Capacity Building & Knowledge Transfer

Measurement comparisons (part II, Calculation of reference values and associated uncertainties)

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Importance of the KCRV (Key Comparison Reference Value)

see: document CIPM MRA-D-05, Sections 2.1.3 and 2.1.4

- *to compute Degrees of Equivalence* of NMIs/DIs participating in CIPM key comparisons.

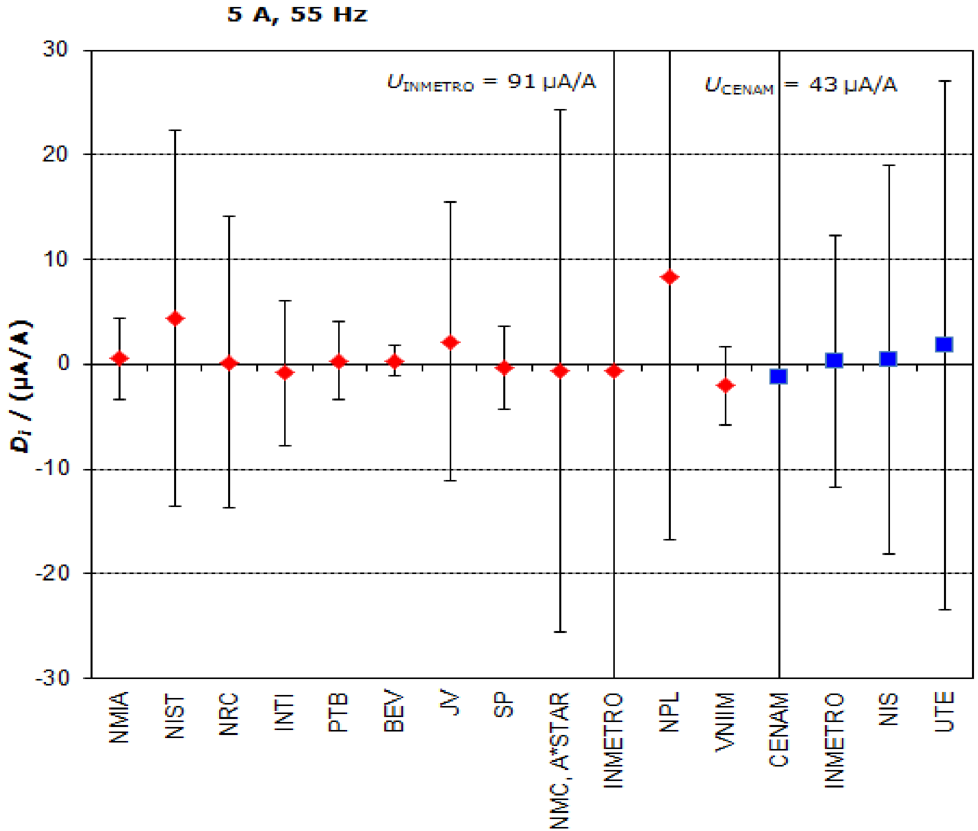
- *to compute Degrees of Equivalence of RMO KC participants*
  - RMO KCs do not have a separate KCRV; they must link their results to the KCRV of the corresponding CIPM comparison

  Supplementary comparisons are not linked to a KCRV, but may calculate a comparison reference value

Supplementary comparisons may have a reference value

"degrees of equivalence relative to a
supplementary comparison reference value
may be computed, but this is not mandatory"

Example: CCEM-K12/SIM.EM-K12, AC-DC current transfer



(2005-2007); Final report 2012

Red diamonds: participants in CCEM-K12

Blue squares: participants in SIM.EM-K12

(2010-2012); Final report 2014

Linking laboratories:

NIST, NRC, INTI

Here are four symbols used to discuss comparisons

- x_i : the measurement result reported by "laboratory i"
- u_i : the standard uncertainty of x_i
- x_{R} : the key comparison reference value
- u_{R} : the standard uncertainty of x_{R}

CCEM-K12, Degrees of Equivalence (an example)

The ***key comparison reference value***, x_R, is computed as the ***weighted average*** of the results of those participants who have an independent realization of primary standards for current AC-DC difference and a low reported uncertainty (see Section 6 of the CCEM-K12 Final Report).

…

The combined standard uncertainty, u_R, of x_R is the standard uncertainty of the weighted average (see equation 2 on page 11 of the Final Report).

For 5 A and 55 Hz, x_R = -0.3 µA/A and $2u_R$ = 1.4 µA/A

The ***degree of equivalence*** of laboratory ***i*** relative to the key comparison reference value is given by a pair of terms both expressed in µA/A: $D_i = x_i - x_R$, and its ***expanded uncertainty*** (k = 2) U_i obtained from equations 4, 5 and 6 on page 11 of the CCEM-K12 Final Report.

There are many ways to calculate a KCRV, for example

- Weighted average (mean) of results (A), or the median (B) are described in this early publication

metrologia

The evaluation of key comparison data

Metrologia, 2002, **39**, 589-595

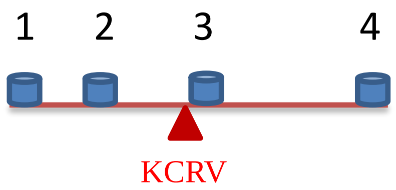
M. G. Cox

Abstract. Guidelines containing two procedures are proposed for the evaluation of key comparison data. They apply to the simple circulation of a single travelling standard around all the participants. The application of the procedures to a specific set of key comparison data provides a key comparison reference value (KCRV) and the associated uncertainty, the degree of equivalence of the measurement made by each participating national institute

- Suppose there are four participants with results:

x_1, x_2, x_3, x_4 ordered from smallest value to largest

Special case (very unlikely): Weighted average if all participants had reported the same uncertainty, u



$$x_R = \frac{x_1 + x_2 + x_3 + x_4}{4}$$

$$u_R^2 = \frac{u^2}{4}$$

$$U_R = 2u_R$$

$$U_{Di} = 2u_{Di}$$

General formula

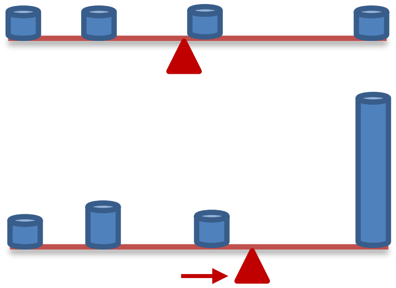
$$u_{D_i} = \sqrt{u^2 - u_R^2}$$

All independent results are used to calculate the KCRV

Each "degree of equivalence" such as $D_1 = x_1 - x_R$ must take account of **correlation** between x_1 and x_R when calculating u_{D1}.

KCRV calculated by weighted average (weighted mean)

When some results are given more "weight" than others, the balance point (KCRV) can change. The balance point depends on the value <u>and</u> the "weight" of each result.



value × weight

$$x_{\mathrm{R}} = \frac{\dfrac{x_1}{u_1^2} + \dfrac{x_2}{u_2^2} + \dfrac{x_3}{u_3^2} + \dfrac{x_4}{u_4^2}}{\dfrac{1}{u_1^2} + \dfrac{1}{u_2^2} + \dfrac{1}{u_3^2} + \dfrac{1}{u_4^2}} = \sum_{i=1}^{4} x_i \left[\frac{u_{\mathrm{R}}^2}{u_i^2} \right]$$

$$\frac{1}{u_{\mathrm{R}}^2} = \frac{1}{u_1^2} + \frac{1}{u_2^2} + \frac{1}{u_3^2} + \frac{1}{u_4^2}$$

$$U_{\mathrm{R}} = 2u_{\mathrm{R}}$$

$$U_{Di} = 2u_{Di}$$

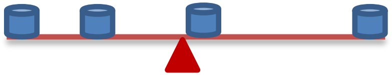
$$D_1 = x_1 - x_{\mathrm{R}}$$

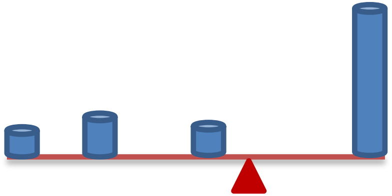
General formula

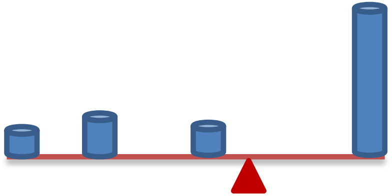
$$u_{D_i} = \sqrt{u_i^2 - u_{\mathrm{R}}^2}$$

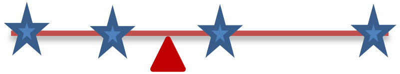
KCRV calculated by median (often used as a check)

When some values are given more "weight" than others, the balance point (KCRV) can change. The balance point depends on the value <u>and</u> its "weight"









median is ***robust***
(not affected by ***outliers***)

Jörg W. Müller
J. Res. Natl. Inst. Stand. Technol. **105**, 551 (2000)
(also Cox, method B. see slide 7)

x_{R} = median of (x_1, x_2, x_3, x_4)

$$u_{\mathrm{R}} \approx \frac{1.9}{\sqrt{N-1}} MAD$$

(*MAD* = median of absolute differences)

Other methods used for calculating KCRV

- Power-moderated mean (PMM)
  - See document CCRI(II)/13-18 (and/or talk to Carine Michotte)

Not the same as weighted mean with equal weights

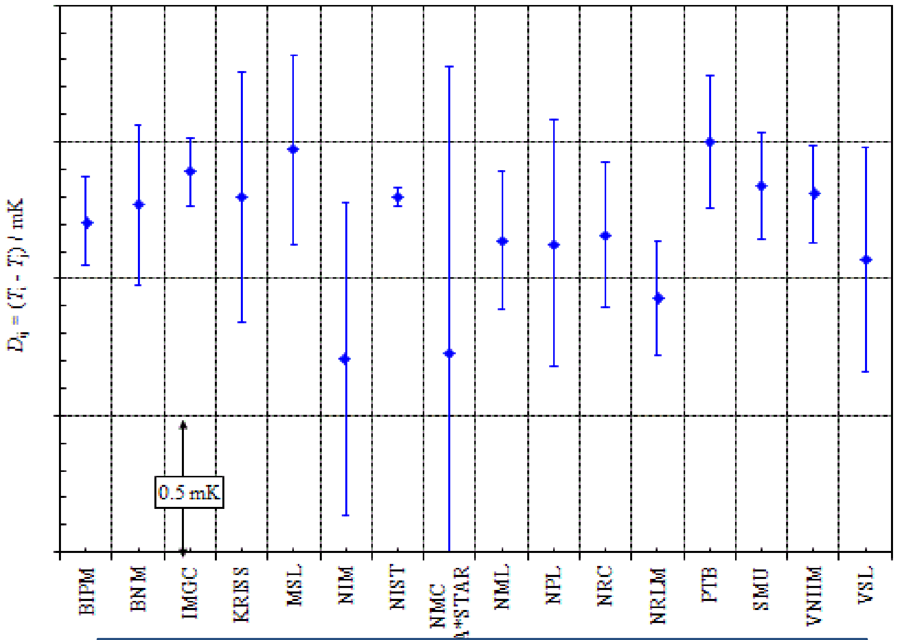
Another way of writing same equations for weighted mean on Slide 9

Table 1. Estimators of mean

Estimator $x_{\mathrm{ref}} = \sum_{i=1}^{N} w_i x_i$	Normalised weight w_i	Mean x_{ref}	Associated standard uncertainty $u(x_{\mathrm{ref}})$
Arithmetic mean	$\frac{1}{N}$ same	$\frac{1}{N}\sum_{i=1}^{N} x_i$ same	$\left[\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N(N-1)}\right]^{1/2}$ different
Weighted mean	$\frac{u^2(x_{\mathrm{ref}})}{u_i^2}$	$u^2(x_{\mathrm{ref}})\sum_{i=1}^{N} \frac{x_i}{u_i^2}$	$\left[\sum_{i=1}^{N} \frac{1}{u_i^2}\right]^{-1/2}$
Mandel-Paule mean	$\frac{u^2(x_{\mathrm{ref}})}{u_i^2 + s^2}$	$u^2(x_{\mathrm{ref}})\sum_{i=1}^{N} \frac{x_i}{u_i^2 + s^2}$	$\left[\sum_{i=1}^{N} \frac{1}{u_i^2 + s^2}\right]^{-1/2}$
PMM	$\frac{u^2(x_{\mathrm{ref}})}{\left(\sqrt{u_i^2 + s^2}\right)^{\alpha} s^{2-\alpha}}$	$u^2(x_{\mathrm{ref}})\sum_{i=1}^{N} \frac{x_i}{\left(\sqrt{u_i^2 + s^2}\right)^{\alpha} s^{2-\alpha}}$	$\left[\sum_{i=1}^{N} \left(\sqrt{u_i^2 + s^2}\right)^{-\alpha} s^{\alpha-2}\right]^{-1/2}$

Problems can occur. For example…

CCT-K3, published 2003



Decision: no KCRV possible (*MRA* T.3)

IOP Publishing | Bureau International des Poids et Mesures Metrologia

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Transfer standard uncertainty can cause inconclusive inter-laboratory comparisons

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Cox's approach uses the following equations to estimate the uncertainties, $u_{x\mathrm{CRV}}$, and u_{d_i}

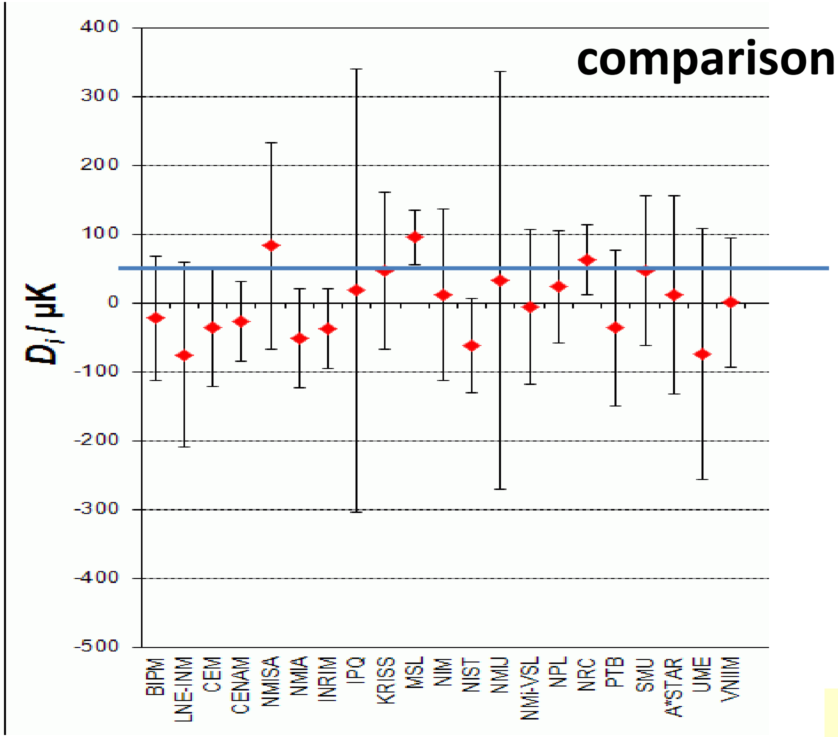
$$\frac{1}{u^2_{x\mathrm{CRV}}} = \frac{1}{u^2_{x_1}} + \frac{1}{u^2_{x_2}} + \ldots + \frac{1}{u^2_{x_n}} \tag{4}$$

and

$$u_{d_i} = \sqrt{u^2_{x_i} - u^2_{x\mathrm{CRV}}}. \tag{5}$$

However, the uncertainty of the reported value (called u_{x_i} by Cox) is *not* simply the uncertainty of the participant's flow reference ($u_{\mathrm{base}\,i}$): it must also include uncertainties introduced by the transfer standard and the repeatability of the reported value at each set point. These extra uncertainty components are often significant relative to the participating labs' base

CCT-K7, results published January 2006



comparison of triple point of water (TPW) cells

In CCT-K7 **NMISA**, **MSL** and **NRC** realized systematically higher temperatures, because they were the only laboratories which based their realizations on the recommendation of the *Supplementary Information for the ITS-90* to use water with the isotopic composition of standard mean ocean water.

Since the publication of CCT-K7 results, the CIPM approved **Recommendation 2 (CI-2005)** "Clarification of the definition of the kelvin, unit of thermodynamic temperature"

Redefinition of the kelvin in 2018 ? TPW will remain important.

Keep in mind...

- Many different statistical techniques have been used to calculate the KCRV of a CC Key Comparison
  - For CCEM-K12, results from ***weighted average***, **arithmetic average**, and **median** are not significantly different.
- There should be a good reason for choosing a particular statistical technique.
  - See what techniques have been used in the past. Why has a particular technique has been chosen?
  - Consult guidance documents; ask questions
  - Can scatter in the results be explained by the participants' uncertainties?

RMO KCs must be linked to the relevant CIPM KCRV

- The link to the KCRV is through *linking laboratories* that have participated in ***both*** the CIPM KC and the corresponding RMO KC.

This means

1. The measurand of the RMO KC should be similar enough to that of the CIPM KC so that a ***link is technically possible***
2. The linking laboratories should have small uncertainties to minimize the added uncertainty of the link. (Try to ensure that $\boldsymbol{u_{Di}}$ will be sufficiently small for all RMO participants.)
3. The number of linking laboratories must be ≥ 1 ! Choose an appropriate number.

The role of the linking lab(s) in calculating $x_{RMO_i} - x_{\mathrm{R}}$

- Principle illustrated by a simple model with one linking lab, *L*:

$$x_{RMO_i} - x_{\mathrm{R}} = x_{RMO_i} - x_L + x_L - x_{\mathrm{R}} = \left(x_{RMO_i} - x_L\right) + \left(x_L - x_{\mathrm{R}}\right)$$

$$\left(x_{RMO_i} - x_L\right) + \left(x_L - x_{\mathrm{R}}\right) = d_{RMO_i,L} + D_L$$

More profound and more useful analyses have been published in *Metrologia* and in Final Reports of RMO KCs.

Finally, role of comparison results in supporting CMCs

- **CIPM MRA, T.7 of Technical Supplement**
  - CMCs are listed in Appendix C of the MRA
  - CMCs *"must be consistent with results given in Appendix B, derived from key comparisons"*: **CMCs must be consistent with Key Comparison results**

- **CIPM MRA-D-04; acceptance criteria for CMCs – Section 3**
  - must be consistent with information from some or all of:
    results of KCs and SCs; past comparisons; knowledge of technical activities (includes publications); peer-assessment reports; RMO participation; *other sources*.

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Thanks for your attention.

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