensemble. Clock readings are then combined at the BIPM through an algorithm designed to optimize the frequency stability and accuracy, and the reliability of the time scale above the level of performance that can be realized by any individual clock in the ensemble. An efficient algorithm is necessary for the statistical generation of a time scale. ALGOS is the algorithm maintained at the BIPM, which produces, monthly, the international reference UTC (Coordinated Universal Time). The calculation of UTC using ALGOS is carried out in three different steps:

- The free atomic time scale EAL (Echelle Atomique Libre) is computed as a weighted average of about 350 free-running atomic clocks spread world-wide. A clock weighting procedure has been designed to optimize the long-term frequency stability of the scale.
- The frequency of EAL is steered to maintain agreement with the definition of the SI second, and the resulting time scale is International Atomic Time TAI. The steering correction is determined by comparing the EAL frequency with that of primary frequency standards.
- Leap seconds are inserted to maintain agreement with the time derived from the rotation of the Earth. The resulting time scale is UTC.

Different algorithms can be considered depending on the requirements on the scale; for an international reference such as UTC, the requirement is extreme reliability and long-term frequency stability. UTC therefore relies on the largest possible number of atomic clocks of different types, located in different parts of the world and connected in a network that allows precise time comparisons between remote sites. Each month the differences between the international time scale UTC and the local time scales UTC(k) for contributing time laboratories are reported at differed time in the official document called BIPM *Circular T* [1]. We present hereafter the basic algorithm for constructing a time scale, and the particular case of UTC.

The original algorithm ALGOS for defining EAL was developed in the 1970s [2, 3, 4] from the equation:

$$EAL(t) = \sum_{i=1}^{N} w_i \left[h_i(t) + h_i'(t) \right] \quad (1)$$

where N is the number of participating clocks, w_i the relative weight of clock H_i , $h_i(t)$ is the reading of clock H_i at time t, and h_i' (t) is the prediction of the reading of clock H_i that serves to guarantee the continuity of the time scale. The weight attributed to a clock reflects its long-term stability, since the objective is to obtain a weighted average that is more stable in the long term than any of the contributing elements [5, 6]. The weights of the clocks obey the relation:

$$\sum_{i=1}^{N} w_i = 1 \qquad (2)$$

Subtracting the same quantity from both sides of eq. (1) we obtain:

$$EAL\,(t) - \sum_{i=1}^{N} w_i h_i(t) = \sum_{i=1}^{N} w_i \left[h_i(t) + h_i'(t)\right] - \sum_{i=1}^{N} w_i h_i(t) \; .$$

Using (2) and rearranging:

$$\sum_{i=1}^{N} w_i \left(EAL\,(t) - h_i(t)\right) = \sum_{i=1}^{N} w_i h_i'(t) \; . \qquad (3)$$

Setting

$$x_i(t) = EAL\,(t) - h_i(t), \qquad (4)$$

equation (3) takes the form:

$$\sum_{i=1}^{N} w_i x_i(t) = \sum_{i=1}^{N} w_i h_i'(t) \; . \qquad (5)$$

The algorithm ALGOS is used in the Time, Frequency and Gravimetry section of the BIPM to generate UTC. Clock weights are determined from the variance of monthly average frequencies, constrained to a maximum value to avoid a few very stable clocks becoming dominant in the scale [6]. The data used by ALGOS take the form of the time differences between readings of clocks, written as:

$$x_{i,j}(t) = h_j(t) - h_i(t). \qquad (6)$$

Eq. (5) in conjunction with the N – 1 equations (6), results in system of N equations and N unknowns:

$$\begin{cases} \sum_{i=1}^{N} w_i x_i(t) = \sum_{i=1}^{N} w_i h_i'(t) \\ x_i(t) - x_j(t) = x_{i,j}(t) \end{cases} \qquad (7)$$

whose solution is:

$$x_j(t) = EAL(t) - h_j = \sum_{i=1}^{N} w_i \left[h_i'(t) - x_{i,j}(t)\right] . \qquad (8)$$

The difference (8) between any clock H_j and EAL depends on the clock weights, the clock frequency prediction, and the measured clock differences. The clock H_j may also represent a UTC(j) time scale; therefore, $x_j(t)$ can also be interpreted as:

$$x_j = EAL - UTC\,(j) \qquad (9)$$

where for simplicity we have dropped the time instant t from the notation.

The clock frequency prediction and weights are fixed by appropriate algorithms based on the clock behavior in the past, and in eq. (8) they can be considered as time-varying deterministic parameters.

From eq. (9), and following the steps described in the preceding section the differences [*TAI* - *UTC*(*j*)] and finally [*UTC* - *UTC*(*j*)] are evaluated.

III. PREDICTION ALGORITHM IN ALGOS

In this section we present the prediction algorithm used in ALGOS; in the generation of a time scale, the prediction of the atomic clock behavior plays an important role, in fact the prediction is useful to avoid or minimize the frequency jumps of the time scale when a clock is added or removed from the ensemble or when its weight changes. Considering two successive intervals of TAI calculation $I_{i-1}\,(t_{i-1}, t_i)$ and $I_i\,(t_i, t_{i+1})$ we impose several conditions on the prediction term at time t_i to avoid or minimize time and frequency jumps in the resulting time scale.

ALGOS operates in post-processing, treating as a whole measurements taken over a basic period of T=30 or 35 days. For each one-month period, the results are the quantities $x_i\,(t_i + nT/6)$ (or $x_i\,(t_i + nT/7)$) with n = 0, ..., 6 (or n = 0, ..., 7) for each clock H_i. The least squares slope of these quantities is referred to as the frequency $B_{i,I_i}(t_i + T)$ of the clock H_i, relative to EAL, for the one-month interval $I_i = [t_i, t_i + T]$. The correction term $h_i'(t)$ [7] for clock H_i is the sum of two terms:

$$h_i'(t) = a_{i,I_i}(t_i) + B_{ip,I_i}(t)(t - t_i) \qquad (10)$$

where $a_i(t_i)$ is the time correction relative to EAL of clock H_i at date t_i :

$$a_{i,I_i}(t_i) = EAL(t_i) - h_i(t_i) = x_i(t_i). \qquad (11)$$

$B_{ip,I_i}(t)$ is the frequency of clock H_i, relative to EAL, predicted for the period $I_i=[t_i, t]$ and $(t - t_i) = nT/6$ (or $nT/7$) with n = 0, ..., 6 (or n = 0, ..., 7). At present, the predicted frequency $B_{ip,I_i}(t)$ for t included in $I_i = [t_i, t_i + T]$ is kept constant for the whole one-month interval $I_i = [t_i, t_i + T]$. It is chosen to be the frequency $B_i(t_i)$ computed for the previous one-month interval $I_{i-1} = [t_i - T, t_i]$, that is to say, the least squares slope of $\{x_i(t_i - T + nT/6)\}$ (or $\{x_i(t_i - T + nT/7)\}$) with n = 0. ..., 6 (or n = 0, ..., 7). This is a one-step linear prediction written as $B_{ip,I_i}(t) = B_i(t_i)$ for $t = t_i + nT/6$ (or $t = t_i + nT/7$), n = 0. ..., 6 (or n = 0, ..., 7). The assumption that clock H_i is most likely to behave over the coming one-month interval as it did on the previous one is linked to the choice of the sample duration T. To evaluate the performance of the linear prediction a test was performed by using three years of clock data (caesium clocks and H-masers) with respect to EAL.

1) Test on the caesium clock data

We have studied the caesium clock behavior for the 3-year period 2006-2008 and we have compared the real values of the difference EAL- caesium with the respective predicted values. The distribution of residuals around zero can indicate if the prediction describes well the behavior of the caesium clocks. We made this analysis on 100 caesium clocks and estimated the mean values of EAL-caesium clock at the end of each 30-day period. In Figure 1 the real values EAL-caesium clock (blue line) and the respective predicted values (red line) are reported. Figure 2 shows the differences between the real and the predicted values with blue lines, and the 1 sigma uncertainty with the red lines. We can observe that the residuals are well distributed around the zero. The mean value is 0.2 ns at 30 days and the standard deviation equal to 21 ns. We conclude that in the case of caesium clocks the linear model is a good predictor of the caesium clock behavior as expected from the physical point of view.

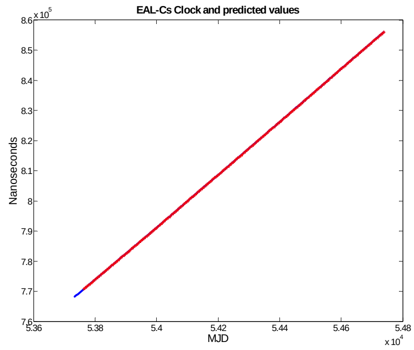

Figure 1. EAL-caesium clock (blue line) and predicted values (red line)

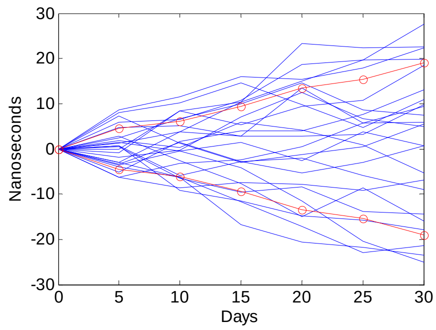

Figure 2. The differences between EAL-caesium clock and the predicted values are reported with the blue lines. With the red lines the 1 sigma uncertainty is reported.

2) *Test on the H-maser clock data*

The same analysis has been carried out for 20 H-masers for the 3-year period 2006-2008; we have compared the real values of EAL – H-maser clocks with the respective predicted values and we have observed the residuals distribution. The mean value of the residuals between (EAL – H-maser) and the predicted values at 30 days is -30 ns and the standard deviation is 40 ns. In Figure 3 the (EAL – H-maser) (blue line) and the predicted values (red lines) are reported. In Figure 2 the differences between (EAL - H-maser) and the predicted values are reported with blue lines. The red lines show the 1 sigma uncertainty. We observe that the residuals are not well distributed around the zero. We conclude that the linear prediction of the frequency does not describe well the behavior of the H-masers as expected from the physical point of view. In fact the frequency of the H-masers is affected by a drift and it is characterized by a quadratic behavior in phase.

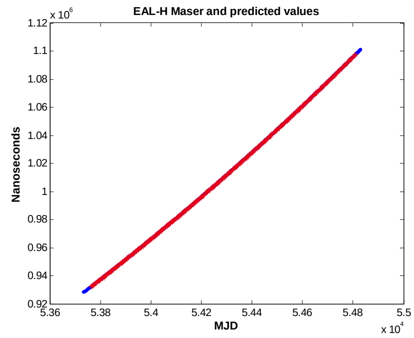

Figure 3. EAL - H-masers (blue line) and predicted values (red line)

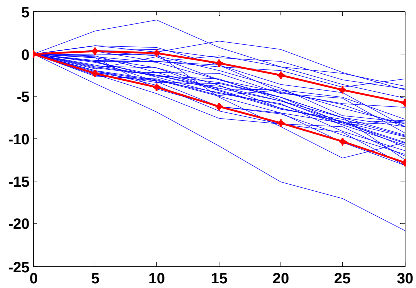

Figure 4. The differences between EAL - H-masers and the predicted values are reported with the blue lines. With the red lines the 1 sigma uncertainty is reported.

IV. TT(BIPM08) AS FREQUENCY REFERENCE

TT(BIPM) provides a stable and accurate reference for characterizing the performance of the frequency of EAL and the frequency drift of the H-masers and the caesium clocks. TT(BIPM) is a time scale optimized for frequency accuracy. It is evaluated annually by making use of all available primary frequency standard data reported to the BIPM by national laboratories [7-9]. Figure 5 plots the relative frequency of EAL with respect to TT(BIPM08), and shows that there is a drift of about $+4*10^{-16}$/month.

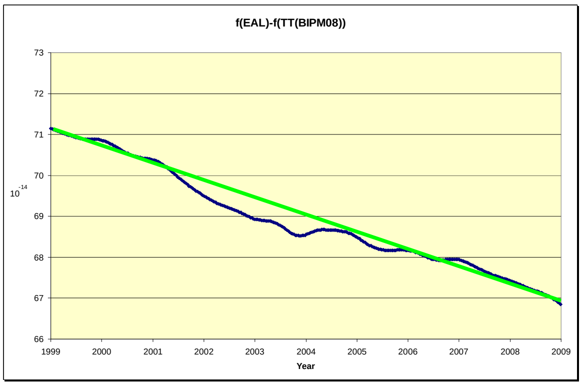

Figure 5. The frequency of EAL respect to TT(BIPM08)

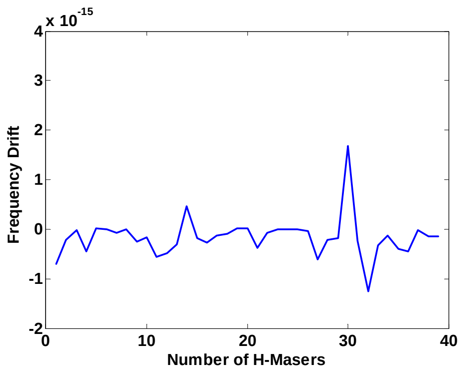

Figure 6. The frequency drift of 40 H-masers respect to TT(BIPM08)

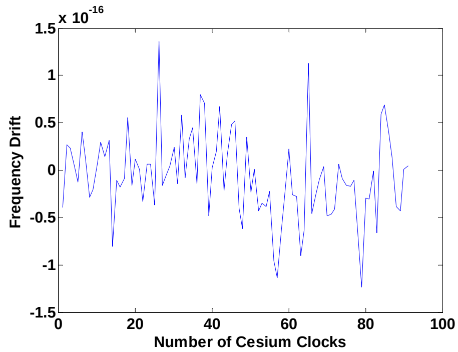

Figure 7. The frequency drift of 100 caesium clocks respect to TT(BIPM08)

We have used TT(BIPM) as the reference for characterizing the frequency drift of 40 hydrogen masers (Figure 6) and of 100 caesium clocks (Figure 7).

The mean value for the frequency drift related to 40 H-masers with respect to TT(BIPM08) is $-4*10^{-16}$/day whereas for the caesium clocks it is $-1*10^{-17}$/day. Caesiums represent about 80% of the weight of the clocks participating in the calculation of EAL, and consequently their impact on the frequency drift of EAL is more significant than that of the H-masers.

V. TEST VERSION OF EAL WITHOUT H-MASERS

To evaluate the effect of the wrongly predicted frequency of the H-masers on EAL we calculated a test version of EAL over the period January 2006 - July 2008. The test version has been calculated removing the H-masers from the clock ensemble. The difference between EAL (without H-masers) and the real EAL was obtained and the result in term of frequency is reported in Figure 8. This difference has a drift of about $1.6*10^{-16}$/month indicating that the frequency drift of the H-masers affects the frequency of EAL. To evaluate this effect we compare the frequency of EAL calculated without H-masers to TT(BIPM08) and we afterwards compare this result to the real behavior of f(EAL - f(TT(BIPM)) as in figure 5. These results are reported in figure 9. The drift of f[EAL(without H-masers) - f(TT(BIPM)] is $2.4*10^{-16}$/month; comparing this value with that of f(EAL)-f(TT(BIPM)) (from section III $4*10^{-16}$/month) we conclude that about 40% of frequency drift of EAL might be due to the frequency drift of the H-masers.

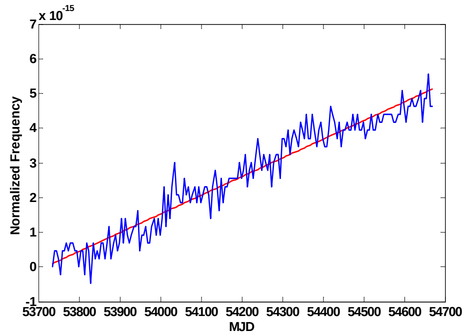

Figure 8. The difference between EAL (without H-masers) and EAL in frequency term is reported.

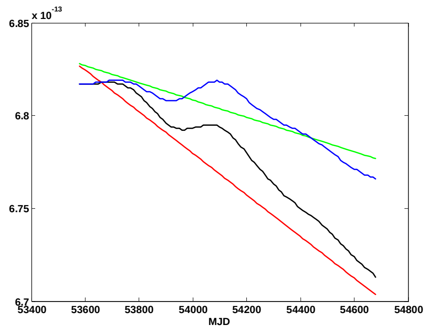

Figure 9. f(EAL)-f(TT(BIPM)) (black line) with its slope (red line) are compared to f(EAL(without H-masers))- f(TT(BIPM)) (blue line) with its slope (green line).

VI. NEW PREDICTION ALGORITHM

To evaluate the effect of the prediction in EAL frequency drift and to better describe the behavior of H-masers we present here a new clock frequency prediction algorithm. Considering two successive intervals of calculation of TAI I_{i-1} (t_{i-1}, t_i) and I_i (t_i, t_{i+1}) we impose several conditions on the prediction term h_i' at time t_i to avoid or minimize time and frequency jumps in the resulting time scale. Instead of the linear term used in ALGOS (10) we consider a quadratic behavior:

$$h_i'(t) = a_{i,I_i}(t_i) + B_{ip,I_i}(t)(t-t_i) + \frac{1}{2}C_{ip,I_i}(t)(t-t_i)^2 \quad (12)$$

To the linear relation (11) we add the last term to describe the frequency drift affecting the H-masers. To evaluate the parameters in (12) we assume at time t_i the following conditions on h_i':

1. no time steps by imposing the continuity to the constant term;
2. no frequency steps by imposing the continuity to the first derivative;
3. no change in frequency drift by imposing the continuity to the second derivative.

We obtain a system of three equations with three variables and solving this system we obtain that the relation (12) can be written in an iterative form:

$$h_i'(t) = a_{i,I_i}(t_i) + B_{ip,I_i}(t)(t-t_i) + \\ + \frac{1}{2}C_{ip,I_{i-1}}(t)(t_i - t_{i-1})(t-t_i) + \frac{1}{2}C_{ip,I_i}(t)(t-t_i)^2 \quad (13)$$

The physical meaning of the terms present in (13) is:

a_{i,I_i} is the time correction relative to EAL of clock H_i at date t_i

B_{ip,I_i} is the frequency of clock H_i, relative to EAL, predicted for the period $[t_i, t]$

C_{ip,I_i} is the frequency drift of the clock H_i, relative to EAL, predicted for the period $[t_i, t]$

$C_{ip,I_{i-1}}$ is the frequency drift of the clock H_i, relative to EAL, predicted for the period $[t_{i-1}, t_i]$.

Two aspects are very important in this relation; the first is that we consider a constant drift during the calculation interval; the second is that with respect to the linear prediction the frequency does not remain constant during the interval. The parameters described above are considered constant during the calculation interval, but to avoid a frequency jump the frequency at the end of an interval and at the beginning of the next one should be the same. The frequency at the end of the interval is expressed by the addition of the second and the third terms in the relation (13):

$$B_{ip,I_i}(t)(t-t_i) + \frac{1}{2}C_{ip,I_{i-1}}(t)(t_i - t_{i-1})(t-t_i) \quad (14)$$

VII. TEST VERSION OF EAL BY USING THE NEW PREDICTION ALGORITHM

To evaluate the effect of the new prediction algorithm a test version of EAL, indicated by EAL′ has been calculated for the period January 2006 - July 2008. The linear prediction (11) for the caesium clocks and the quadratic prediction (13) for the H-masers have been used. The weighting algorithm has not been changed so the ensemble of H-masers represents about 12% of the total weight and the caesium clocks about 80%. The frequency drift for the H-masers has been evaluated with respect to EAL′ on one month past period.

We start by testing if the quadratic model for the frequency prediction describes the behaviour of the H-masers applying the analysis described in section III. We consider an ensemble of 20 H-masers (same than above) and we compare the real values of (EAL'- H-masers) with the predicted values. The mean value of the residuals at 30 days is 2 ns and the standard deviation equal to 45 ns. We conclude that a quadratic model predicts the frequency of the H-masers better than the linear prediction, considering that in section III the mean value at 30 days of the residuals for a linear prediction model is -30 ns. Figure 10 shows the differences between (EAL′ - H-masers) and the respective predicted values with blue lines; the red lines show the standard deviation. The residuals are well distributed around zero. To evaluate the effect of the new prediction algorithm on the EAL frequency drift the difference between EAL′ and EAL was obtained and the result in term of frequency is reported in Figure 11. This difference shows a frequency drift of about 8.2×10^{-17}/month, indicating that the linear frequency prediction for all clocks used in ALGOS contributes to the EAL frequency drift. To calculate this contribution we compare the frequency of EAL′ to TT(BIPM) and afterwards we compare this result to f(EAL)-f(TT(BIPM)). These results are reported in Figure 12. The frequency drift of f(EAL)-f(TT(BIPM)) is $4*10^{-16}$/month whereas that of f(EAL′) - f(TT(BIPM)) is $3.2*10^{-16}$/month. These results seem to indicate that about 20% of the frequency drift of EAL might be due to the linear prediction algorithm applied to H-masers.

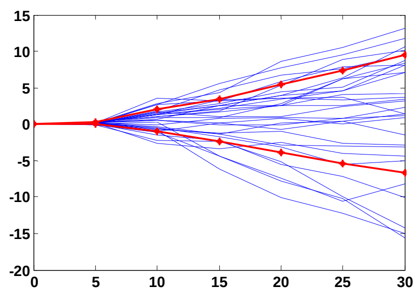

Figure 10. The differences between EAL′- H-masers and the predicted values are reported with the blue lines. With the red lines the 1 sigma uncertainty is reported.

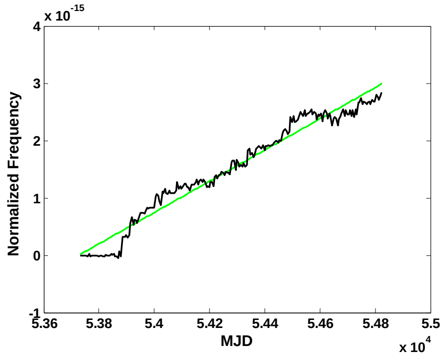

Figure 11. The difference between EAL´ and EAL in frequency term is reported.

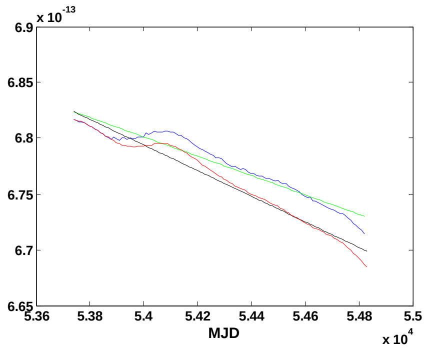

Figure 12. f(EAL)-f(TT(BIPM08)) (red line) with its slope (black line) are compared to f(EAL´)- f(TT(BIPM08)) (blue line) with its slope (green line).

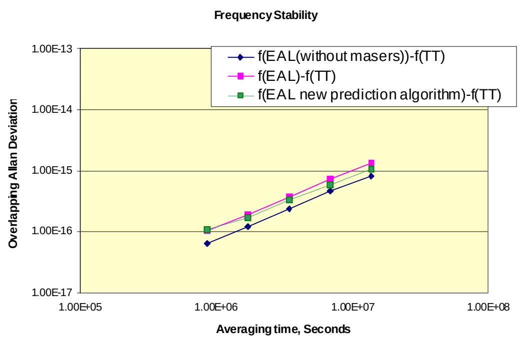

Figure 13. Stability analysis of f(EAL)-f(TT(BIPM)) (pink line), f(EAL(without H-masers))- f(TT(BIPM)) (blue line) and f(EAL´)- f(TT(BIPM)) (green line).

We conclude this analysis by reporting in Figure 13 the stability analysis of f(EAL)-f(TT(BIPM)), f(EAL(without H-masers))- f(TT(BIPM)) and f(EAL´)- f(TT(BIPM)).

Considering that TT and EAL are correlated we can only study the long term stability. That the time scale calculated without H-masers is the most stable, this is due to the non-predicted frequency drift of the H-masers in the current EAL algorithm. The new algorithm, with the quadratic prediction of the frequency of the H-masers improves the stability performance of EAL.

VIII. CONCLUSIONS

The effect of the linear prediction algorithm has been studied for different types of clocks in TAI. ALGOS predicts the clock frequency with a linear model that is well adapted to the caesium clock, but not to the H-maser clock. A test version of EAL without H-masers has been calculated to evaluate the effects of the equal modelling of the clock frequencies. A new mathematical expression for the prediction of the H-maser frequency is proposed taking into account the drift. Tests over a 3-year period have been performed applying the linear prediction to the caesium clocks and the quadratic prediction to the H-masers. A version of EAL on the basis of the proposed frequency prediction for H-masers, but with the classical clock weighting has been evaluated. The results seem to indicate that non-modeling of the frequency drift of H-masers could be responsible for 20% of the drift of EAL. In this test one month of past data has been used to evaluate frequency drift but a longer period could be tested. EAL still shows a significant drift; further work needs to be done on EAL weighting algorithm.

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