System identification and uncertainty analysis for challenging measurement applications: a case study in micro-Newton level force measurement

BIPM workshop on metrology for dynamic measurement

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Outline

- What is dynamic measurement?
  - Definition
  - Features and properties of a dynamic measurement system
  - What's the problem?
  - How to deal with it
- Case study: micro-Newton thrust measurement
  - Requirements
  - Instrument design
  - Testing
- Summary of observations and issues

What is dynamic measurement?

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- Definition:

  A measurement is considered dynamic if the relationship between the measurand and the sensor output is frequency dependent.

- The term dynamic does not apply to all time-varying signals
- Not necessarily limited to rapid rates of change in measurand. e.g. seismometers often have a low-frequency cut-off.

Features and properties

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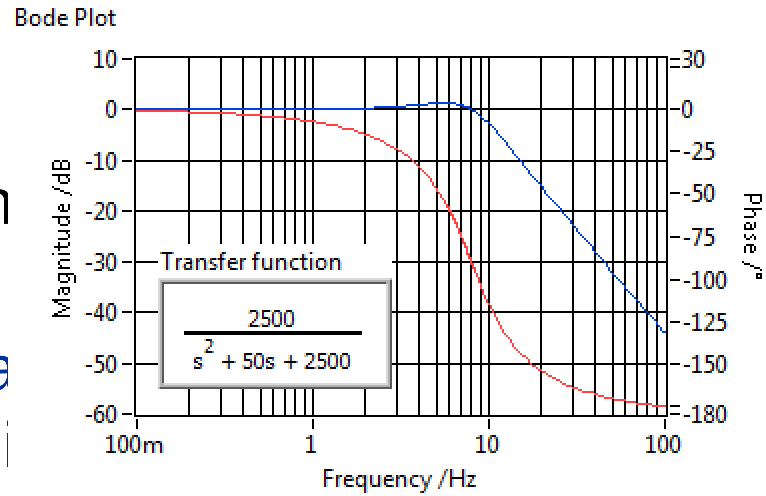
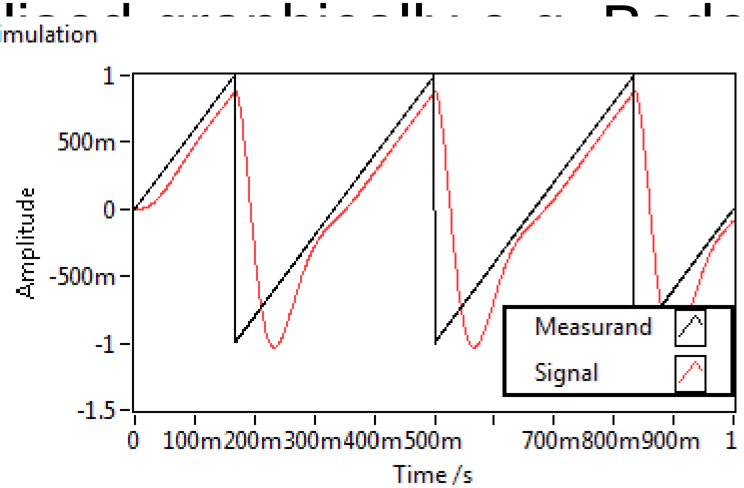


Measuran ... Signal

- Sensor ha ... n the relationshi ... t) and measurand (input)
  - Can be described by a transfer function (equation)
  - Visu ... plot



What's the problem?

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- We want our sensor output signal to represent the measurand as closely as possible.
- The frequency response of the sensor has introduced a **systematic effect**.
- GUM says we must compensate all known **systematic effects** and associate uncertainties with these compensations.
- This requires a calibration process
  - Determine relationship between measurand and output signal
  - Establish correction factor(s)/coefficient(s)
  - Estimate uncertainty associated with corrections

Solution

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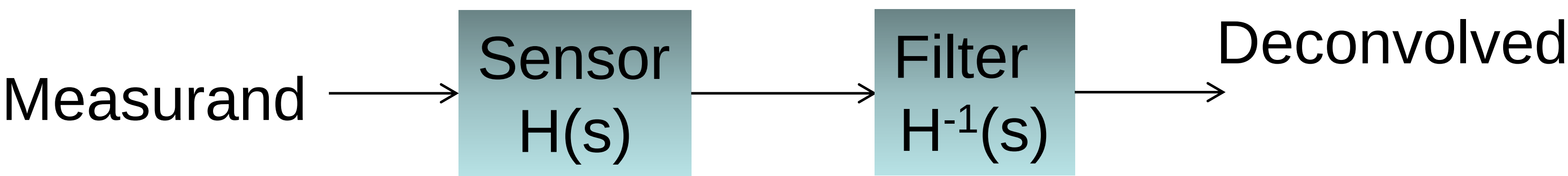

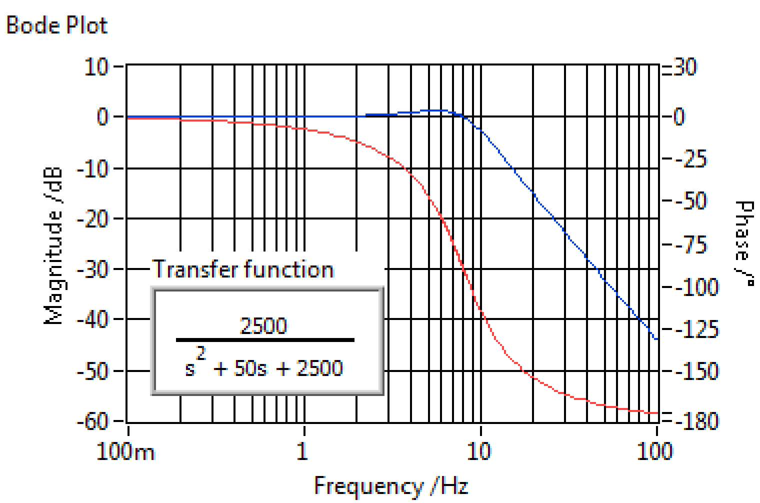

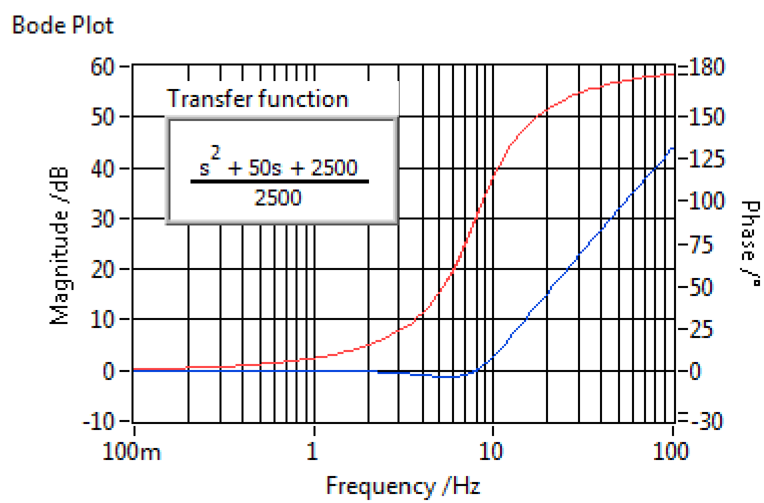

Inverse filter design

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- Use system ID techniques to determine the transfer function of the sensor, H(s).
  - Physical model
  - “Black box” model
  - “Grey box” model
- For Black box or Grey box model, perform experiment to determine model parameters
- Invert the transfer function to obtain the inverse filter, $H^{-1}(s)$.
- But,
  - Inverse filter may be unstable, will amplify noise
    - low-pass filter
  - Uncertainties and errors associated with the model
    - **Validate** the model

Low-pass filter

- Design of filter is an issue:
  - Cut-off frequency
  - Filter order
- Does it matter?
  - Trade-off between error reduction and uncertainty
- Guidance needed

System ID/Model validation tools

- Easy to use software tools available, e.g.
  - LabVIEW control design and simulation toolkit
  - MATLAB system identification toolkit
- Transfer function parameter estimates provided, but not always uncertainty estimates
  - We need uncertainty estimates
  - How do we get them?
- Model validation tools available
  - More guidance on use to these tools is needed
- These tools cover the engineering aspects but the requirements for metrology are lacking

Propagate uncertainties

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- Errors in the model
  - Incomplete i.e. un-modelled effects
- Uncertainties in the model/filter parameters
- How do we propagate these uncertainties (frequency domain) through to the deconvolved signal (time domain)?
  - Monte Carlo?
  - Some other technique?
  - HELP!

A subtlety: Correlation

- Inverse filter is a finite impulse response (FIR) or infinite impulse response (IIR) filter. So all output data samples are inherently **correlated** with all previous samples.
- GUM says we need to take correlations into account.
  - How?
- e.g. Deriving an uncertainty associated with a time averaged output signal needs to take this correlation into account otherwise uncertainty estimates could be wrong!
- Note: the above applies whenever filtered data is averaged.

Summary

- System ID to determine sensor transfer function
  - Uncertainties in model & model parameters
- Derive and apply inverse filter and accompanying LPF
  - Uncertainty in filter parameters
  - Inherent correlation
- Propagating uncertainties through to time domain is an issue.
- When calibrating a dynamic sensor, how/what do we report?
  - Inverse filter coefficients and uncertainties?
  - Include correlation coefficients?
  - On paper or electronically?
  - Guidance on use of calibration data?
  - …

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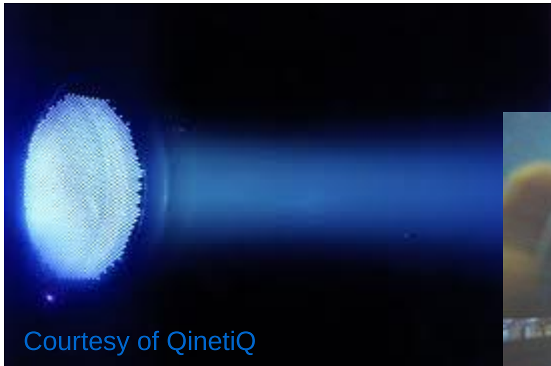
Case study: µN thrust balance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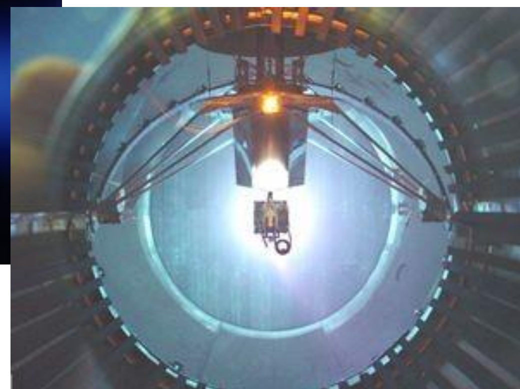


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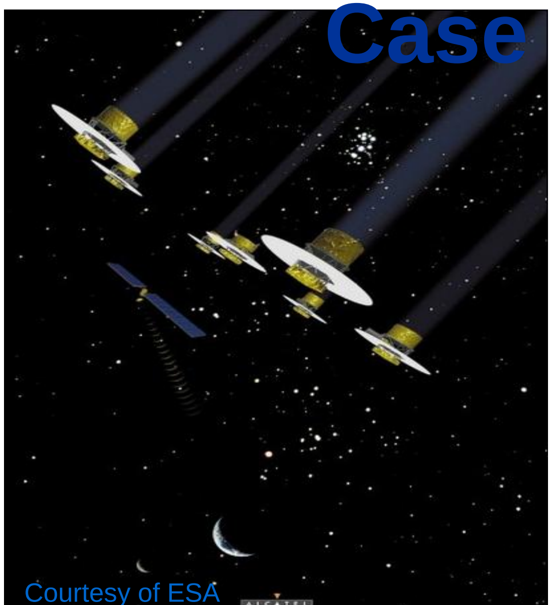

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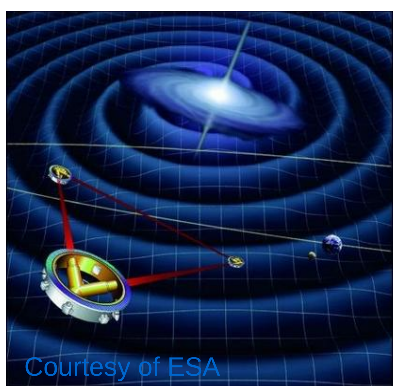
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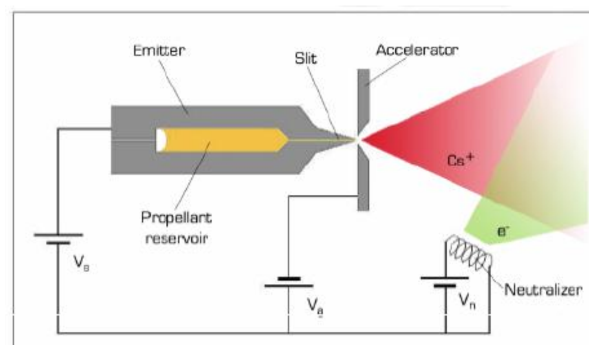

Micro-thrusters

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- Thrusters operating in the range 0.1 μN to 500 mN
- Used by spacecraft, for example, for
  - fine attitude control
  - drag compensation
  - station keeping
  - formation flying
- Thrust generated by accelerating ions or gas/liquid
  - Electric Propulsion
  - Cold gas

Micro-thruster performance requirements

- LISA Pathfinder requires thruster performance:
  - Range 0.3 µN to 150 µN
  - Resolution 0.3 µN
  - Noise below 0.1 µN/√Hz between 10^{-2} Hz and 10 Hz
    1 µN/√Hz between 10^{-3} Hz and 10^{-2} Hz
- **Traceable** measurements with rigorously evaluated **uncertainties** are required to verify thruster performance

NPL/ESA micro-newton thrust balance requirements

- Primarily for cold-gas thrusters
- 0 µN to 500 µN thrust range
- 0 Hz to 10 Hz measurement bandwidth
- Noise floor below 1 µN/√Hz
- Traceable to international standards
- Rigorous uncertainty evaluation, target 1 µN (*k* = 2)

NPL/ESA micro-newton thrust balance requirements

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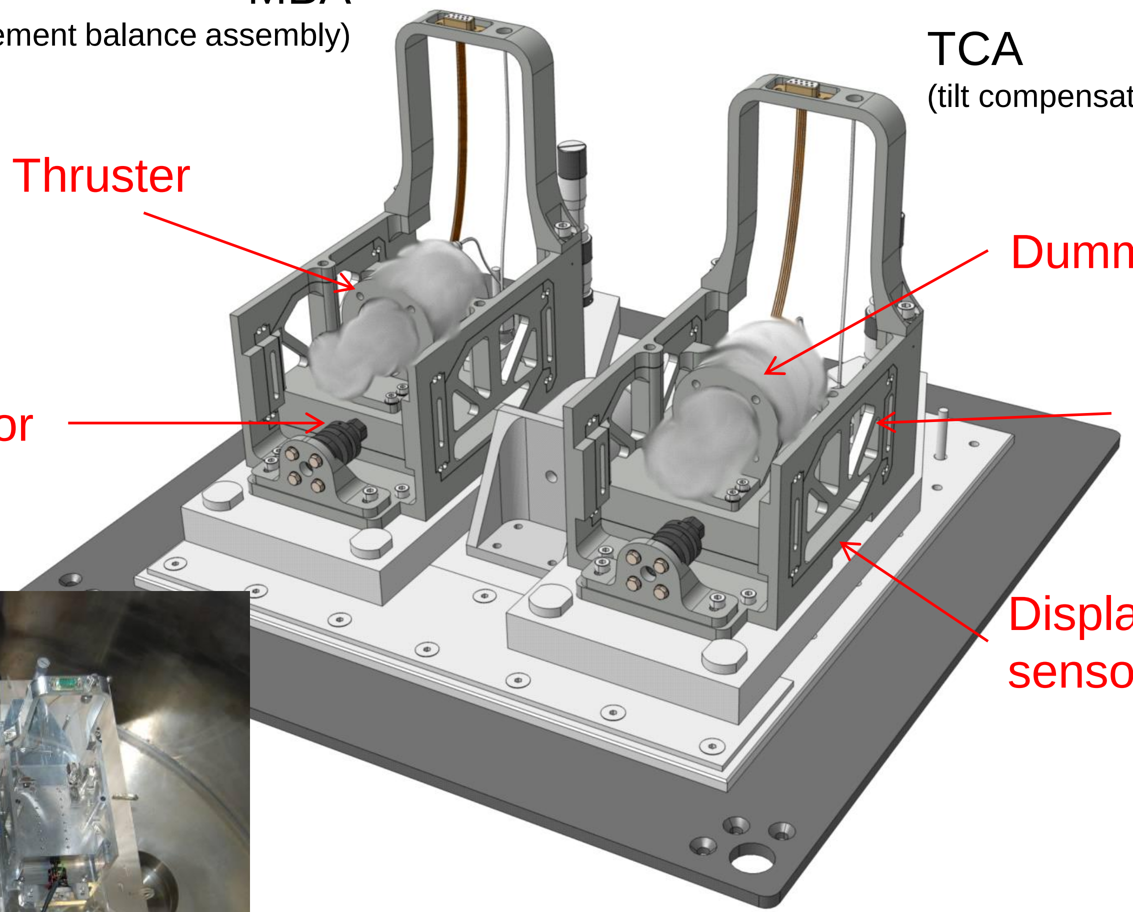
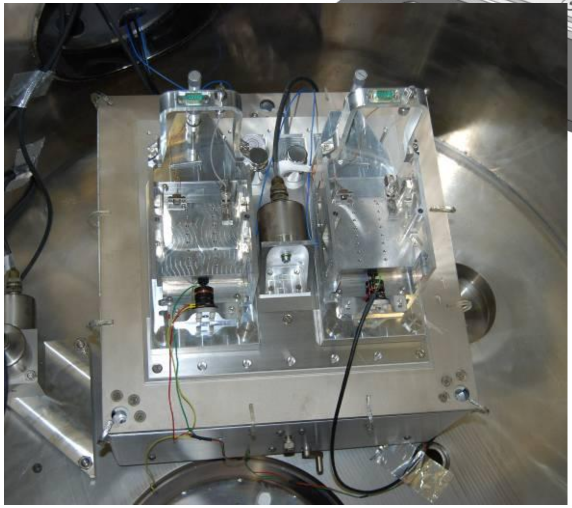
- Current focus is on '**static**' thrust measurement
- Next goal is to measure thruster dynamics…
- … and thrust noise.

Thrust balance design





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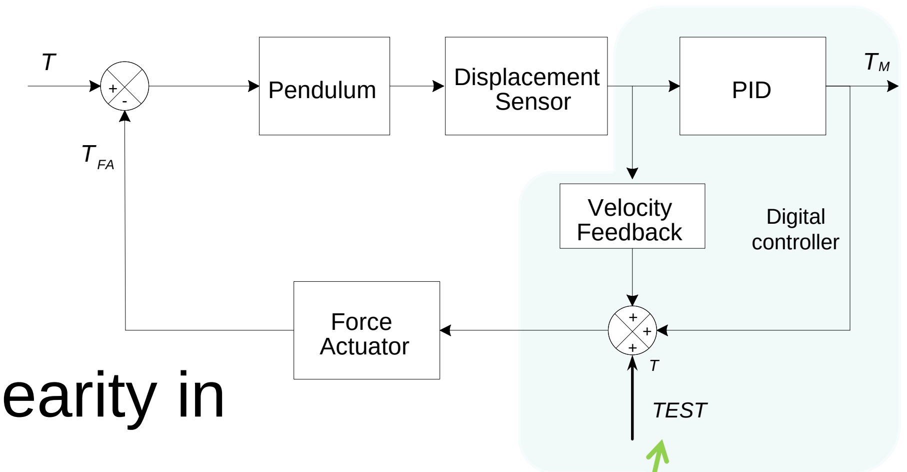
Thrust balance principles

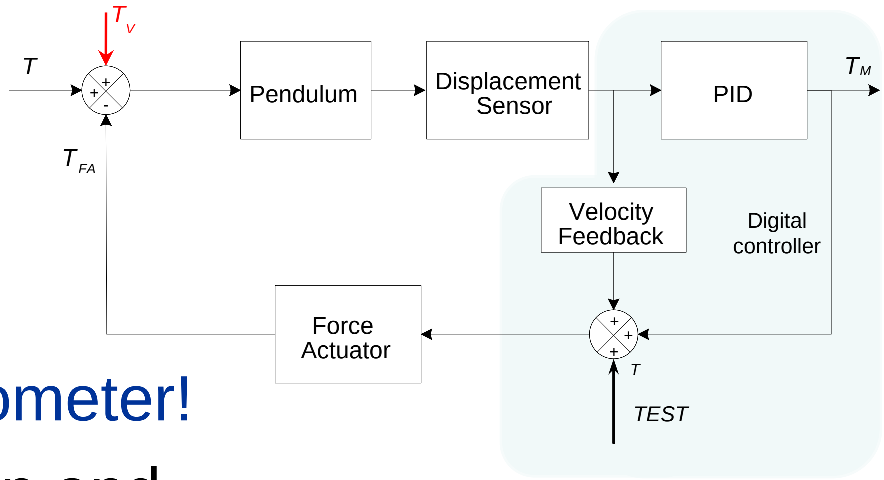


- Pendulum mechanism
- Null-displacement, force feedback control
    - Overcomes non-linearity in mechanism
    - Sensitivity determined by force actuator (in steady-state)
    - Traceability through force actuator calibration
    - **Test input** for system ID and uncertainty verification

Important feature of the design - an accurately known "physical" input.

Thrust balance principles

- Pendulum mechanism
- Null-displacement, force feedback control
- Sounds just like a seismometer!
  - Sensitive to vibration and tilt
  - Use a ‘dummy’ matched pendulum to compensate for the vibration signal



Thrust balance principles

- Pendulum mechanism
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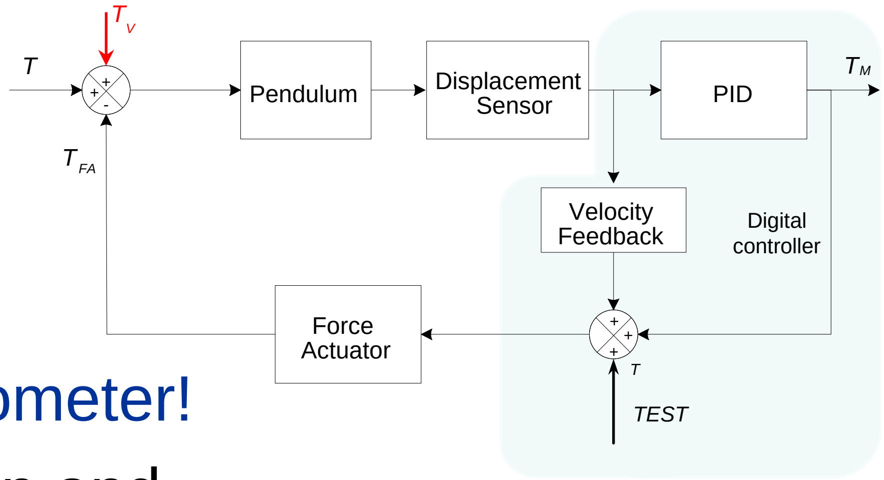

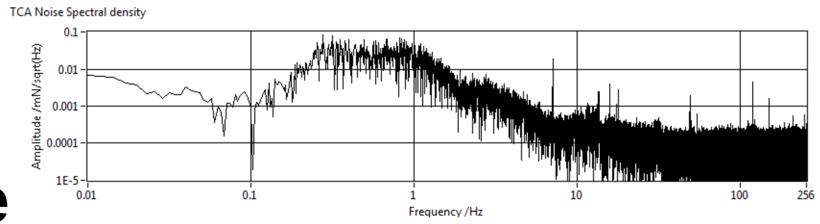

This is a dynamic measurement!

Dynamic compensation:

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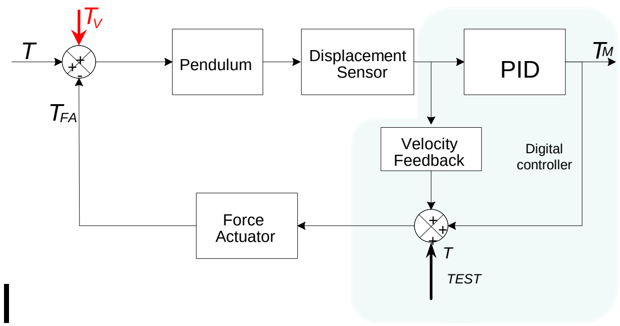
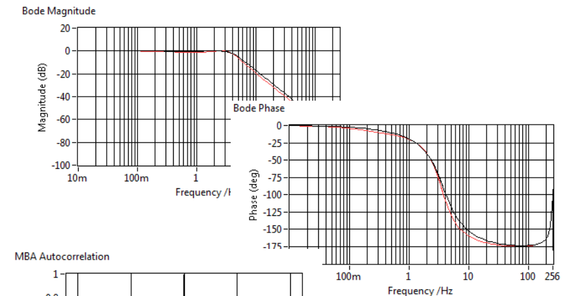
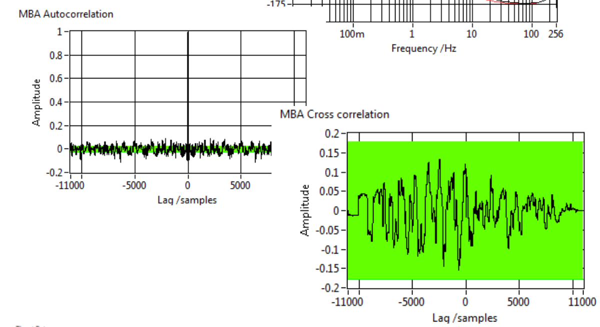
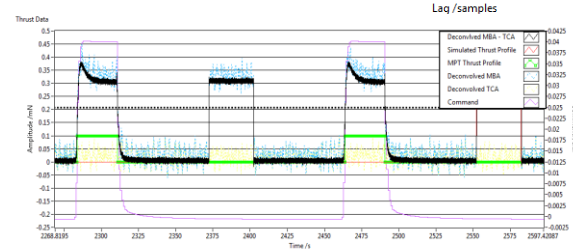
- Derive grey box model from physics
- For each pendulum (MBA and TCA):
  - Perform system ID test
  - Fit grey box model to data to obtain unknown parameters of grey box model
  - Perform validation test on both pendulum models
  - Design inverse filter for each pendulum
- Perform thrust measurements
  - Record thrust/vibration data and apply inverse filters
- Subtract deconvolved TCA signal from deconvolved MBA signal to compensate for vibration



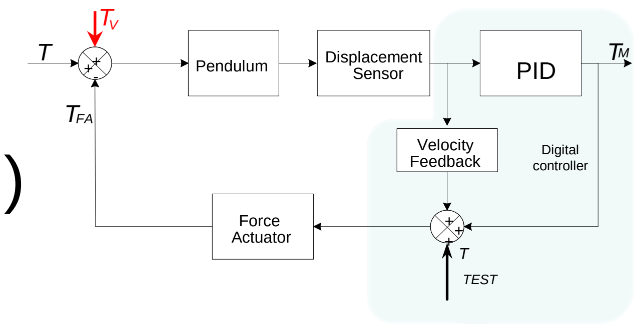






Dynamic compensation:
1. System model

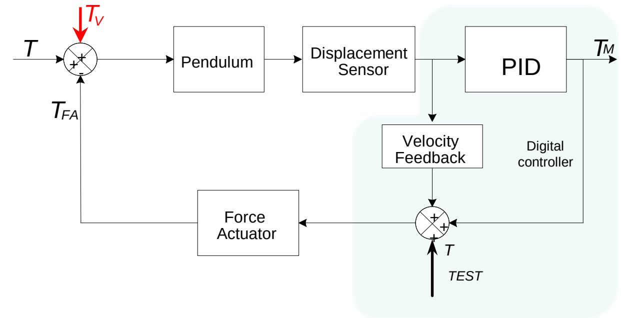
- Grey box model
  - Some parameters knows (green)
  - Some unknown or known within limits (red)

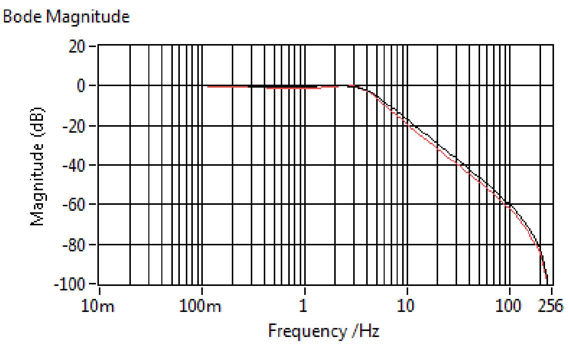


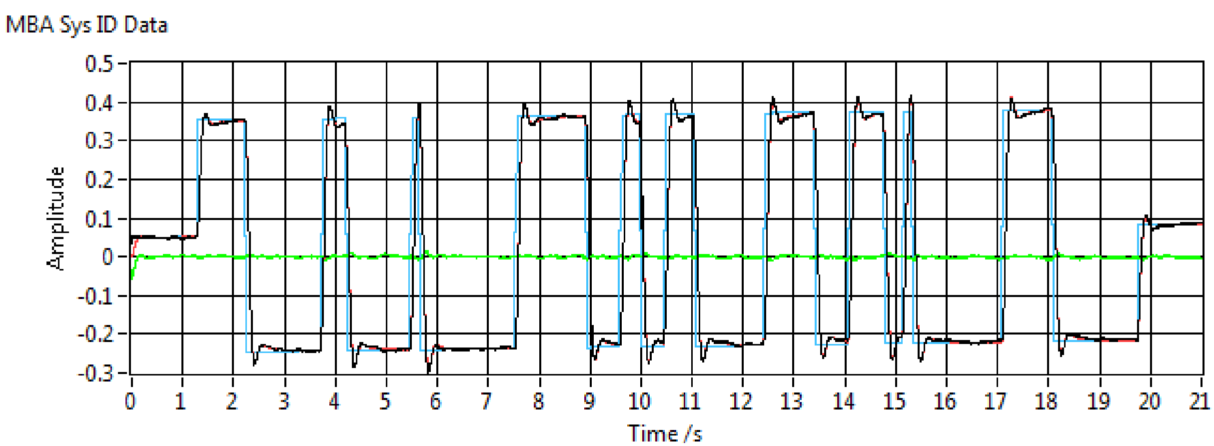
Parameter	Symbol	Units
Measured thrust (balance output)	T_M	µN
Applied thrust (to be determined)	T	µN
Pendulum length	l_p	m
Force actuator sensitivity	k_a	µN/V
Displacement sensor sensitivity	k_p	V/µN
Proportional gain (PID)	k_c	V/V
Velocity feedback gain	k_d	V·s/V
Effective mass of pendulum + load	M_e	kg
Integration time constant (PID)	t_i	s
Derivative time constant (PID)	t_d	s
Angular stiffness of pendulum	k	N·m/rad
Damping coefficient of pendulum	ξ	N·s/m

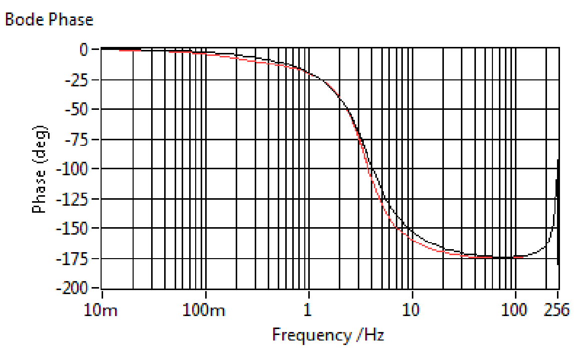
Dynamic compensation: 2. System ID

- Apply known stimulus via the test input
- Record stimulus and response (output)
- Fit grey box model to data.
- But currently, **no uncertainties given**.



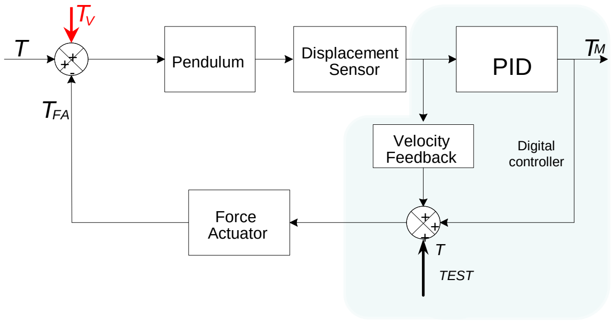


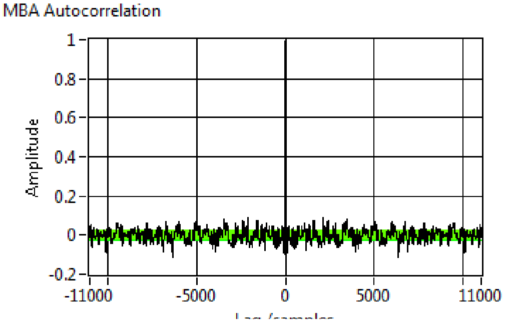


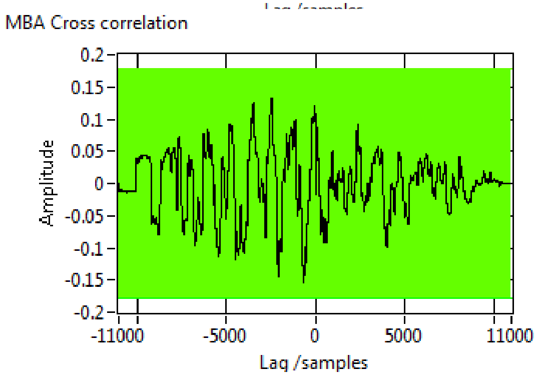


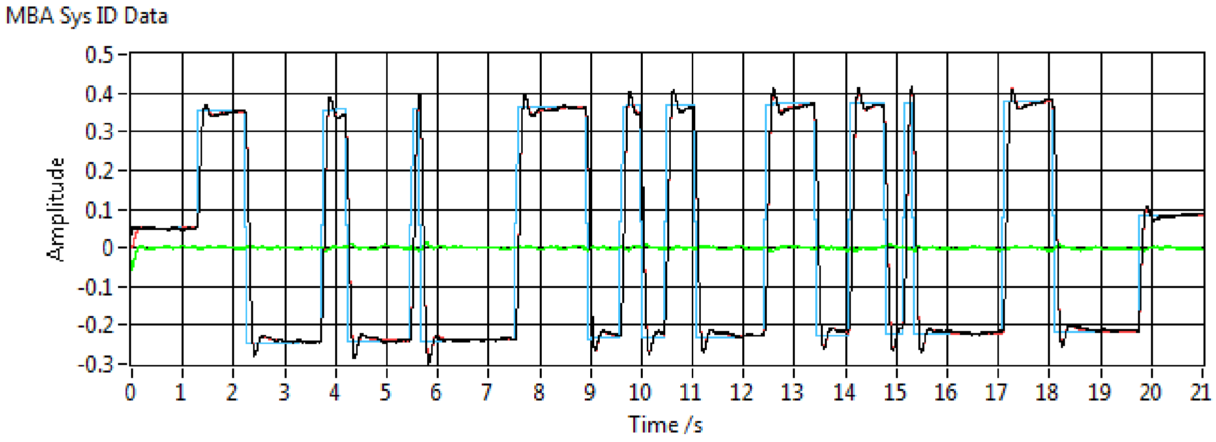
Dynamic compensation:
3. Model verification

- Apply known stimulus via the test
- Record stimulus and response
- Compute prediction error
- Autocorrelation of prediction error
- Cross correlation between prediction error and stimulus



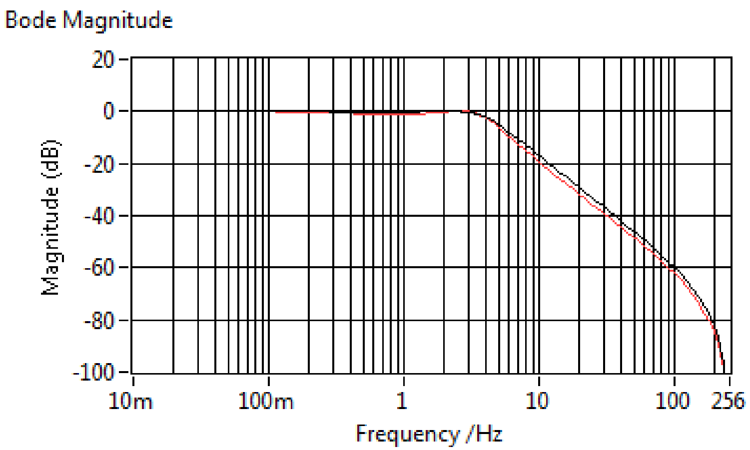


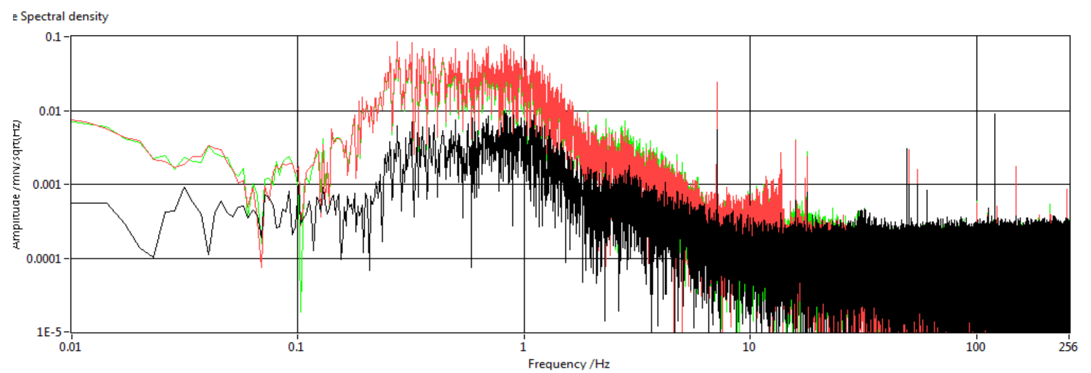


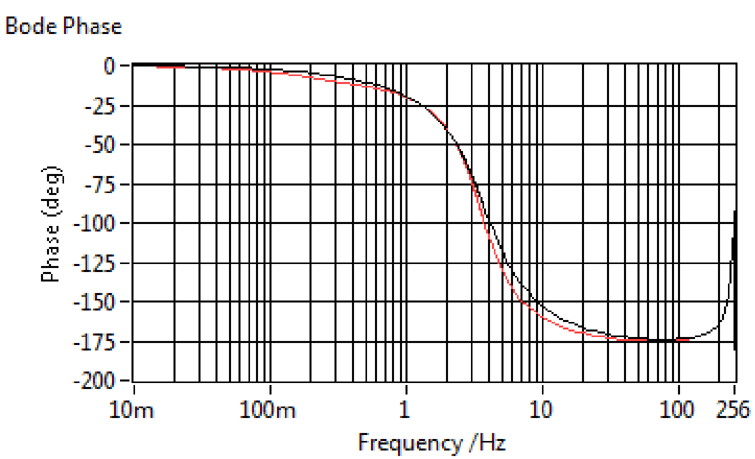


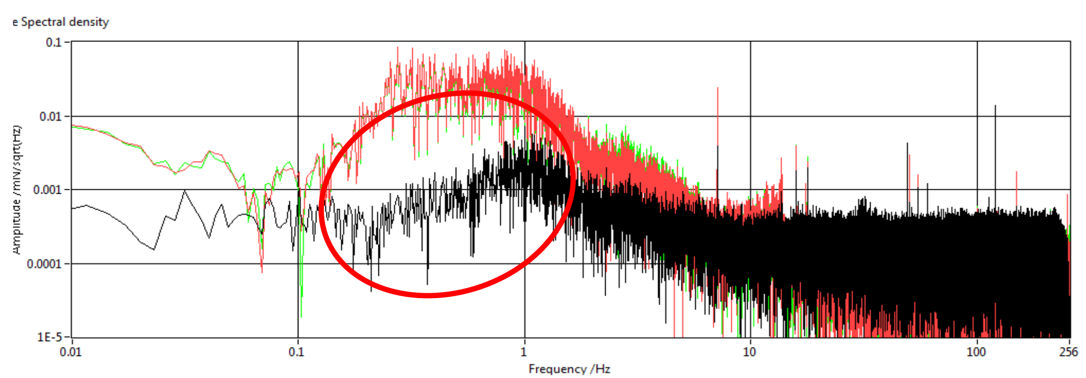
Dynamic compensation:

- Vibration compensation



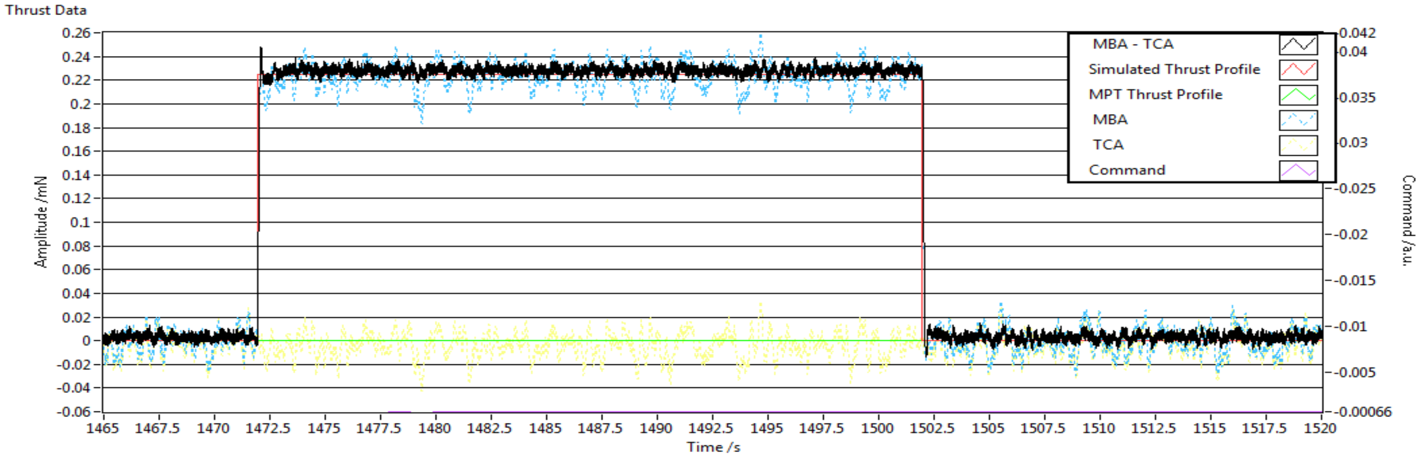


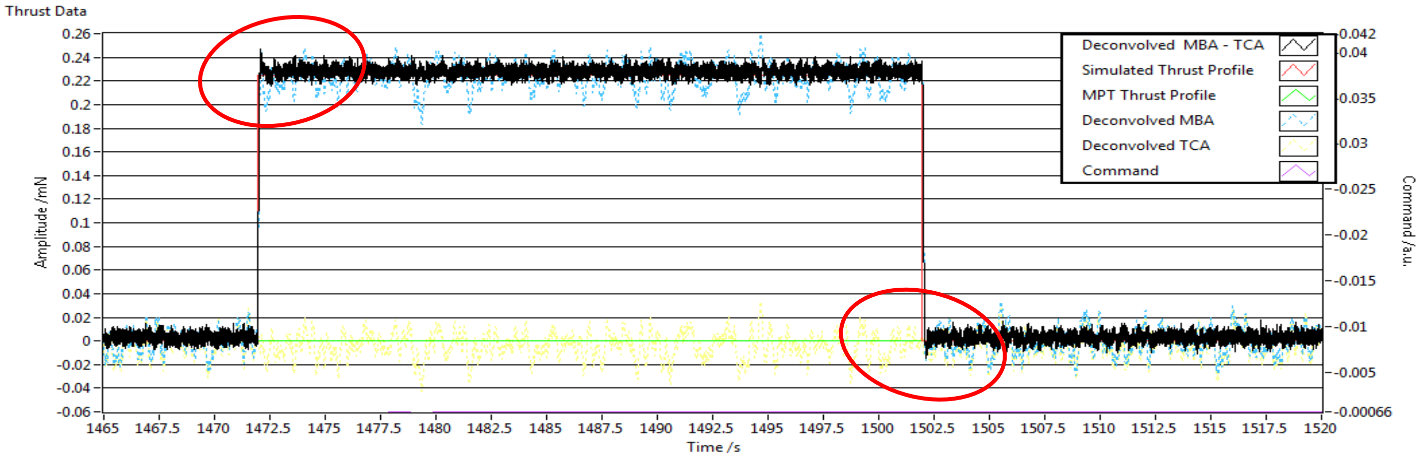




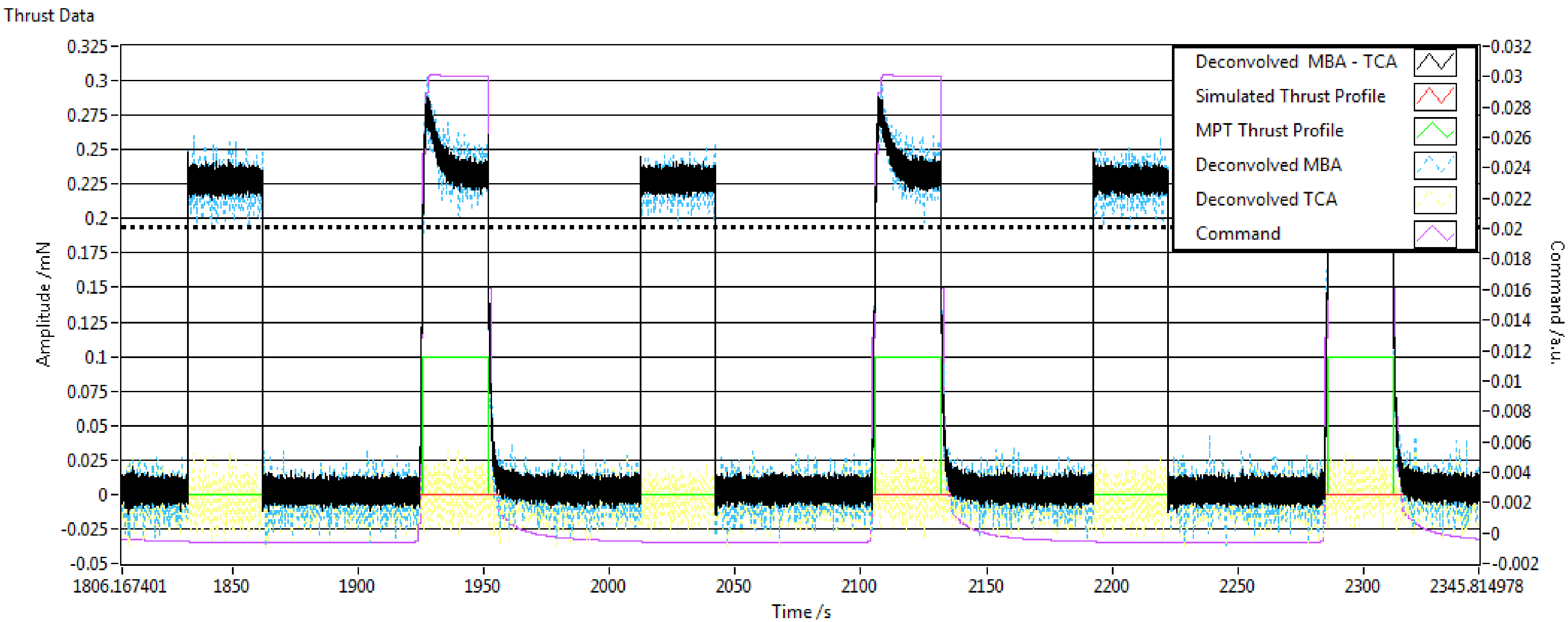
Dynamic compensation:

- TF compensation



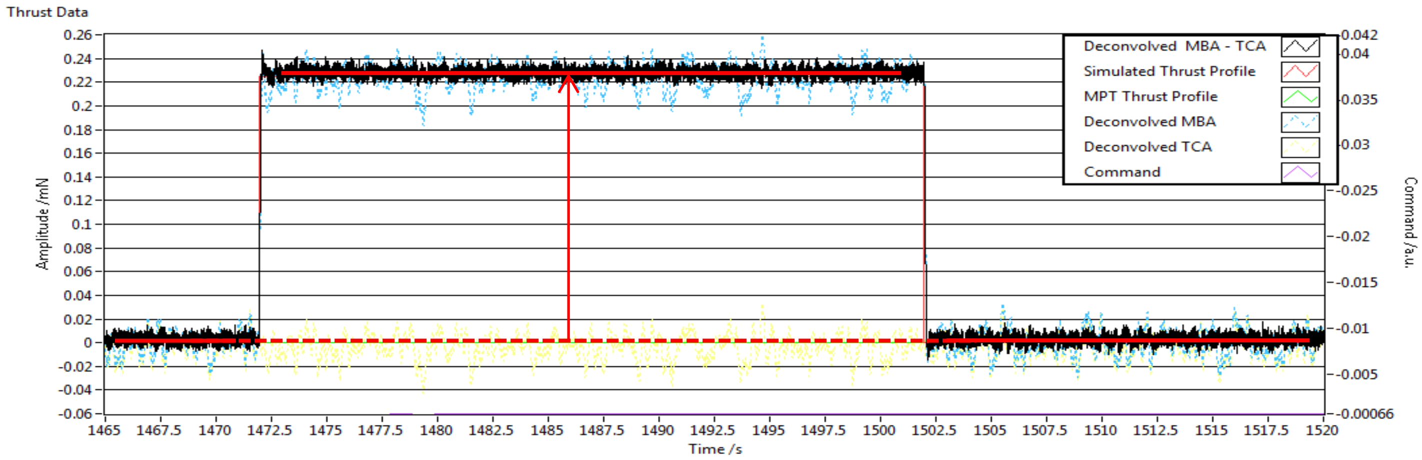


‘Static’ thrust measurement



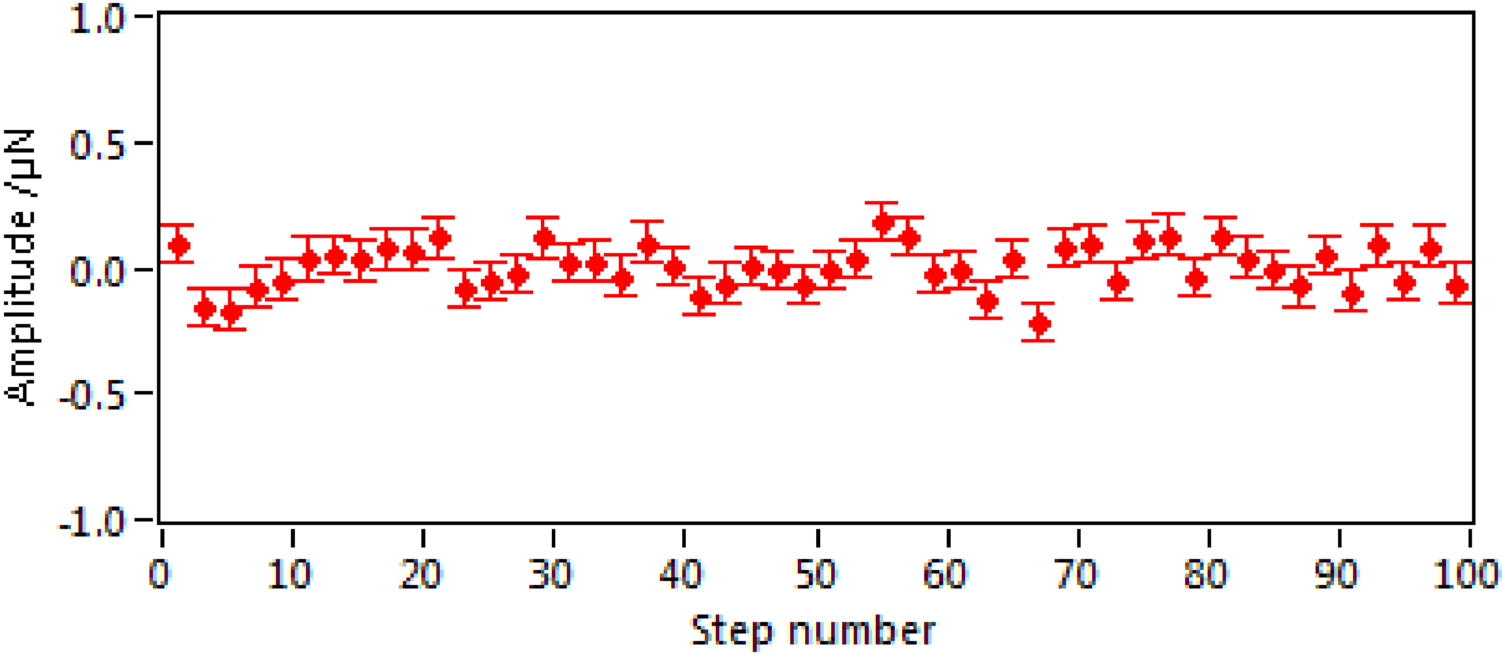
- Interleaved simulated and real thrust steps (deconvolved)
- Need to account for residual noise (after vibration compensation)

Thrust measurement



- Fit parallel lines through data when thrust is OFF and ON.
- Standard least-squares analysis gives us the magnitude of the thrust step (separation of parallel lines) and an uncertainty.

Thrust measurement



- Simulated thrust steps repeated 50 times.
- Plotted as errors (deviation from known thrust).
- Uncertainty shown by error bars plotted at k = 2.
- **Calculated uncertainty is not consistent with observed variation! Why?**

Thrust measurement

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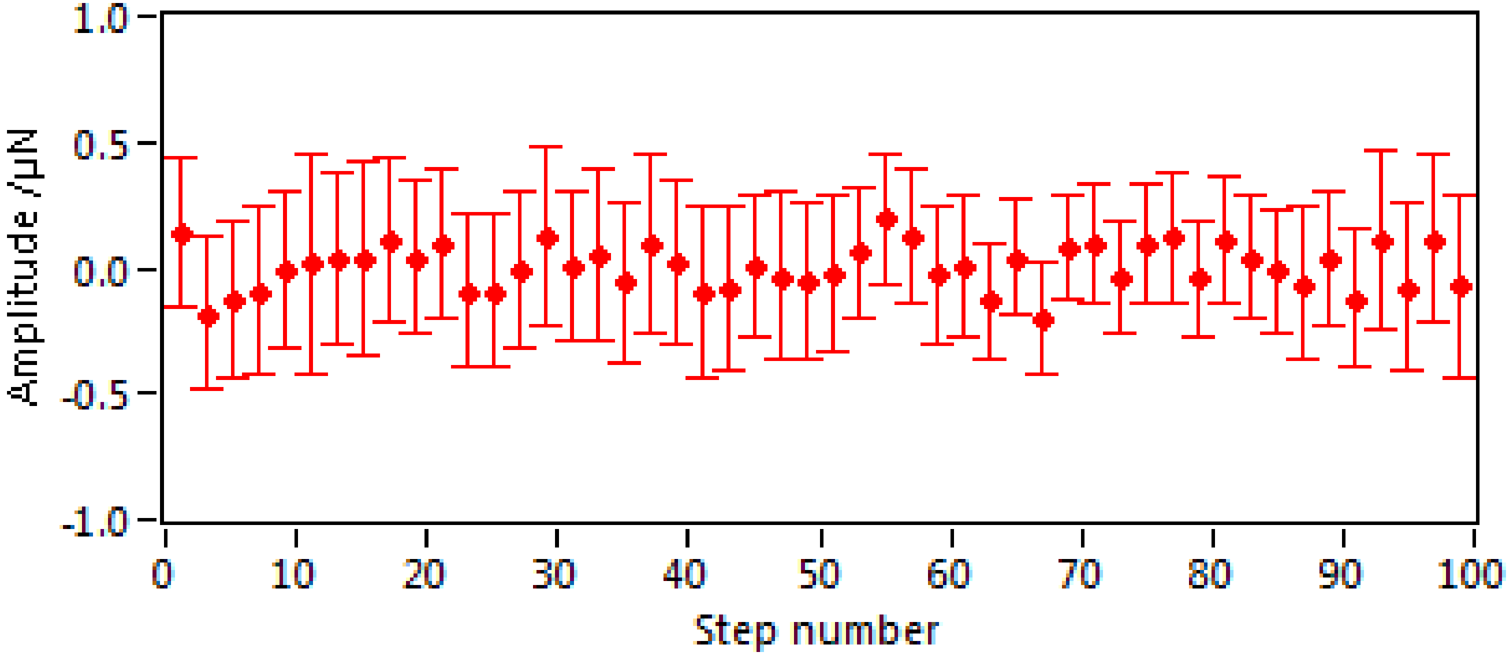
- **Calculated uncertainty is not consistent with observed variation! Why?**
- The standard statistical analysis assumes noise is:
  - White (broadband)
  - Random
  - Stationary
  - Un-correlated
- Remember the inverse filter? This correlates the data.
- Use empirical (AR) model to compensate for correlation

Thrust measurement



- Calculated uncertainties are more consistent with observed variation
- But this is an empirical solution based on our current best guess of the cause of inconsistency
- More work needed…

Next steps

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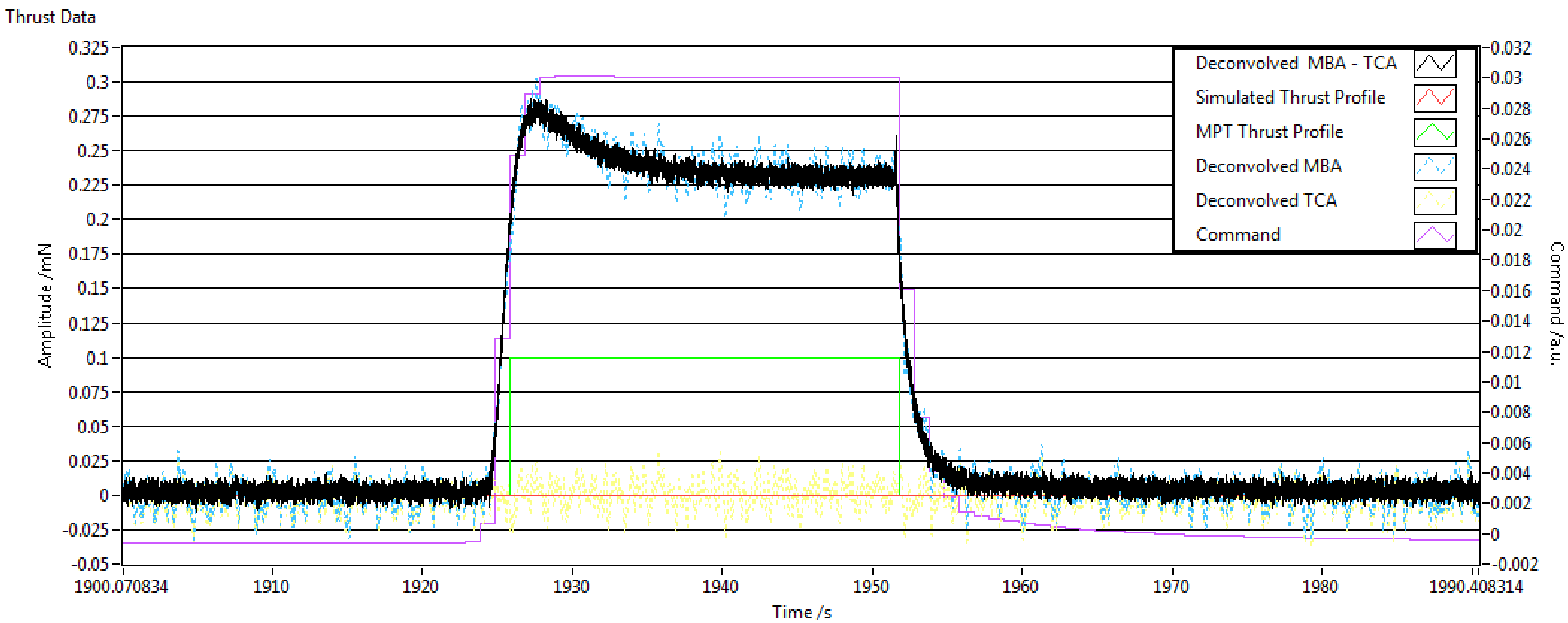
- Thruster dynamics
- Thrust Noise



Summary & Observations

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- GUM does not explicitly cover dynamic measurement
- Dynamic measurement/calibration is a complex process
- Relies on several assumptions that need to be validated, LTI, model, *etc*
- potential pit-falls, correlation
- Software tools exist, e.g. for system ID, but cover engineering aspects not metrology
- Scientific literature on the subject is scarce and needs to be adapted for a wider audience