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The fundamental constants of physics:
what are they and what is their role in redefining the SI

Fundamental Strings
and
Fundamental Constants

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Pre-preamble

In the end we always deal, in physics, with pure numbers, i.e. with ratios of quantities having the same physical dimensions.

These do not depend on the units we choose.

Yet, it is convenient to have some units to compare any given physical quantity with.

Which ones are necessary or redundant, which are more or less convenient, more or less fundamental or anthropological is much a matter of personal (or a given community's) taste.

Steven Weinberg's definition of Fundamental Constants

(The Constants of Physics,
Phil Trans. R. Soc. Lon. A310 (1983) 249)

adopted hereafter

"The list of fundamental constants depends on who is compiling the list…."

(e.g. hydrodynamicist, atomic physicist, etc.)

Weinberg's list: "a list of constants whose value we cannot calculate with precision in terms of more fundamental constants, not just because the calculation is too complicated, but because we do not know of anything more fundamental.

The membership of such a list thus reflects our present understanding of fundamental physics…"

How does/did such a list change together with our theories?

Sometimes, in order to form such dimensionless ratios we use some conceptual progress allowing us to compare things that did not look comparable.

Example: temperature can be compared with energy. The constant k_B is the conversion factor representing an important step in our understanding of thermodynamics.

Once the physics is clear, k_B can be set to 1, FAPP.

Preamble

Basically all the theories we use can be formulated, both at the classical and at the quantum level, in terms of an action principle and a Lagrangian. Alternatively, we can use an Hamiltonian language or just "equations of motion"

For clarity I will use the first of these

$$S = \int dt \; L(q_i(t), \dot{q}_i(t), \ldots)$$

Classical: normalization of S is irrelevant

Quantum: the pure number S/h matters.

Redefinitions: physically irrelevant for both

Outline

PART 1

- Classical (non)relativistic point-particles
- Quantum relativistic point particles
- Classical and quantum electromagnetism
- Classical and quantum gravity

PART 2

- Classical Strings have only c
- Quantum Strings have c and l_s
- String's low energy effective theory and dimensionless constants.

PART 1

Classical non-relativistic point-particles

A system of NR interacting point-particles

$$S_{NRPP} = \int dt \left[\frac{1}{2} \sum_i m_i \left(\frac{dX_i}{dt} \right)^2 - V(X_i) \right]$$

V=0: Only mass ratios are relevant.

V ≠ 0: We can use m_1 as unit of mass.. but why not m_2, or a cc of water at such and such T?

Units of space, time, velocity largely arbitrary (e.g. solar year, earth diameter,...)

Classical relativistic point-particles

A system of rel. interacting point-particles

$$S_{RPP} = -\sum_i m_i c^2 \int dt \sqrt{1 - \frac{1}{c^2}\left(\frac{dX_i}{dt}\right)^2} - \int dt V(X_i)$$

Much as before: units of space, time, largely arbitrary (e.g. solar year, earth diameter,...) but a fundamental unit of velocity has emerged: c as an absolute upper limit to v!

Is c like k_B? I tend to believe that it is not...

We can use units where c=1 but we cannot say that space is time in disguise...

Quantum point-particles

Introducing Planck's constant the relevant quantity (in the relativistic case) is:

$$\frac{1}{\hbar}S_{RPP} = -\sum_i \omega_i \int dt \sqrt{1 - \frac{1}{c^2}\left(\frac{dX_i}{dt}\right)^2} - \frac{1}{\hbar}\int dt V(X_i) \quad ; \quad \omega_i \equiv \frac{m_i c^2}{\hbar}$$

We could now use ω_1 to define a unit of time but then why not ω_2? Better use h to define a unit of action (or angular momentum).

c & h are basically the units used by particle physicists together with a reference energy (or mass/length/time), the eV (from ElMag!)

Classical Electromagnetism

Are other fundamental units (besides c =1) needed? Consider the Maxwell action (dropping pure numbers and indices):

$$S_{CED} = -\epsilon_0 \int d^4x F^2 - q \int dx A \quad ; \quad F = dA$$

$$[qA] = [p] \quad ; \quad [\epsilon_0] = [q]^2 [pl]^{-1} \ \Rightarrow [q]^2 = [\epsilon_0][pl]$$

Define new A & q by: $\tilde{A} \equiv \sqrt{\epsilon_0} A \ ; \ \tilde{q} = q/\sqrt{\epsilon_0}$

$$S_{CED} = - \int d^4x \tilde{F}^2 - \tilde{q} \int dx \tilde{A} \quad ; \quad \tilde{F} = d\tilde{A}$$

$$[\tilde{A}] = [p]^{\frac{1}{2}} [l]^{-\frac{1}{2}} \quad ; \quad [\tilde{q}] = [pl]^{\frac{1}{2}} \quad ; \quad [\tilde{F}^2] = [p][l]^{-3}$$

NB: only mechanical units!

$$S_{CED} = -\int d^4x \tilde{F}^2 - \tilde{q}\int dx \tilde{A} \quad ; \quad \tilde{F} = d\tilde{A}$$

$$[\tilde{A}] = [p]^{\frac{1}{2}}[l]^{-\frac{1}{2}} \quad ; \quad [\tilde{q}] = [pl]^{\frac{1}{2}} \quad ; \quad [\tilde{F}^2] = [p][l]^{-3}$$

Quantum Electromagnetism

Classically we can rescale the action

Rather than an arbitrary rescaling consider the one relevant for quantization:

$$S_{QED} = \frac{1}{\hbar}S_{CED} = -\int d^4x \hat{F}^2 - \hat{q}\int dx \hat{A} \; ; \; [\hat{A}] = [l]^{-1} \; ; \; [\hat{q}] \sim \sqrt{\alpha}$$

$$\hat{A} = \hbar^{-1/2}\tilde{A} = \hat{q} = \hbar^{-1/2}\tilde{q}$$

Theorists sometimes prefer to include q in the def. of A and write the QED action in the form

$$S_{QED} = -\frac{1}{\alpha}\int d^4x \bar{F}^2 - \int dx \bar{A} \quad ; \quad \bar{A} \equiv \hat{q}\hat{A}$$

Closer to gravity and string theory cases and also somehow to original (ε_0 ->1/α)

Only c and h units needed, no new fundamental unit of length has emerged (A is a non-universal inverse length, can provide eV...)

Classical Gravity

$$S_{CGR} = -\frac{1}{G}\int d^4x\sqrt{-g}\,(R+2\Lambda) - \sum_i m_i \int d\tau\sqrt{-\dot{X}_i^{\mu}\dot{X}_i^{\nu}g_{\mu\nu}(X_i(\tau))}$$

$$[G] = [l][m]^{-1}$$ and $g_{\mu\nu}$ is dimensionless

In GR it is convenient to rescale the action by an overall factor G. This amount to redefine masses/energies transforming them into lengths (c=1), the gravitational radii R_g associated with them.

The dimensionless ratio of R_g and a physical size is extremely relevant (e.g. distinguishes a normal star from a BH). Yet CGR (w/ $\Lambda = 0$) has no fundamental mass or length scale. Is G a conversion factor like k_B?

$$S_{CGR} = -\frac{1}{G} \int d^4x \sqrt{-g}\, (R + 2\Lambda) - \sum_i m_i \int d\tau \sqrt{-\dot{X}_i^\mu \dot{X}_i^\nu g_{\mu\nu}(X_i(\tau))}$$

$[G] = [l][m]^{-1}$ and $g_{\mu\nu}$ is dimensionless

With Λ GR acquires a length scale ($[\Lambda] = [l]^{-2}$). But this could hardly satisfy (at least at present) Weinberg's criterion for being called fundamental (we don't even know whether it's the correct explanation for dark energy!)

Quantum Gravity

If we proceed as in QED, in Q-Gravity only the combination $Gh = l_P^2$ appears in S/h.

Quantum gravity, even for $\Lambda = 0$, has a fundamental length scale.

Comparing a grav. radius to l_P is of substance.

In the quantum-particle discussion we associated masses w/ inverse lengths (m -> m/h). This remains so if we adopt GR units of mass (m->Gm). Then l_P^2 plays the role of h & the wavelength associated with a mass m is just l_P^2/Gm.

We can now compare many length scales: size R, grav. radius R_g, wavelength λ, Λ, and l_P.
We may thus distinguish particles and stars from black holes, classical from quantum BHs etc.

Nothing would be lost of classical or quantum physics by having replaced E w/ GE and h w/ l_P^2.

Only a small problem: there is no known consistent way to quantize GR (UV-divergences)

But there is a quantum theory of gravity (and other interactions) known as String Theory. What happens there to FCs?

PART 2

Classical Strings:

The action of fundamental relativistic strings is:

$$S_{CST} = T \int d\sigma d\tau \sqrt{-(\dot{X}^\mu \dot{X}^\nu g_{\mu\nu})(X'^\rho X'^\sigma g_{\rho\sigma}) + (\dot{X}^\mu X'^\nu g_{\mu\nu})^2} \equiv T \int d(Area)$$

Note the analogy/natural extension of the point-particle part of the CGR action:

$$S_{CGR} = -\frac{1}{G} \int d^4x \sqrt{-g}\, (R + 2\Lambda) - \sum_i m_i \int d\tau \sqrt{-\dot{X}_i^\mu \dot{X}_i^\nu g_{\mu\nu}(X_i(\tau))}$$

and the fact that T (the string tension) has the same dimensions as those of 1/G.

$$S_{CST} = T \int d\sigma d\tau \sqrt{(\dot{X}^\mu \dot{X}^\nu g_{\mu\nu})(X'^\mu X'^\nu g_{\mu\nu}) - (\dot{X}^\mu X'^\nu g_{\mu\nu})^2} \equiv T \int d(Area)$$

$$S_{CGR} = -\frac{1}{G} \int d^4x \sqrt{-g}\ (R + 2\Lambda) - \sum_i m_i \int d\tau \sqrt{-\dot{X}_i^\mu \dot{X}_i^\nu g_{\mu\nu}(X_i(\tau))}$$

But there are important differences:

1. There is no sum over species (only a single string and a single T)
2. There is no 4-volume term. Yet more analogies:

Classically we can rescale S_{CST} by a factor $1/T$ and express masses/energies in length units. Giving the length of a string is like giving its mass! But, as in CGR, there is no fundamental unit of length in CST! There is only c as unit of velocity as in CRPPs.

Can we say that, in CST, T is just a conversion factor like k_B? It looks more like G...

If a string of a given mass/length moves in a non-trivial geometry then the ratio of its size and the characteristic scale of the geometry (e.g. the Hubble radius in cosmology) does matter.
Yet there is no single length scale worth defining a fundamental unit.

What about the gravitational radius of a string and its relation with its proper size? This question we can only answer after having found where is G in string theory (see below).

Quantum Strings

At the quantum level $S_{CST}/h \sim Area/l_s^2$ $\quad l_s^2 = \hbar/T$
contains a fundamental unit of length (Cf. $l_P^2 = Gh$)

This length scale is ubiquitous in QST:

- Is the typical size of a light quantum string
- It's a typical mass (in 1/T units) of an excited string (there is also a massless sector)
- It's, up to a (half) integer ≤ 2, the angular momentum (in length x mass/T units) of the massless strings (a quantum miracle)
- It's the minimal/typical size of extra dimensions
- It's the minimal size of a stringy black hole
- Last but not least: it's QST's UV cutoff!

As in quantum gravity if we measure masses/ energies in length units the natural dimensions of Planck's constant are those of an area.

$$l_s^2 = \hbar/T \quad ; \quad l_P^2 = \hbar G$$

The big difference is that QST is supposedly a complete theory of all particles and interactions (even though it's far from clear whether it is a realistic theory).

It is certainly a quantum theory of gravity avoiding the UV problems of QGR. As already mentioned, unlike l_P in QGR, l_s plays the role of a finite UV cutoff (nice analogy with $G_F^{-1/2}$ and M_W in SM)

String theory's effective action and its (dimensionless) constants

One can work out an effective action of QST describing the interactions of the lightest (in first approx. massless) strings at low energy (i.e much below the string scale).

It has the typical form of a QFT eff. action (i.e. includes quantum corrections) but differs from it in a number of ways:

- It has higher-derivative corrections (times the appropriate power of l_s) that become important as one approaches the string scale
- These modify the field theoretic short distance behavior and eliminate UV divergences
- It contains no adjustable dimensionless parameters. These are replaced by scalar fields, called moduli, which are often massless in perturbation theory but are hopefully fixed at the end.
- α is one of them, $l_P/l_s \sim (GT)^{1/2}$ is another, actually related one. This gives: $l_s \sim 10\ l_P$

- It includes extra dimensions (basically 6) which can be compact. Their sizes in string units are themselves moduli (frozen at l_s ?).
- It allows for space-time dep. constants (on which we have often strong bounds)
- Should the moduli acquire a potential these dimensionless constants will be dynamically fixed and the corresponding moduli will be massive
- In the opposite case some dimensionless constants will be arbitrary but then the corresponding moduli will be massless and mediate new long range interactions threatening the Equivalence Principle (very well tested UFF).

Conclusion

Since we have no idea whether a particular solution of QST describes the real world we should not take what I have said too literally.

It represents, however, what could happen to fundamental units if we had a good finite quantum relativistic theory of all elementary particles and fundamental interactions.

I believe that, in such a theory, there will be room for the limiting speed c associated with relativity and for a fundamental length(time) L(L/c) associated with quantum mechanics providing, like Planck's h in 1900, a high-frequency cutoff.

THANK YOU