ISTITUTO NAZIONALE DI RICERCA METROLOGICA

EURAMET Project 870

(EURAMET.L-K3.1)
Final Report

1. Introduction

The metrological equivalence of national measurement standards and of calibration certificates issued by national metrology institutes is established by a set of key comparisons chosen and organized by the Consultative Committees of the CIPM or by the regional metrology organizations in collaboration with the Consultative Committees.
At its meeting in October 2005 the EUROMET TC group for Length has approved a proposal of a comparison of measurements of precision polygons between INRIM (formerly IMGC, Italy), IPQ (Portugal) and LNMC (Latvia), later Metrosert (Estonia) has been added to the comparison.

2. Organisation

2.1 Participants

Laboratory	Address	Contact person /tel/fax/e-mail
INRIM	Istituto Nazionale di Ricerca Metrologica Strada delle cacce, 73 IT-10135 Torino Italy	Marco Pisani +39 011 3919 961 +39 011 3919 959 m.pisani@inrim.it
IPQ	Instituto Português da Qualidade (IPQ) Laboratório Central de Metrologia Rua António Gião, 2 PT-2829-513 Caparica Portugal	Fernanda Saraiva +351 21 294 81 60 +351 21 264 81 88 fsaraiva@mail.ipq.pt
LNMC	Latvian National Metrology Centre 157, K. Valdemara Str. Riga, LV-1013 Latvia	Edite Turka +371 7 362 086 +371 7 362 805 edite.turka@lnmc.lv
Metrosert	AS Metrosert Aru 10 10317 Tallinn Estonia	Lauri Lillepea tel: +372 681 48 10 fax: +372 681 48 18 lauri.lillepea@metrosert.ee

Table 1: *Participating laboratories*

2.2 Schedule

The schedule was designed to fit with the preferences of the laboratories for scheduling the measurements and any changes to the schedule, after the start of the circulation, were discussed and agreed among the participants.

Laboratory	Date of measurements
INRIM	January 2008
IPQ	February 2008
LNMC	March 2008
Metrosert	April 2008
INRIM	May 2008

Table 2: *Time schedule of the comparison as in the protocol*

Unfortunately, because of troubles with IPQ calibration facility the schedule has been changed a couple of times. This caused the end of the measurements to be shifted by 7 months. The following table is the final circulation schedule.

Laboratory	Date of measurements	Results received – date of certificate
INRIM		
Metrosert	January 2008	15.05.2008
LNMC	Received on 17.03.2008	25.06.2008
IPQ	28.04.2008-12.05.2008 (6-sided)	
INRIM	June-July 2008	
IPQ	19.11-2008-27.11.2008 (12-sided)	19.12.2008
INRIM	December 2008	

Table 3: *The real time schedule of the comparison*

3. Standards

The artifacts to be measured consisted of two polygons: a 6-sided polygon (INRIM POLST 03) and a 12-sided polygon (Matrix POLST 01)
The 6-sided polygon manufactured by ZG Optique was made of glass with aluminum coated faces measuring 50 (horizontal) by 25 (vertical) mm. The faces are marked clockwise from 1 to 6. The piramidality error of the polygon is between - 6 ‘’ and + 6 ‘’.
The 12-sided polygon, manufactured by Matrix, was made of steel. The faces are marked clockwise from 0° to 330° and measure 12 by 12 mm. The piramidality error of the polygon is between 0 ‘’ and 10 ‘’ where the sign + indicates the normal to the faces inclined downward.

The standards were supplied in a custom made transport case, manufactured from aluminium and steel, containing high density foam, sculpted to make a tight fit with the two polygons, to prevent any motion and generation of excessive bending forces.

4. Measurement instructions and reporting of results

Before calibration, the polygons had to be inspected for damage of the measurement surfaces. Any scratches, rusty spots or other damage had to be documented by a drawing using forms appended to the instructions.
The measurement quantity was the angle between the projections of the normal to the faces on the measurement plane, which is parallel to the polygon base.
The uncertainty of measurement had to be estimated according to the ISO Guide for the Expression of Uncertainty in Measurement.

5. Measurement methods and instruments used by the participants

Different instrument and techniques were used to perform the measurements
As for the movement of the polygon, each participant used a his own device, while to measure the deviation from nominal angle all laboratories used autocollimators. The details of these instruments are reported in Table 4.

Laboratory	**Autocollimator/ interferometer**	**Table**
INRIM	Elcomat 3000 (Moller-Wedel)	Moore 1440
IPQ	Elcomat HR (Moller-Wedel)	Tekniker
LNMC	Autocollimator	Goniometer GS-2 (Arsenal)
Metrosert	Elcomat 3000 (Moller-Wedel)	Eimeldingen DP-400

Table 4: *Instrumentation and methods*

6. Stability and condition of the gauges

6.1 Stability

In order to check the stability of the artifacts over the comparison, a calibration of the both polygons before the start of the comparison is necessary.

As for the 12-sided polygon, many measurements concerning its historic stability were available and let us conclude that the nominal angles of the polygon did not change substantially for years. Unfortunately, when the polygon was measured again in July (before the last measurement), four faces showed a difference between the preliminary values of the order of the expanded uncertainty. This behavior was confirmed by the last measurements performed at the end of the circulation in December. Hence, probably something happened to the polygon between the previous measurement (which dated back to 2003) and the circulation. Since we were not able to identify when and why this happened, the mean value of all the data was evaluated.

No preliminary measurements of the 6-sided polygon were present.

6.2 Condition of the gauges

Both polygons presented some minor defect on faces (scratches) which have been documented with pictures before the circulation. The participating laboratories were asked to document any anomaly and or damages on the measurement surfaces when receiving the polygons.
The participants did not notice any damage worthy of note (except for the previously documented ones), when they examined the polygons at the start of the comparison.

7. Measurement results, as reported by participants

In Tables 5 and 6, all measurement results for the two polygons are given along with their combined standard uncertainties, as reported by the participants. Results reported are the measured deviations from the nominal differential angles in seconds of arc. The positive sign indicates that the real angle is greater than the nominal value and vice versa.

Face	INRIM	IPQ	LNMC	METROSERT
0°-30°	1.41	1.35	1.7	1.60
30°-60°	-3.17	-3.08	-2.9	-3.29
60°-90°	3.31	3.24	3	3.13
90°-120°	-3.76	-3.72	-3.8	-3.79
120°-150°	0.69	0.68	0.5	0.86
150°-180°	-0.04	-0.09	-0.2	-0.10
180°-210°	2.05	2.13	1.8	2.20
210°-240°	-2.08	-2.16	-2.2	-2.20
240°-270°	2.07	2.05	1.9	1.97
270°-300°	3.28	3.31	3.6	3.36
300°-330°	-1.10	-1.01	-0.6	-1.29
330°-0°	-2.65	-2.70	-2.8	-2.45
U (k=2)	0.30	0.26	1.2	0.71

Table 5: *12-sided polygon: deviation from nominal angle (expressed in arcseconds)*

Face	INRIM	IPQ	LNMC	METROSERT
1-2	4.68	4.60	4.6	4.70
2-3	-0.69	-0.61	-0.9	-0.70
3-4	-3.71	-3.71	-3.8	-3.79
4-5	1.25	1.21	1.4	1.26
5-6	0.06	0.15	0.1	0.12
6-1	-1.59	-1.64	-1.4	-1.59
U (k=2)	0.20	0.32	1.5	0.65

Table 6: *6-sided polygon: deviation from nominal angle (expressed in arcseconds)*

Figures 1 and 2 show the deviations from nominal angle with expanded uncertainty bars

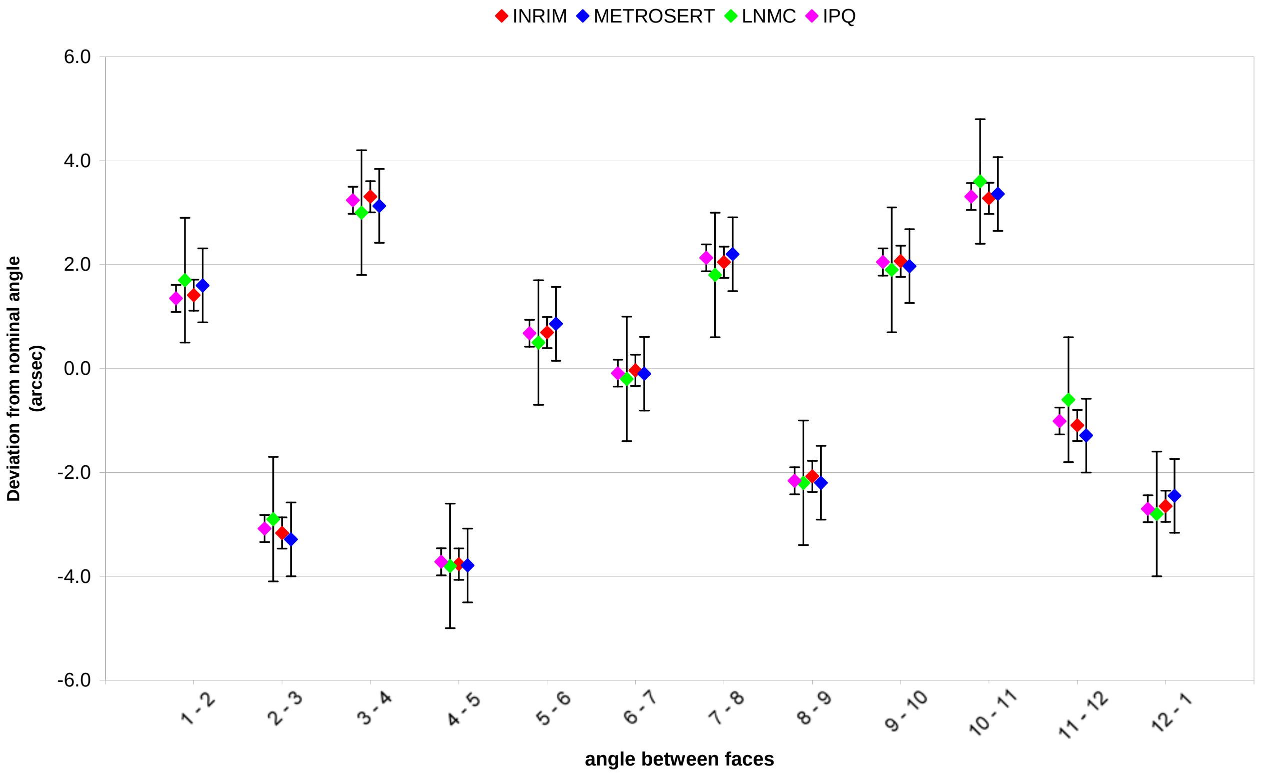

Fig.1: Results for 12-sided polygon

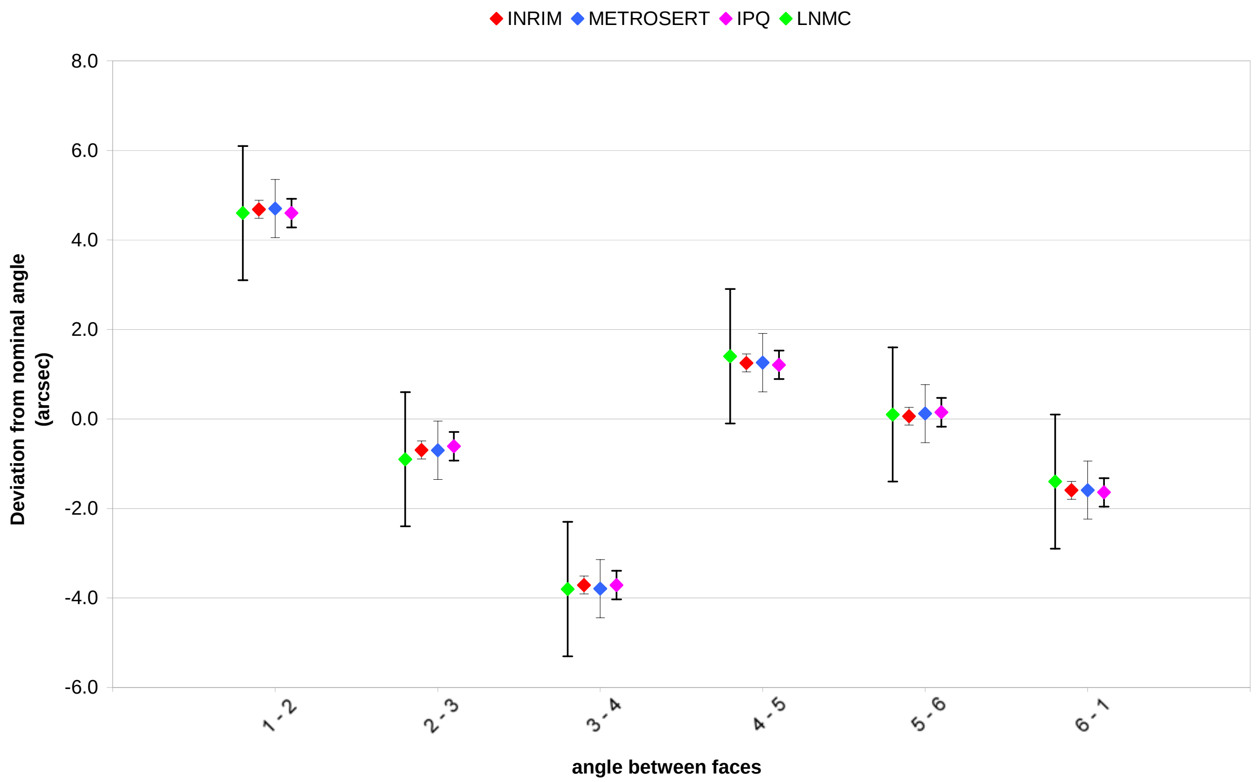

Fig. 2: Results for 6-sided polygon

8. Measurement uncertainty

8.1 Model equation

The participants were asked to estimate the uncertainty of measurement according to ISO *Guide to the Expression of Uncertainty in Measurement.*
Because of the non uniformity in the uncertainty budget of the different laboratories (caused by the difference between calibration stations) it is not possible to compare the single uncertainty terms. The uncertainty budget of each laboratory is reported in the each one's measurement protocol.

In Table 7 and 8, the uncertainty given by each laboratory is reported. The numerical values are standard uncertainties given in second of arc.

	INRIM	IPQ	LNMC	Metrosert
u_c	0.15	0.13	0.6	0.355

Table 7: *standard uncertainty for 12-sided polygon*

	INRIM	IPQ	LNMC	Metrosert
u_c	0.1	0.16	0.75	0.327

Table 8: *standard uncertainty for 6-sided polygon*

9. Analysis of the reported results and reference values

In order to assess the agreement between the data provided by each institute, a statistical analysis has been performed.
From Table 8 and Figures 1 and 2, it is clear that the uncertainties quoted by the participants are different from one participant to another. Thus analysis via use of the simple arithmetic mean as an estimator of the true mean is not suitable and instead, the weighted mean should be used.

The participating institutes are identified by the index *i*=1,…,4. The institutes' measurement result for each angle of the two polygons is denoted by x_i *i*=1,…,4 and the standard uncertainty associated with these values is denoted by $u(x_i)$ *i*=1,…,4.
Under the hypothesis of independent and normally distributed measurements, the weighted mean of the institutes measurements and the associated standard deviation were calculated according to the following formulas:

$$\bar{x} = \frac{\sum_i x_i \cdot u^{-2}(x_i)}{\sum_i u^{-2}(x_i)}, \quad u(\bar{x}) = \sqrt{\frac{1}{\sum_i u^{-2}(x_i)}} \qquad (1)$$

12-sided polygon	$\bar{x}$
0°-30°	1.40
30°-60°	-3.12
60°-90°	3.25
90°-120°	-3.74
120°-150°	0.69
150°-180°	-0.07
180°-210°	2.09
210°-240°	-2.13
240°-270°	2.05
270°-300°	3.31
300°-330°	-1.05
330°-0°	-2.67
$u(\bar{x})$	0.09

6-sided polygon	$\bar{x}$
1-2	4.66
2-3	-0.67
3-4	-3.72
4-5	1.24
5-6	0.09
6-1	-1.60
$u(\bar{x})$	0.08

Table 9: *weighted mean of the measurements and associated standard deviation*

A first check for statistical consistency of the results based on chi-squared test was carried out to decide whether to accept the weighted mean as the key comparison reference value (KCRV) or not. To this aim, the observed chi-squared was calculated as follows:

$$\chi^2_{obs} = \sum_i \frac{(x_i - \bar{x})^2}{u^2(x_i)} \qquad (2)$$

and compared with the tabulated value of $\chi^2(\nu)$, with ν = 4-1=3 degrees of freedom. The chi-squared test at 95% significance level was carried out successfully, therefore the weighted mean was accepted as KCRV.

Hence, the degree of equivalence, i.e. the deviation of each laboratory's result from the KCRV, is determined simply as $x_i - \bar{x}$. The uncertainty of this deviation is calculated as a combination of the uncertainties of the result, $u(x_i)$, and the uncertainty of the weighted mean. The uncertainty of the deviation from the weighted mean is given by equation (3), which includes a minus sign to take into account the correlation between the two uncertainties.

$$u(x_i - \bar{x}) = \sqrt{u^2(x_i) - u^2(\bar{x})} \qquad (3)$$

Finally, a check for statistical consistency of the results with their associated uncertainties can be made by calculating the E_n value for each laboratory, where E_n is defined as the ratio of the deviation from the weighted mean, divided by the uncertainty of this deviation:

$$E_n = \frac{x_i - \bar{x}}{\sqrt{u^2(x_i) - u^2(\bar{x})}} \qquad (4)$$

The results of this analysis are presented in Tables 10 and 11 and in Figures 3 and 4.

Face	INRIM		IPQ		LNMC		METROSERT	
	$x_i - \bar{x}$ /“	E_n	$x_i - \bar{x}$ /“	E_n	$x_i - \bar{x}$ /“	E_n	$x_i - \bar{x}$ /“	E_n
0°-30°	0.013	0.107	-0.050	-0.557	0.300	0.506	0.200	0.583
30°-60°	-0.043	-0.364	0.044	0.485	0.224	0.378	-0.166	-0.485
60°-90°	0.054	0.461	-0.012	-0.137	-0.252	-0.426	-0.122	-0.357
90°-120°	-0.020	-0.175	0.024	0.267	-0.056	-0.094	-0.046	-0.134
120°-150°	0.000	-0.004	-0.013	-0.144	-0.193	-0.326	0.167	0.488
150°-180°	0.037	0.315	-0.018	-0.199	-0.128	-0.216	-0.028	-0.082
180°-210°	-0.048	-0.410	0.036	0.396	-0.294	-0.496	0.106	0.309
210°-240°	0.055	0.468	-0.029	-0.319	-0.069	-0.116	-0.069	-0.201
240°-270°	0.019	0.159	0.003	0.035	-0.147	-0.248	-0.077	-0.224
270°-300°	-0.031	-0.267	0.003	0.030	0.293	0.494	0.053	0.154
300°-330°	-0.043	-0.363	0.043	0.471	0.453	0.764	-0.237	-0.693
330°-0°	0.016	-0.132	-0.034	-0.380	-0.134	-0.227	0.216	0.630
$u(x_i - \bar{x})$/”	0.117		0.090		0.593		0.342	

Table 10: *degree of equivalence and E_n values for each laboratory (12-sided polygon)*

Face	INRIM		IPQ		LNMC		METROSERT	
	$x_i - \bar{x}$ /“	E_n	$x_i - \bar{x}$ /“	E_n	$x_i - \bar{x}$ /“	E_n	$x_i - \bar{x}$ /“	E_n
1-2	0.021	0.371	-0.061	-0.447	-0.061	-0.082	0.039	0.122
2-3	-0.019	-0.321	0.064	0.466	-0.226	-0.303	-0.026	-0.082
3-4	0.006	0.109	0.006	0.042	-0.084	-0.113	-0.074	-0.236
4-5	0.008	0.146	-0.033	-0.239	0.157	0.211	0.017	0.054
5-6	-0.027	-0.470	0.062	0.447	0.012	0.015	0.032	0.100
6-1	0.010	0.166	-0.037	-0.269	0.203	0.272	0.013	0.041
$u(x_i - \bar{x})$/”	0.058		0.138		0.746		0.315	

Table 11: *degree of equivalence and E_n values for each laboratory (6-sided polygon)*

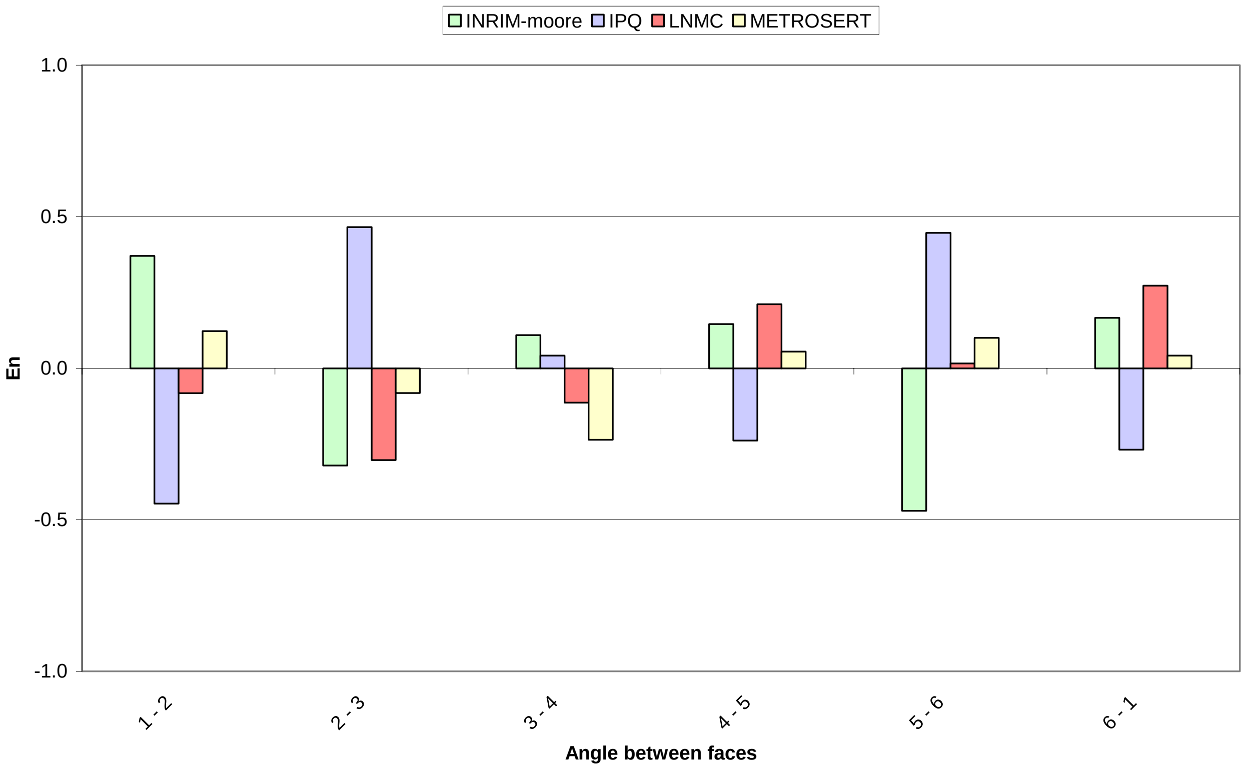

Fig. 3: E_n values for 6-sided polygon

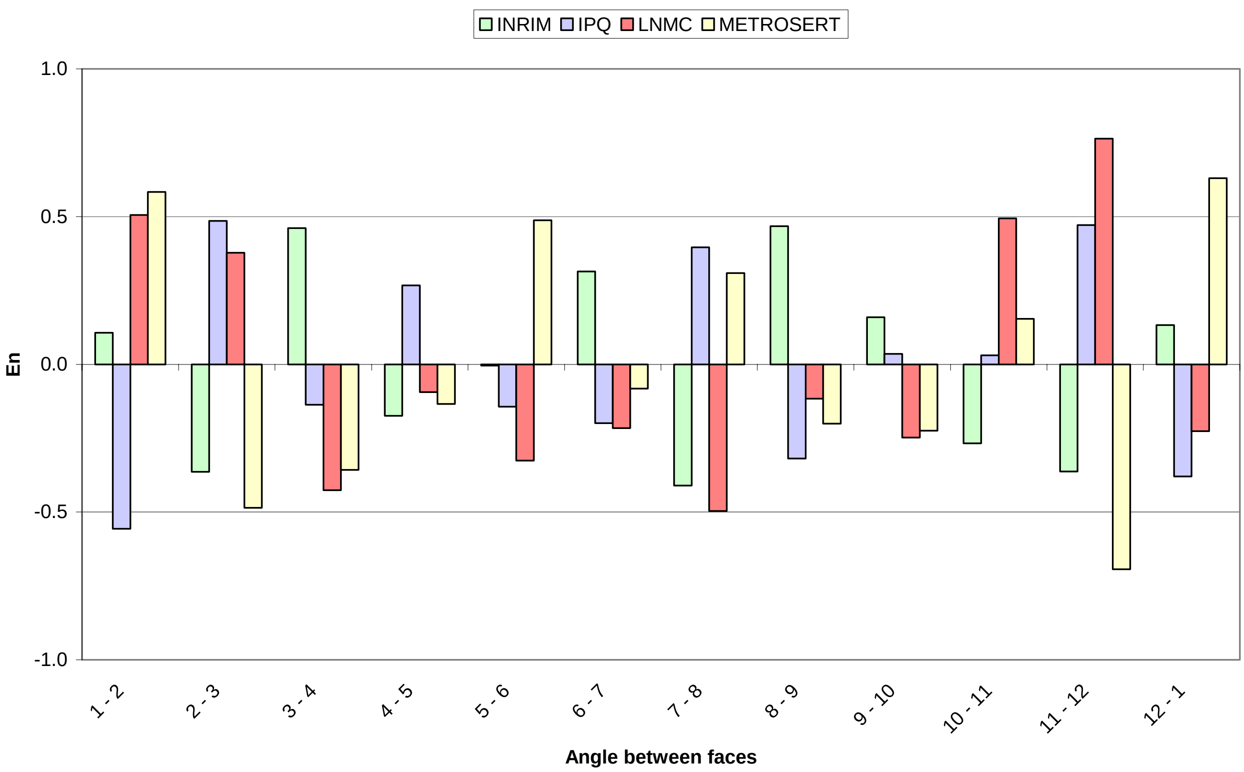

Fig. 4: E_n values for 12-sided polygon

Examination of Figure 3 and 4 indicates that the E_n ratio has a magnitude less than 1 for each angle of the two polygons.

10. Conclusion

The comparison was based on two polygons with extremely different features (geometry, material and reflectivity), furthermore polygons were in condition of normal use (with non perfect reflecting surfaces because of wear). Each participant have used his own calibration station each one different from the others. The results show a good agreement between the laboratories since the E_n values are smaller than 1 for each angle of the two polygons for all laboratories. We can conclude that the measurement procedure and the evaluation of the measurement uncertainty can be positively confirmed by this result.

11. References

[1] Guidelines for CIPM key comparisons 1st March 1999
[2] ISO, Guide to the expression of uncertainty in measurements, (1993)
[3] M.G. Cox, The evaluation of key comparison data, *Metrologia*, 2002, **39**, 589-595