Convolution in Metrology
3 case studies not covered by the GUM

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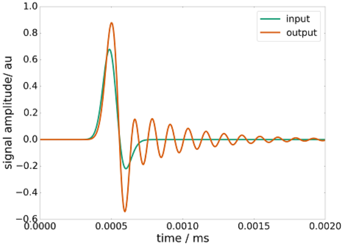
Problem setting



A quantity is called *dynamic* if its variation as a function of time must be accounted for explicitly for its intended use.

A measurement is called *dynamic* when at least one of the involved quantities is dynamic.

Dynamic Metrology

accelerometers **drop weight** **shock tube** **flow** **periodic force** **shock force** **torque** **sampling oscilloscopes** **power grids** **spectrometry** **gas grids**

Problem setting

Measurement model

Convolution

$$Y(t) = \int_{-\infty}^{t} h(t-\tau) X(\tau) \mathrm{d}\tau$$

measurement

measurand

How to obtain an estimate of the measurand?

$\rightarrow$ de-convolution

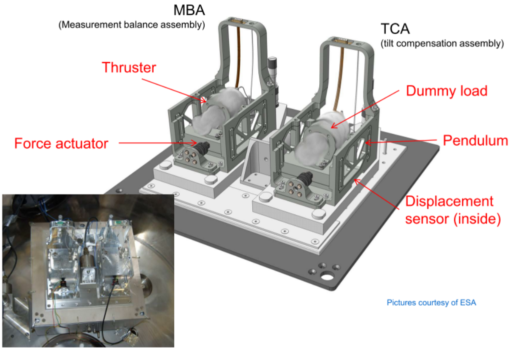
1. Introduction

2. Case studies

3. Conclusions

Case study: thrust balance

- micro-thrusters operate in range from 0.1 μN up to 500 mN
- applied for spacecraft altitude control, drag compensation, etc.
- calibration measurements very much affected by environmental noise



Case study: thrust balance

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- applied for spacecraft altitude control, drag compensation, etc.
- calibration measurements very much affected by environmental noise

MBA	thrust force + environmental noise
TCA	environmental noise only

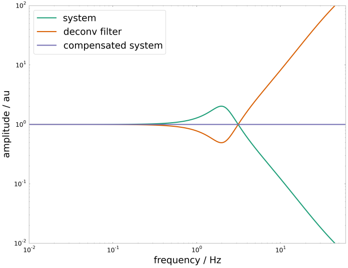
Method

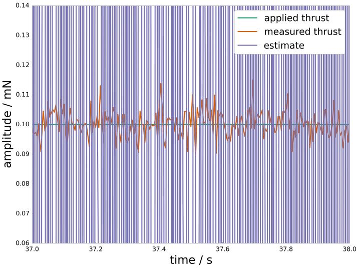
1. deconvolve TCA response to noise ($\rightarrow$ 'true' noise)
2. deconvolve MBA response
3. remove noise in deconvolved MBA response

Case study: thrust balance

deconvolution approach

1. determine system frequency response from calibration measurements
2. fit a digital filter to its reciprocal
3. apply filter to the output signal

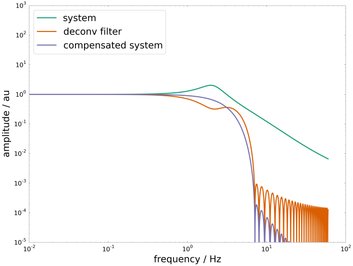


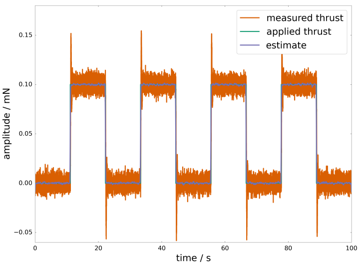


Case study: thrust balance

deconvolution approach

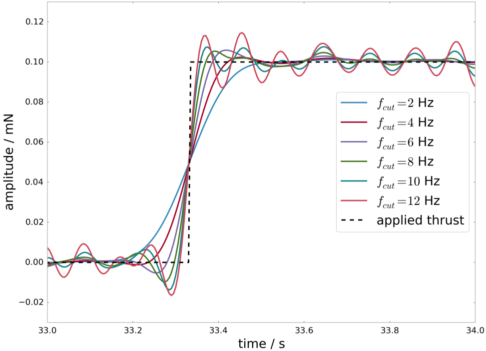
1. determine system frequency response from calibration measurements
2. fit a digital filter to its reciprocal
3. apply filter to the output signal
4. apply additional low-pass filter





Case study: thrust balance

lowpass filter cut-off frequency



What is the uncertainty contribution of the induced systematic error?

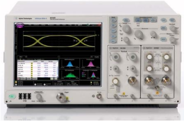
The answer is informed by prior knowledge about the measurand.

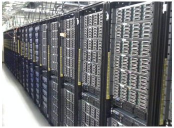
Case study: sampling oscilloscope

Electronics

Fiber optics

Test and Measurement





Data storage
and
Computing

Wireless communications

Internet applications

Case study: sampling oscilloscope

Necessity of dynamic calibration

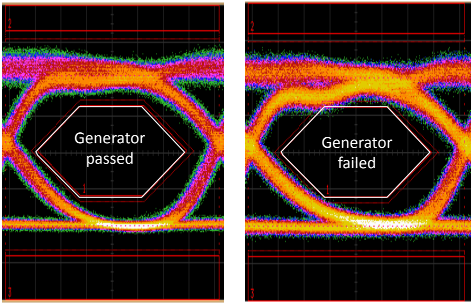


Figure: Eye diagram for characterizing electric signals
same device under test – different oscilloscopes

Case study: sampling oscilloscope

deconvolution approach

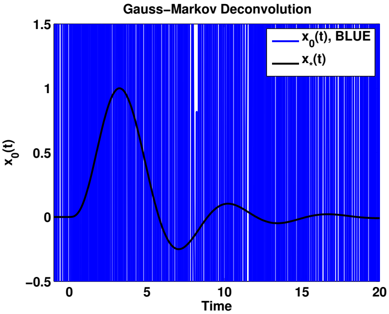
1. determine system impulse response from calibration measurements
2. calculate convolution matrix
3. solve linear estimation problem

measurement model

$$\mathbf{y} = \mathbf{H}\mathbf{x}_* + \mathbf{n} \qquad \mathbf{n} \sim N(0, \boldsymbol{\Sigma}_n)$$

best linear unbiased estimator (BLUE)

$$\mathbf{x}_0 = \left(\mathbf{H}^*\boldsymbol{\Sigma}_n^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^*\boldsymbol{\Sigma}_n^{-1}\mathbf{y}$$



Case study: sampling oscilloscope

deconvolution approach

1. determine system impulse response from calibration measurements
2. calculate convolution matrix
3. solve regularized linear estimation problem

$$\mathbf{x}_\lambda = \left(\mathbf{H}^*\mathbf{H} + \lambda^2\mathbf{L}^*\mathbf{L}\right)^{-1}\mathbf{H}^*\mathbf{y}$$

interpretation as generalized filter

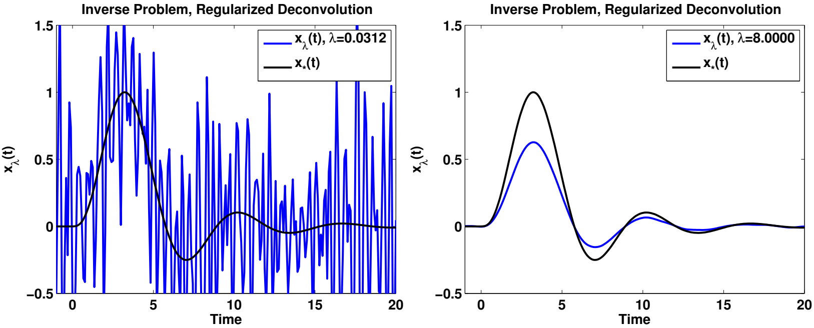
$$\mathbf{x}_\lambda = \mathbf{R}_\lambda \mathbf{H}^{-1}\mathbf{y}$$

$$\mathbf{R}_\lambda = \left(\mathbf{H}^*\mathbf{H} + \lambda^2\mathbf{L}^*\mathbf{L}\right)^{-1}\mathbf{H}^*\mathbf{H}$$

$\Rightarrow$ parameter λ controls bias-variance trade-off

Case study: sampling oscilloscope

choice of λ

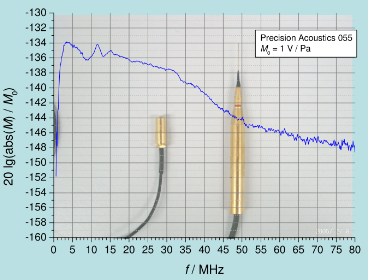


What is the uncertainty contribution of the induced systematic error?

The answer is informed by prior knowledge about the measurand.

Case study: hydrophone

- measurement of pressure generated by medical ultrasound devices
- assessment of pressure peaks to ensure patient safety
- current standards consider quasi-static or approximate dynamic methods

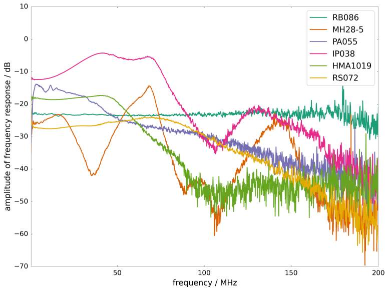


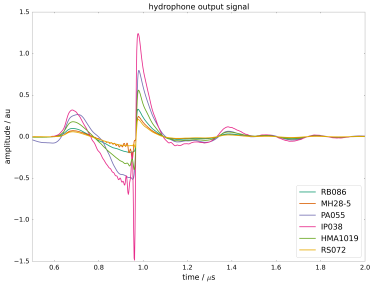
Case study: hydrophone

deconvolution approach

1. determine frequency response from measurements
2. design low-pass filter based on prior knowledge
3. carry out deconvolution in frequency domain

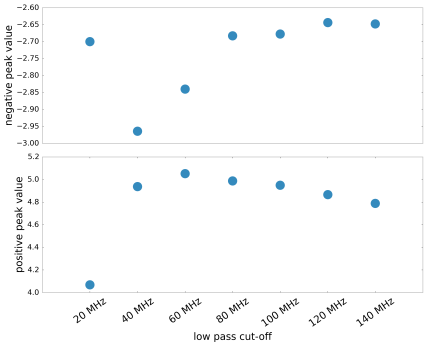
$$X_0(j\omega) = \frac{Y(j\omega)}{H(j\omega)} L(j\omega)$$





Case study: hydrophone

design additional low-pass filter $X_0(j\omega) = \frac{Y(j\omega)}{H(j\omega)} L(j\omega)$



What is the uncertainty contribution of the induced systematic error?

The answer is informed by prior knowledge about the measurand.

Comparison

	frequency	deconvolution approach	regularization approach
thrust balance	$\approx$ 5 Hz	digital filter	visual inspection
hydrophones	$\approx$ 40 MHz	division in freq. domain	prior knowledge
oscilloscopes	$\approx$ 20 GHz	linear estimation model	data dependent

Generic estimation problem

Determine trade-off between reduced variance and increased systematic error.

Generic uncertainty problem

Quantification of the uncertainty contribution of the introduced systematic error requires prior knowledge about the measurand.

Incorporation of prior knowledge not considered by the GUM

Summary

- Dynamic Metrology is emerging field in almost all application areas
- Typical task in Dynamic Metrology is deconvolution
- Different approaches, but similar mathematical challenges
- Deconvolution requires some kind of regularization
- Regularization introduces systematic error
- Uncertainty contribution of regularization requires prior knowledge about the measurand

Some further challenges in Dynamic Metrology

- Estimation of large covariance matrices from measurement
- Transfer of large covariance matrices (calibration certificates)
- Utilization of noise structures other than normal distributed
- Standard software tools for uncertainty evaluation not applicable
- System models often based on differential equations
- Dynamic estimation requires dynamic calibration (new approaches!)
- Classical key comparisons not suitable

Conclusion

Dynamic measurements are becoming increasingly relevant throughout metrology, but there is almost no guidance in the GUM.