Force Key Comparison CCM.F-K1.a and CCM.F-K1.b
5 kN and 10 kN

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Content

Foreword

This report describes the Key Comparison, named CCM.F-K1.a and CCM.F-K1.b, for force with loads of 5 kN and 10 kN. The Draft A, reporting the measurement results of the key comparison, has been accepted in force expert group meeting in Pretoria 23.3.2004. The part A is the Part 1 of preliminary draft B. There have been several discussions to find the best way for determination of the reference value for force values 5 kN and 10 kN in the key comparison CCM.F-K1.a and CCM.F-K1.b. According to the meeting held in Mexico, Queretaro by CENAM 3. – 5.12.2007 the reference values has been calculated in chapter 2 for each single transducer and in chapter 3 as one reference value for 5 kN and second reference value for 10 kN. To get a better consistency a linear model for the drift of transducers has been applied. In the evaluation of results the paper from M.G. Cox, “The evaluation of key comparison” (Metrologia, 2002, 39, 589-595).

I thank all participants for the good co-operation in this intercomparison and specially the group of laboratories to be able to realize this paper.

Aimo Pusa

Chapter 1
Intercomparison and measured data

1.1 General

The CCM force expert group, chaired by Prof. Manfred Peters of PTB, in October 1998 in Sydney, made decisions about the CIPM force key comparisons. These were to be split into four ranges, a) 5 kN – 10 kN, b) 50kN – 100 kN, c) 500 kN – 1000 kN, and d) 2 MN – 4 MN, with the respective pilot laboratories being a) MIKES-Raute, Finland, b) NPL, United Kingdom, c) PTB, Germany, and d) NIST, USA. This report gives the results for key comparison a) (5 kN and 10 kN), officially identified by CIPM as CCM.F-K1.a (scheme A) and CCM.F-K1.b (scheme B).

Participants in the comparison

There were 16 laboratories including the pilot, listed in table 1.

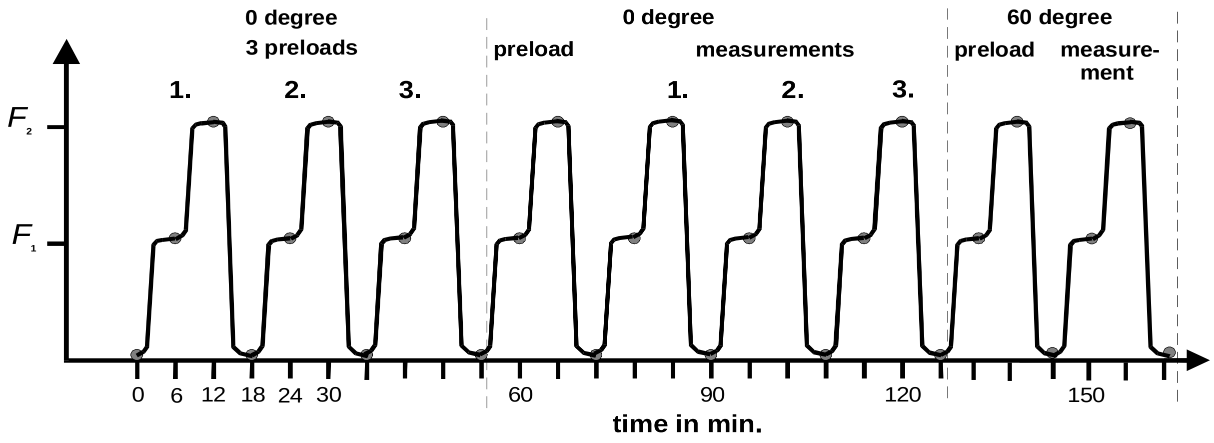
Country	Institute	Number	Country	Institute	Number
Australia	CSIRO	15	Japan	NMIJ/AIST	5
Belgium	MD	2	R.o. Korea	KRISS	12
China	NIM	10	Mexico	CENAM	6
Finland	MIKES-Raute	0	Netherlands	NMI-VSL	13
Germany	PTB	3	Singapore	PSB	14
Spain	CEM	4	Turkey	UME	7
France	LNE	11	United Kingdom	NPL	8
Italy	INRIM	1	USA	NIST	9

Table 1.1 Participating countries and laboratories, including the code number used in the report.

Principles of the comparison

The purpose of key comparisons is to compare the units of measurement as realized throughout the world. In the area of force, the way this is done is by the use of high quality load cells subjected to similar loading profiles in national force standard machines (FSMs), following a strict measurement protocol and using similar instrumentation. The CCM Force Working Group proposed the following loading schemes:

Scheme A



Scheme B

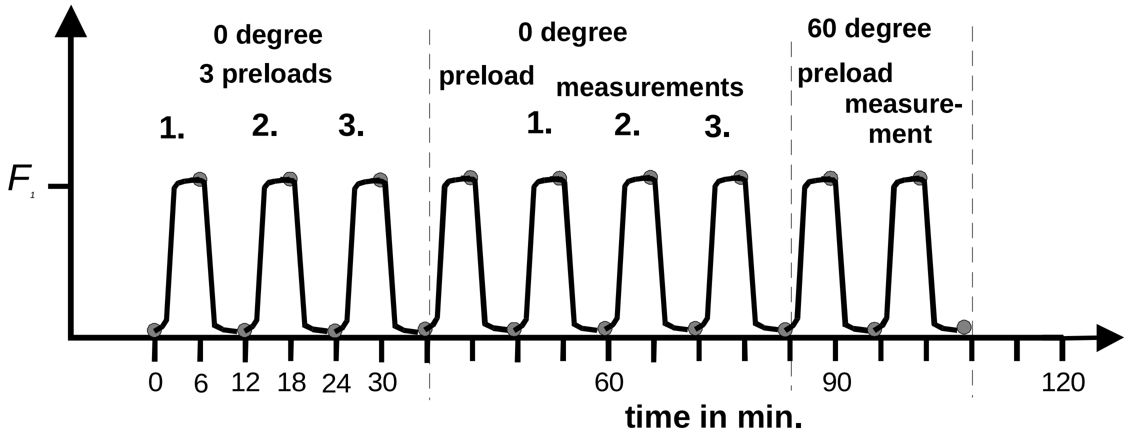


Figure 1.1. Loading scheme for both set of transducers, forces 5 kN and 10 kN (scheme A) and force 5 kN (scheme B).

The force transducer is rotated through 720° in both schemes. One preload and one measurement (as at 60° in Figure 1) is carried out at 120°, 180°, 240°, 300°, 360°/0°, 60°, 120°, 180°, 240°, 300°, and 360°.

The comparison is carried out using four transducers, two with nominal capacity 10 kN for Scheme A and two with nominal capacity 5 kN for Scheme B, identified as Tr1/10 kN, Tr2/10 kN, Tr1/5 kN, and Tr2/5 kN. Both transducers starting with Tr1 are from one manufacturer and the two starting with Tr2 are from another manufacturer. The construction principles of the two transducer types are different, and they have been selected as having the best characteristics for this comparison work.

Realisation of the comparison

The comparison is made in a star format; the transducers come back to the Pilot after each participating laboratory´s measurements. One complete measurement cycle (Pilot – Participating Laboratory – Pilot) is called a loop. The first measurement by the Pilot is called the A-measurement and the second measurement by the Pilot, after the participating laboratory, is called the B-measurement.

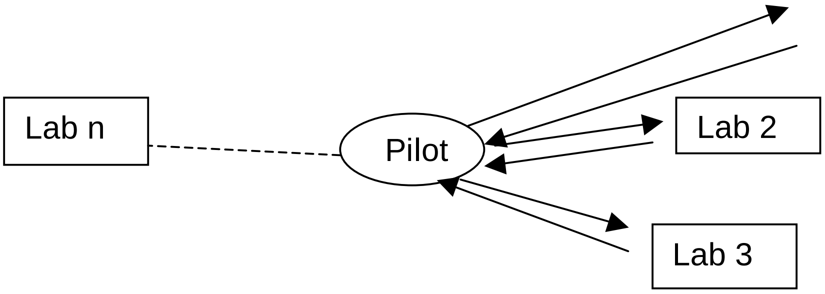


Figure 1.2. Principle of the star type comparison

Limitations of the comparison

Due the fact that there is no real reference value to circulate (as the transfer transducers do not provide constant values), the following facts should be accepted:

- every measurement loop is independent of the others,
- numerical values of different loops are not easily comparable,
- only relative deviations can be compared,
- there is no absolute numerical reference value.

Uniformity of the measured values

In practice, it is not possible to calibrate the DMP40 measurement instruments used (one at each laboratory), against one reference standard. The uniformity of the DMP40s used was confirmed with reference to a BN100 calibrator unit. Each participating laboratory measured the indication of their DMP40 against the signal of BN100, which is stable to better than approximately $4\cdot10^{-6}$. The Pilot monitored the signal of the BN 100 against two instruments in their laboratory.

The resulting correction value to be used by the participating laboratory is calculated as:

$$c_{\mathrm{DMP,L}} = x_{\mathrm{DMP,\,P}} - x_{\mathrm{DMP,\,L}} \qquad (1.1)$$

Where;

$c_{\mathrm{DMP,L}}$ = Correction value for DMP40 of the participating laboratory

$x_{DMP,\,L}$ = Indication of the DMP40 at the participating laboratory with the signal of BN100, calculated for each transducer from two measurements, before and after the comparison measurement,

$x_{DMP,\,P}$ = Indication of the DMP40 at the Pilot laboratory with the signal of BN100, also calculated from two measurements.

The corrected deflection value to be used is calculated as:

$$x''_{\mathrm{L}} = x'_{\mathrm{L}} + C_{\mathrm{DMP,L}} \qquad (1.2)$$

x'_{L} = Measured deflection value of the laboratory

x''_{L} = Deflection value to be used (with BN 100 correction)

1.2. Characteristics of the transducers

Creep effect

To minimise the influence of creep, a relatively long reading period of 6 minutes was selected. There are two important elements of the creep:

- the creep effect should be small enough to eliminate the uncertainty of the time of reading,
- the creep effect is constant during every loading.

The aim was to have equal loading times for each laboratory, but this was not possible due to the fact that the machines did not have similar capabilities. The loading times varied from 20 s to 125 s and all of transducers had constant creep after 3 min and 55 s, which was the shortest time after loading by one laboratory for the taking of readings. The pilot checked the loading time with transducer 10 kN Nr 2, which has the worst creep, and the difference between loading times of 40 s and 125 s gave a difference of only $1\bullet10^{-6}$, which is less than any measurement uncertainty. Table 1.1 shows the creep effect as numerical values.

Transducer	Total creep value 6 min after loading the force [nV/V]	Rel. change of creep between 4 min. and full time (6 min) [rel. creep/min]
TR1/10 kN	-20	$5{,}0 \cdot 10^{-7}$
TR2/10 kN	75	$1{,}9 \cdot 10^{-6}$
TR1/5 kN	45	$2{,}5 \cdot 10^{-7}$
TR2/5 kN	-25	$6{,}3 \cdot 10^{-7}$

Table 1.2. Numerical values of the creep of the transducers.

The numerical values indicate that the influence of a change in the reading time by a few seconds is not significant to the uncertainty of measurement.

Temperature effect of the sensitivity

The effect of temperature sensitivity can be an important factor if the environmental temperature at the participating is not the same as that at the pilot laboratory. The temperature sensitivity of each transducer was determined by taking measurements at two different temperatures which differed by 15 °C. (The uniformity of the temperature scale between Pilot and participant laboratories is based on the assumption that every participant has traceability to their national temperature scale with uncertainty of less than 0,5 °C).

Transducer	Relative temperature coefficient/K	Uncertainty of the value (*k*=*2*)/K
TR1/10 kN	$8{,}24 \cdot 10^{-6}$	$10 \cdot 10^{-7}$
TR2/10 kN	$4{,}6 \cdot 10^{-7}$	$9 \cdot 10^{-7}$
TR1/5 kN	$4{,}4 \cdot 10^{-6}$	$5 \cdot 10^{-7}$
TR2/5 kN	$1{,}9 \cdot 10^{-6}$	$3 \cdot 10^{-7}$

Table 1.3. Temperature coefficients of each transducer.

Stability of the transfer transducers

a) Stability of sensitivity over the complete period of the key comparison

Based on the fact that the quality of the comparison is dependant upon the three measurements during the loop, the stability of the transducers is extremely important. The following figures show the stability of the transducers by giving the measurements made by the Pilot. The Pilot values are compared against the mean value calculated from all measurements.

$$\overline{x'} = \frac{\sum\limits_{i=1 \ldots n_1} x'_P + \sum\limits_{i=1 \ldots n_2} x'_L}{n_1 + n_2} \qquad \begin{array}{l}\text{(for 5 kN; } n_1=13,\ n_2=8\text{)} \\ \text{(for 10 kN; } n_1=14,\ n_2=10\text{)}\end{array} \qquad (1.3)$$

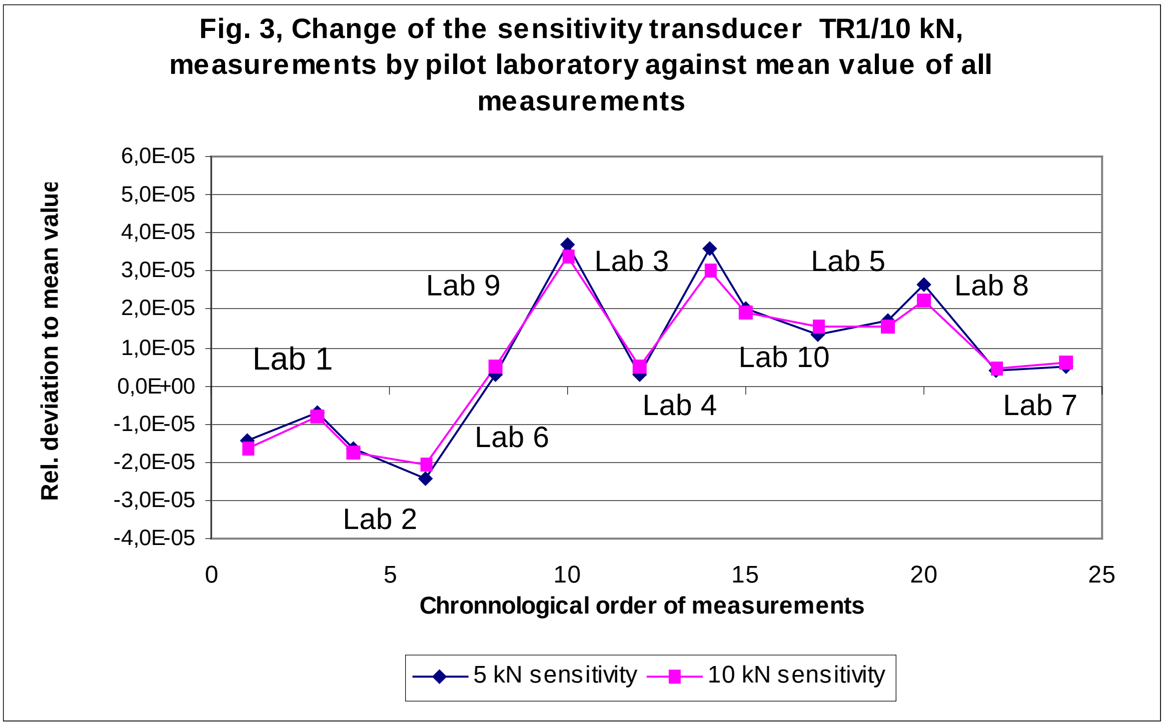


Figure 1.3. Stability of transducer TR1/10 kN measured by the Pilot - between the points measured by the Pilot, the participating laboratories are indicated.

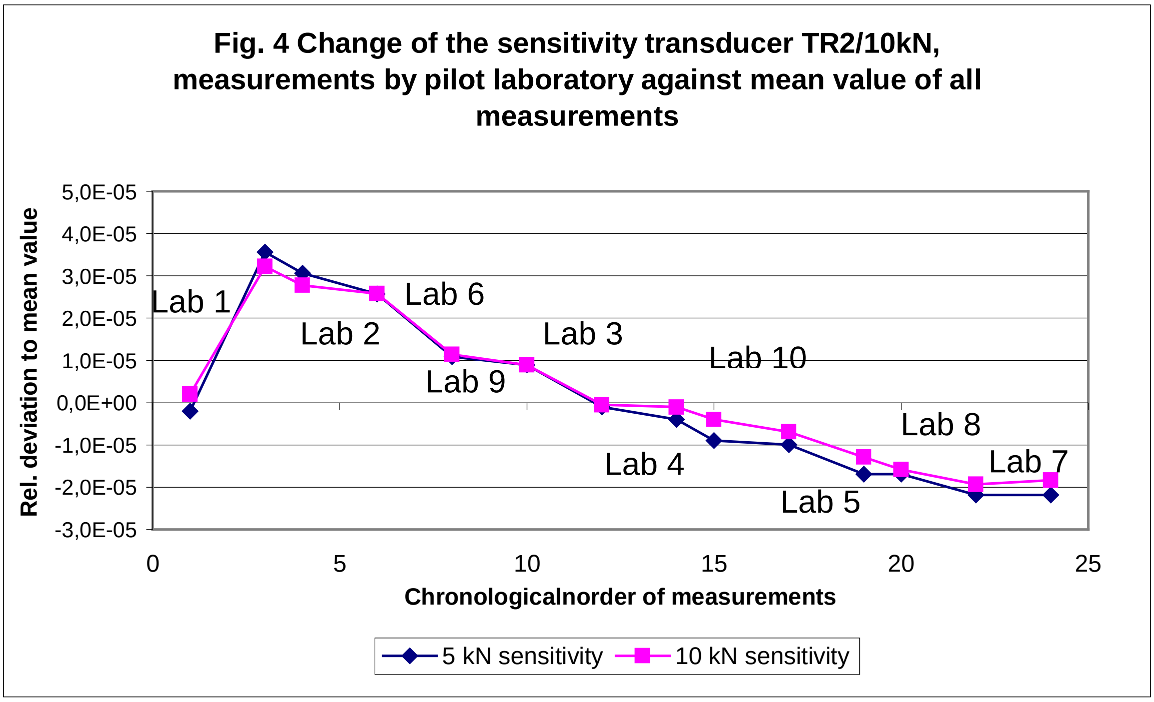


Figure 1.4. Stability of transducer TR2/10 kN measured by the Pilot - between the points measured by the Pilot, the participating laboratories are indicated.

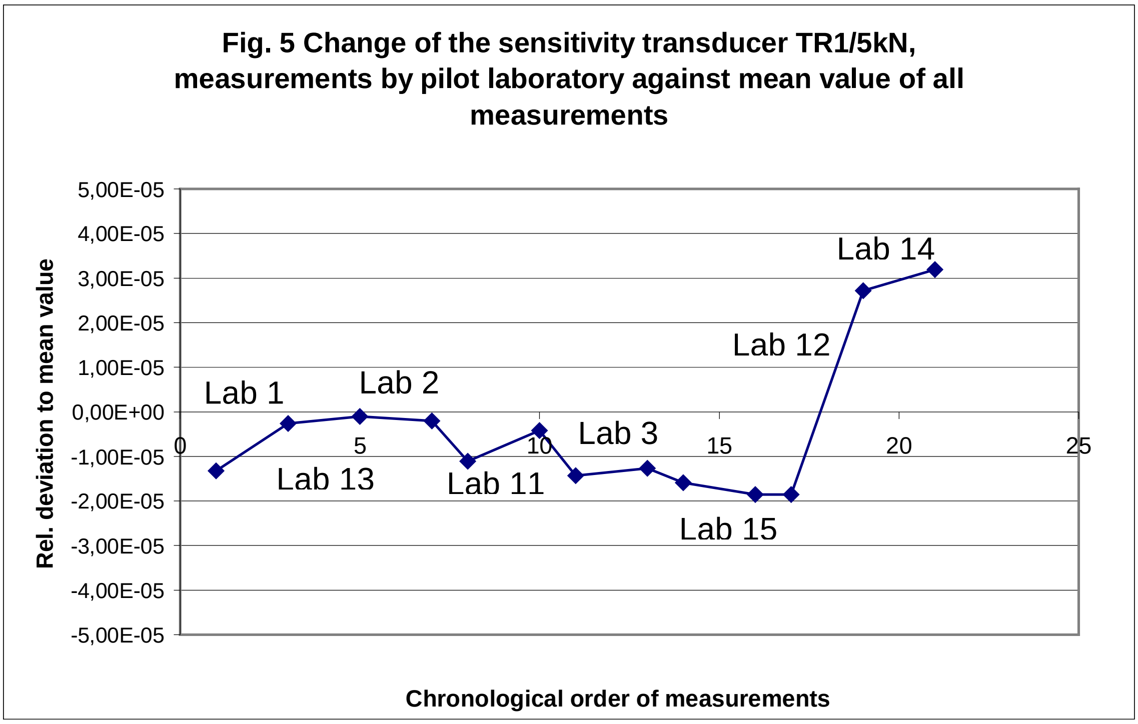


Figure 1.5. Stability of transducer TR1/5 kN measured by the Pilot - between the points measured by the Pilot, the participating laboratories are indicated.

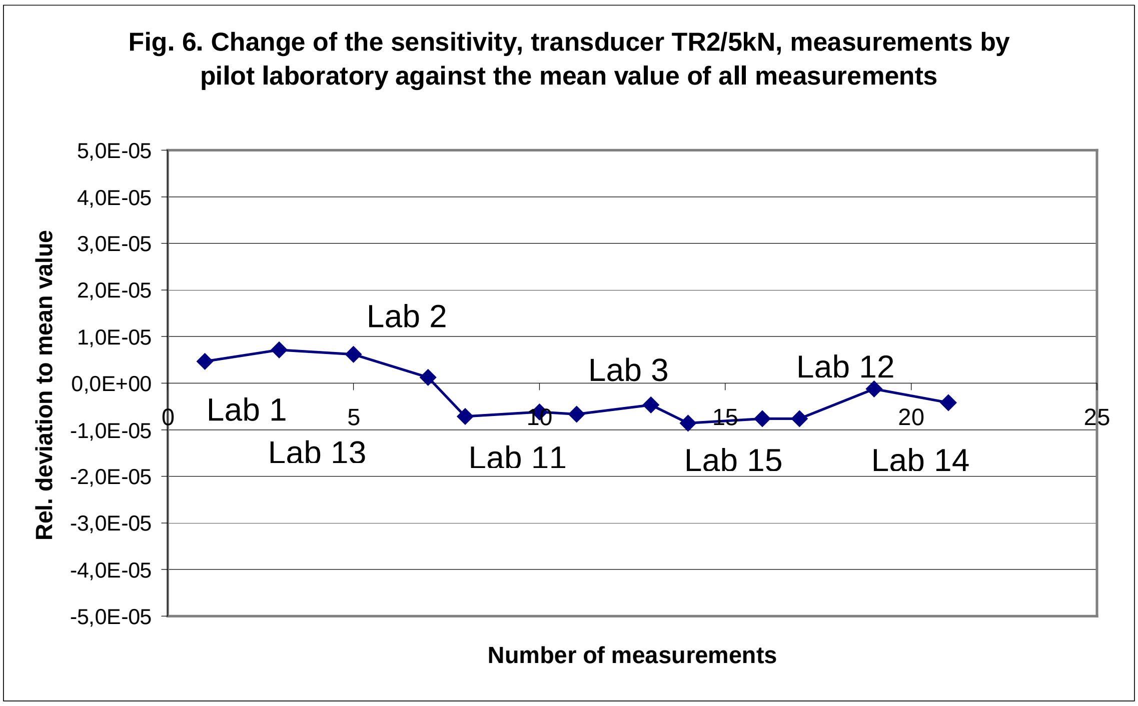


Figure 1.6. Stability of transducer TR2/5 kN measured by the Pilot - between the points measured by the Pilot, the participating laboratories are indicated.

Tables 1.4 and 1.5 show the chronological order of the measurements.

Measuring laboratory	Chronolog. order	Date of calibration	
		Tr1 10 kN	Tr2 10 kN
Pilot	1	1.2.2000	31.1.2000
INRIM	2	17.2.2000	16.2.2000
Pilot	3	7.3.2000	8.3.2000
Pilot	4	28.3.2000	21.3.2000
MD-Belgium	5	11.4.2000	6.4.2000
Pilot	6	25.4.2000	26.4.2000
CENAM-Mexico	7	13.5.2000	15.5.2000
Pilot	8	27.5.2000	26.5.2000
NIST-USA	9	7.6.2000	8.6.2000
Pilot	10	22.6.2000	21.6.2000
PTB-Germany	11	6.7.2000	7.7.2000
Pilot	12	18.7.2000	22.7.2000
CEM-Spain	13	31.7.2000	1.8.2000
Pilot	14	17.8.2000	17.8.2000
Pilot	15	28.9.2000	26.9.2000
NIM-China	16	16.10.2000	15.10.2000
Pilot	17	1.11.2000	31.10.2000
NMIJ/AIST-Japan	18	17.11.2000	16.11.2000
Pilot	19	28.11.2000	28.11.2000
Pilot	20	8.3.2001	8.3.2001
NPL-UK	21	14.3.2001	16.3.2001
Pilot	22	26.3.2001	27.3.2001
UME-Turkey	23	13.4.2001	16.4.2001
Pilot	24	26.4.2001	27.4.2001

Table 1.4. Chronological order of the measurements by the pilot and participating laboratories for transducers Tr1 10 kN and Tr2 10 kN.

Measuring laboratory	Chronolog. order	Date of calibration	
		Tr1 5 kN	Tr2 5 kN
Pilot	1	25.1.2000	27.1.2000
INRIM-Italy	2	7.2.2000	6.2.2000
Pilot	3	26.2.2000	27.2.2000
VSL-Netherlands	4	9.3.2000	7.3.2000
Pilot	5	16.3.2000	17.3.2000
MD-Belgium	6	5.4.2000	13.4.2000
Pilot	7	28.4.2000	27.4.2000
Pilot	8	11.9.2000	8.9.2000
LNE-France	9	20.9.2000	21.9.2000
Pilot	10	4.10.2000	5.10.2000
Pilot	11	19.10.2000	18.10.2000
PTB-Germany	12	6.11.2000	2.11.2000
Pilot	13	13.11.2000	14.11.2000
Pilot	14	23.11.2000	24.11.2000
NMIA-Australia	15	7.12.2000	11.12.2000
Pilot	16	28.12.2000	27.12.2000
Pilot	17	3.1.2001	4.1.2001
KRISS-R.o.Korea	18	16.1.2001	17.1.2001
Pilot	19	31.1.2001	30.1.2001
NMC-Singapore	20	13.2.2001	14.2.2001
Pilot	21	5.3.2001	17.3.2001

Table 1.5. Chronological order of the measurements by the pilot and participating laboratories for transducers Tr1/5 kN and Tr2/5 kN.

b) Stability in one loop

Figures 1.7 to 1.12 show the stability of the Pilot´s measurements as relative deviations between their A and B measurements.The relative deviation between the values of the A and B measurements is called drift and calculated as follows:

$$drift = \frac{X_{PA} - X_{PB}}{X_{PA}} \qquad (1.4)$$

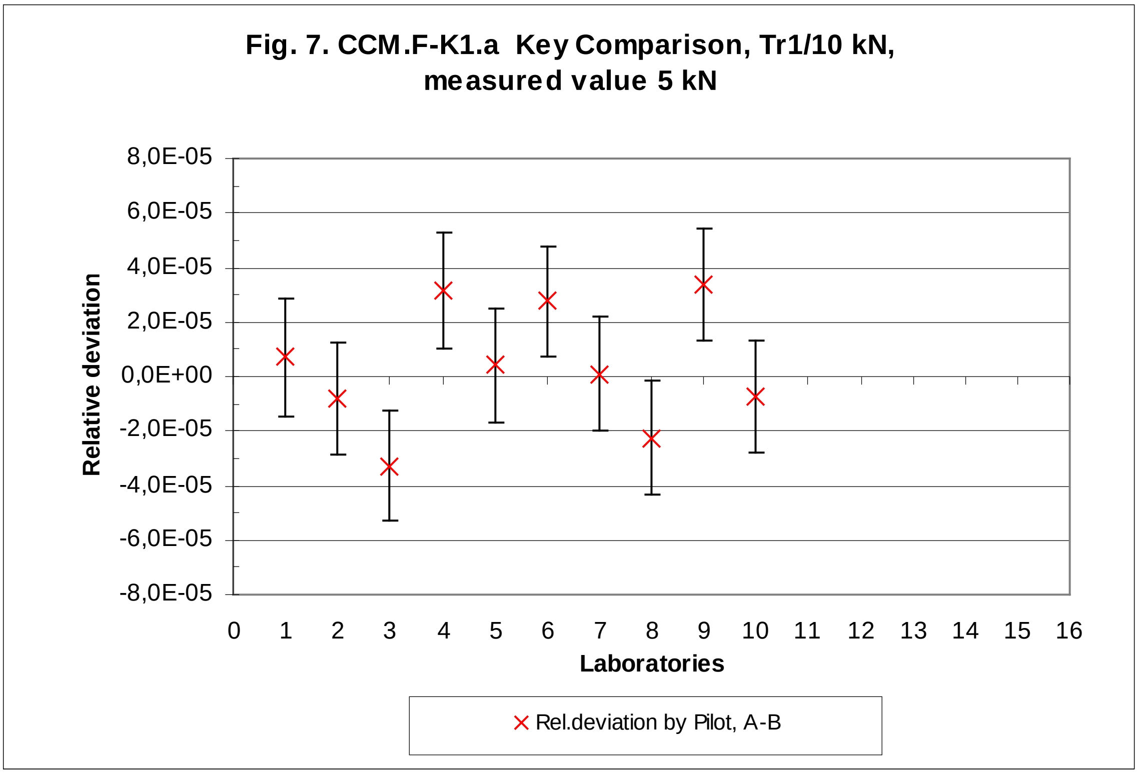


Figure 1.7. Transducer TR1/10kN, 5 kN load, relative deviations between pilots A and B measurement with the relative expanded uncertainty of pilot laboratory.

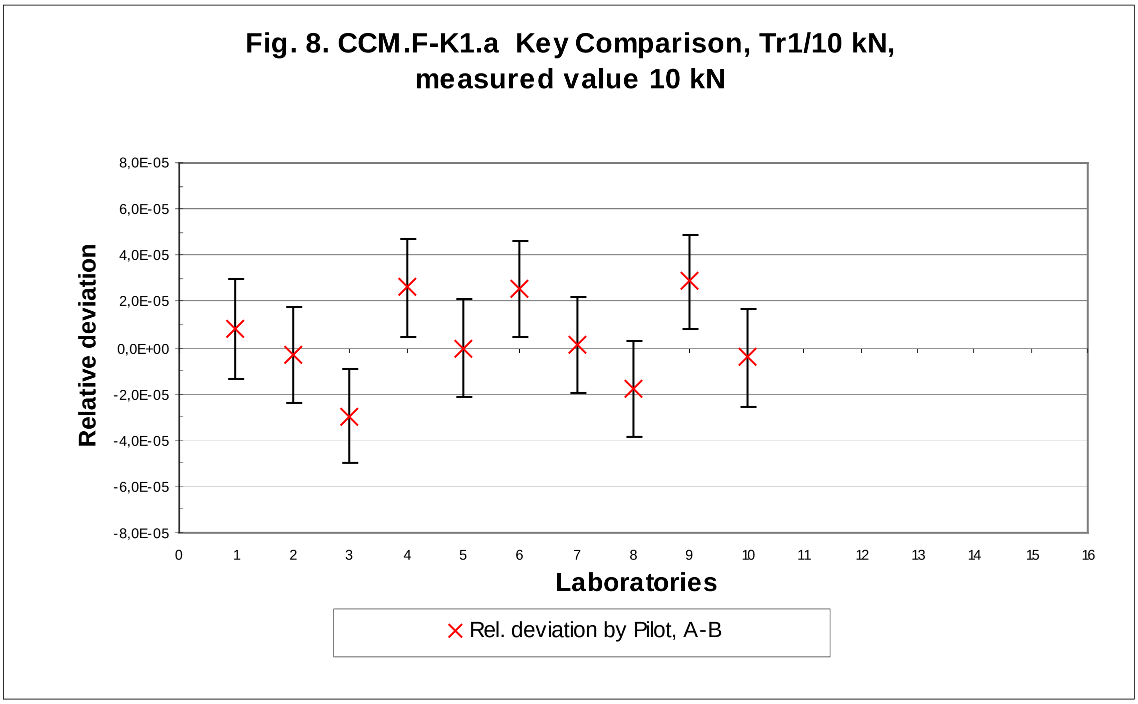


Figure 1.8. Transducer TR1/10kN, 10 kN load, relative deviations between pilots A and B measurement with the relative expanded uncertainty of pilot laboratory.

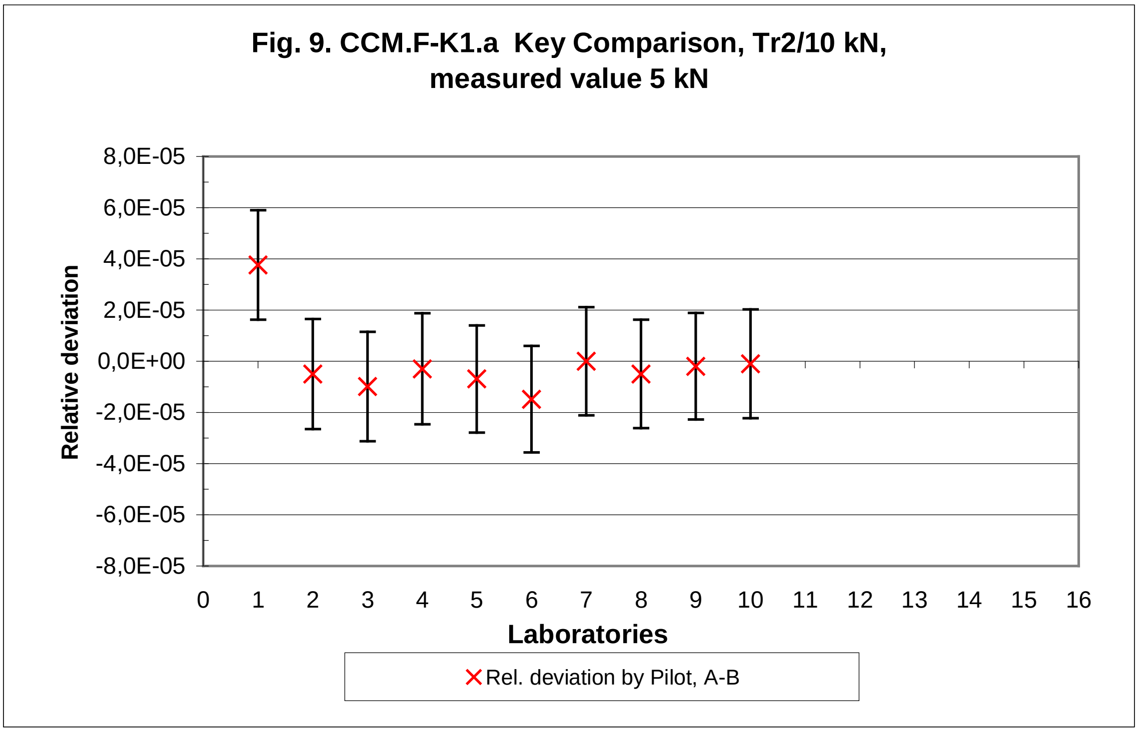


Figure 1.9. Transducer TR2/10kN, 5 kN load, relative deviations between pilots A and B measurement with the relative expanded uncertainty of pilot laboratory.

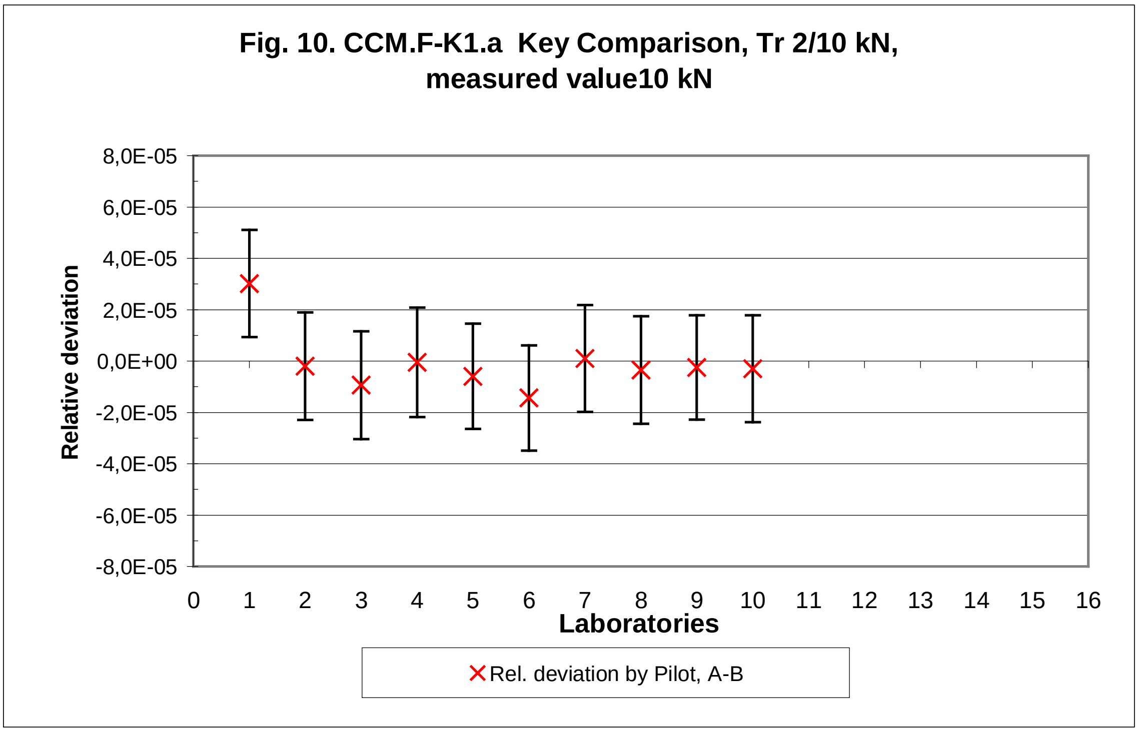


Figure 1.10. Transducer TR2/10kN, 10 kN load, relative deviations between pilots A and B measurement with the relative expanded uncertainty of pilot laboratory.

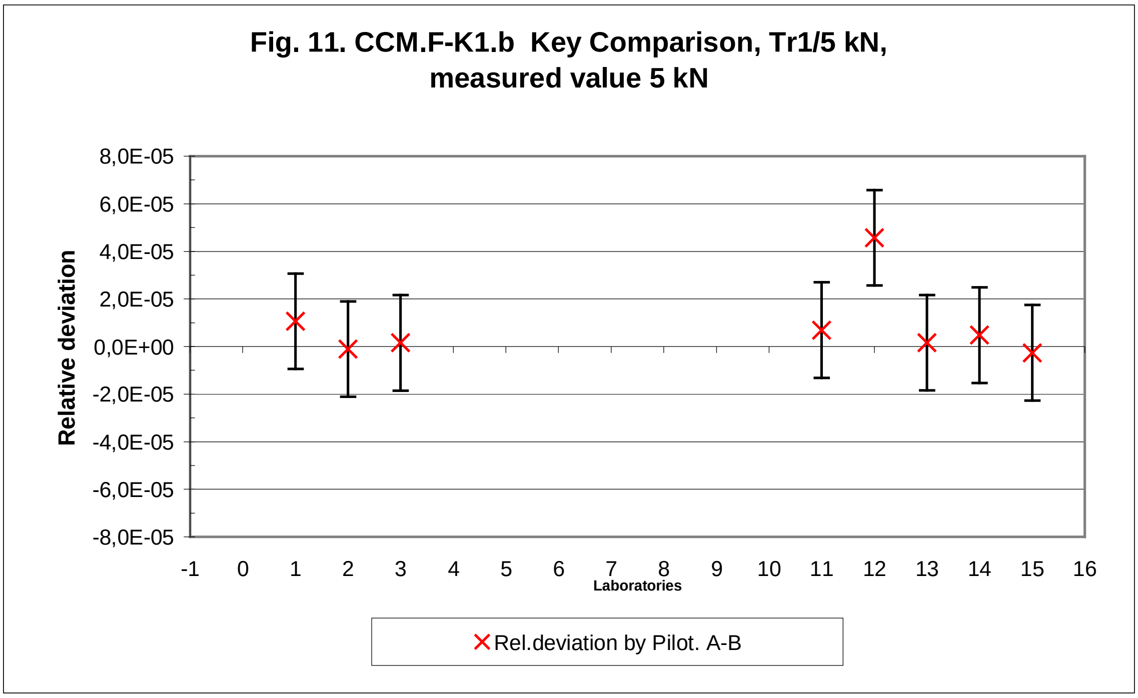


Figure 1.11. Transducer ”TR1/5kN”, 5 kN load, relative deviations between pilots A and B measurement with the relative expanded uncertainty of pilot laboratory.

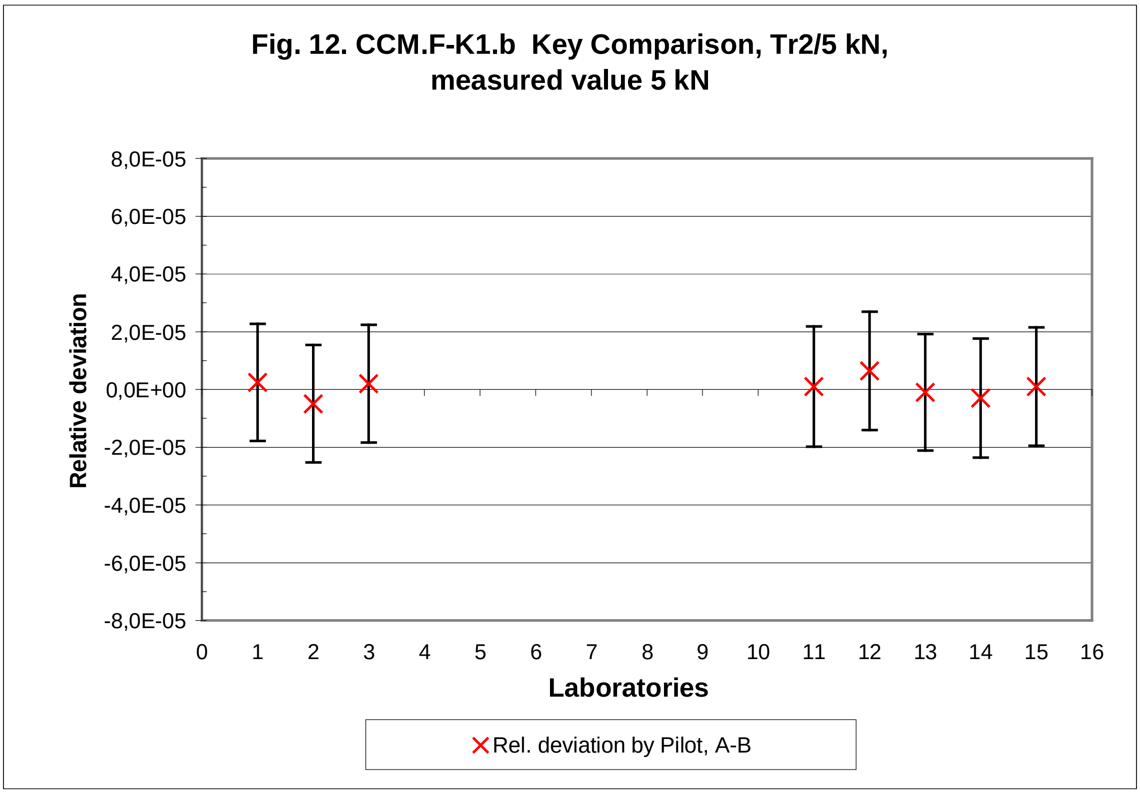


Figure 1.12. Transducer ”TR2/5kN”, 5 kN load, relative deviations between pilots A and B measurement with the relative expanded uncertainty of pilot laboratory.

Figures 1.7 and 1.8 demonstrate that Tr1 10 kN shows a continuous random change of sensitivity and Figure 1.11 that Tr1 5 kN made one significant change of sensitivity. Table 1.6 shows the stability of the transducers at the Pilot laboratory as numerical values. The mean value gives the absolute deviation between the Pilot´s A and B measurements from all measurement loops. From all these measurements, the standard deviation has been calculated, as well the standard deviation of the mean values.

Transducer	Load 5 kN			Load 10 kN		
	Mean value	Standard deviation	Standard deviation of the mean value	Mean value	Standard deviation	Standard deviation of the mean value
TR1/10 kN	$1{,}76 \cdot 10^{-5}$	$2{,}24 \cdot 10^{-5}$	$7{,}5 \cdot 10^{-6}$	$1{,}44 \cdot 10^{-5}$	$1{,}92 \cdot 10^{-5}$	$6{,}4 \cdot 10^{-6}$
TR2 /10 kN	$8{,}51 \cdot 10^{-6}$	$1{,}43 \cdot 10^{-5}$	$4{,}8 \cdot 10^{-6}$	$7{,}67 \cdot 10^{-6}$	$1{,}19 \cdot 10^{-5}$	$4{,}0 \cdot 10^{-6}$
TR1/5 kN	$9{,}37 \cdot 10^{-6}$	$1{,}57 \cdot 10^{-5}$	$5{,}5 \cdot 10^{-6}$			
TR2/5 kN	$2{,}71 \cdot 10^{-6}$	$3{,}49 \cdot 10^{-6}$	$1{,}2 \cdot 10^{-6}$			

Table 1.6. Mean values of relative deviations and associated standard deviations between the Pilot´s A and B measurements .

1.3. Results of the measurements

1.3.1 Measured deflections and results of the measurements

Used symbols

x_{PA} = Measured deflection by pilot for A measurement
x_{PB} = Measured deflection by pilot for B measurement
x_{P} = Used deflection value for pilot for loop n
x'_{L} = Measured deflection value of the laboratory
$c_{\mathrm{DMP,L}}$ = Correction value for DMP40 of the participating laboratory
x_{L} = Used deflection value with correction of the laboratory (BN 100 and possible other corrections e.g. temperature, extrapolation.)
w_{PA} = Relative expanded uncertainty of X_{PA}
w_{PB} = Relative expanded uncertainty of X_{PB}
w_{P} = Relative expanded uncertainty of X_{P}
w_{L} = Relative expanded uncertainty of X_{L}

Country		***Trans.***	***Load***	x'_L	$c_{DMP, L}$	x_L	w_L	x_{PA}	w_{PA}	x_{PB}	w_{PB}
	Nr	Identif.	kN	mV/V	10^{-6}·mV/V	mV/V	$*10^{-5}$	mV/V	$*10^{-5}$	mV/V	$*10^{-5}$
Italy	1	Tr1/10 kN	5	0,979722	0,0	0,979722	2,0	0,979689	2,14	0,979696	2,08
		Tr1/10kN	10	1,959214	-1,0	1,959213	2,0	1,959181	2,09	1,959197	2,05
		Tr2/10 kN	5	1,010011	1,5	1,010013	2,0	1,00998	2,15	1,010018	2,08
		Tr2/10kN	10	2,020018	-0,5	2,020018	2,0	2,019995	2,13	2,020056	2,08
		Tr1/5 kN	5	1,882006	-0,5	1,882006	2,0	1,881986	2,01	1,882006	2,01
		Tr2/5 kN	5	2,027862	0,5	2,027862	2,0	2,027819	2,03	2,027824	2,02
Belgium	2	Tr1/10 kN	5	0,979694	1,0	0,979694	7,4	0,979687	2,15	0,979679	2,08
		Tr1/10kN	10	1,959190	2,0	1,959192	7,1	1,959179	2,09	1,959173	2,05
		Tr2/10 kN	5	1,010005	2,0	1,010007	7,3	1,010013	2,08	1,010008	2,10
		Tr2/10kN	10	2,020039	2,0	2,020041	7,1	2,020047	2,10	2,020043	2,06
		Tr1/5 kN	5	1,882044	2,0	1,882047	7,1	1,882009	2,01	1,882007	2,01
		Tr2/5 kN	5	2,027847	2,0	2,027849	7,1	2,027822	2,02	2,027812	2,03
Germany	3	Tr1/10 kN	5	0,979686	2,0	0,979688	2,2	0,979739	2,14	0,979707	2,17
		Tr1/10kN	10	1,959178	1,5	1,959180	2,1	1,959279	2,10	1,959221	2,13
		Tr2/10 kN	5	1,009981	2,5	1,009984	2,1	1,009991	2,05	1,009981	2,11
		Tr2/10kN	10	2,019989	2,5	2,019992	2,1	2,020009	2,05	2,019990	2,11
		Tr1/5 kN	5	1,881989	-2,0	1,881987	2,1	1,881984	2,01	1,881987	2,02
		Tr2/5 kN	5	2,027788	-1,5	2,027787	2,1	2,027796	2,04	2,027800	2,04
Spain	4	Tr1/10 kN	5	0,979680	-2,5	0,979678	2,1	0,979707	2,17	0,979738	2,10
		Tr1/10kN	10	1,959175	-7,5	1,959168	2,0	1,959221	2,13	1,959272	2,06
		Tr2/10 kN	5	1,009976	-2,0	1,009974	2,0	1,009981	2,11	1,009978	2,05
		Tr2/10kN	10	2,019980	-5,5	2,019975	2,0	2,019990	2,11	2,019989	2,06
Japan	5	Tr1/10 kN	5	0,979668	8,5	0,979677	1,8	0,979716	2,09	0,97972	2,07
		Tr1/10kN	10	1,959153	15,5	1,959169	1,6	1,959243	2,05	1,959243	2,06
		Tr2/10 kN	5	1,009952	7,9	1,009960	1,8	1,009972	2,09	1,009965	2,09
		Tr2/10kN	10	2,019940	12,6	2,019953	1,3	2,019977	2,08	2,019965	2,10

Table 1.5a. Summary of the measured data, part I.

Country		***Trans.***	***Load***	x'_{L}	$c_{DMP, L}$	x_{L}	w_{L}	x_{PA}	w_{PA}	x_{PB}	w_{PB}
	Nr	Identif.	kN	mV/V	10^{-6}·mV/V	mV/V	*10^{-5}	mV/V	*10^{-5}	mV/V	*10^{-5}
México	6	Tr1/10 kN	5	0,979705	-5,5	0,979699	2,9	0,979679	2,08	0,979706	2,08
		Tr1/10kN	10	1,959223	-7,5	1,959215	3,2	1,959173	2,05	1,959223	2,03
		Tr2/10 kN	5	1,010004	2,5	1,010007	2,5	1,010008	2,10	1,009993	2,04
		Tr2/10kN	10	2,020005	5,5	2,020011	2,8	2,020043	2,06	2,020014	2,04
Turkey	7	Tr1/10 kN	5	0,979688	2,5	0,979690	1,0	0,979707	2,11	0,979708	2,12
		Tr1/10kN	10	1,959180	6,0	1,959186	2,1	1,959222	2,08	1,959225	2,10
		Tr2/10 kN	5	1,009950	3,5	1,009953	1,6	1,00996	2,07	1,00996	2,06
		Tr2/10kN	10	2,019924	8,0	2,019932	2,2	2,019952	2,07	2,019954	2,08
UK	8	Tr1/10 kN	5	0,979703	2,5	0,979706	1,6	0,979729	2,12	0,979707	2,11
		Tr1/10kN	10	1,959207	10,5	1,959219	1,3	1,959257	2,09	1,959222	2,08
		Tr2/10 kN	5	1,009983	2,0	1,009985	1,3	1,009965	2,09	1,00996	2,10
		Tr2/10kN	10	2,019979	11,0	2,019990	1,5	2,019959	2,07	2,019952	2,07
USA	9	Tr1/10 kN	5	0,9796831	4,0	0,979687	1,2	0,979706	2,08	0,979739	2,14
		Tr1/10kN	10	1,9591891	9,0	1,959198	1,2	1,959223	2,03	1,959279	2,10
		Tr2/10 kN	5	1,009971	5,0	1,009976	1,2	1,009993	2,04	1,009991	2,05
		Tr2/10kN	10	2,019979	8,0	2,019987	1,2	2,020014	2,04	2,020009	2,05
China	10	Tr1/10 kN	5	0,979681	7,5	0,979689	1,4	0,979723	2,12	0,979716	2,09
		Tr1/10kN	10	1,959201	13,0	1,959214	1,4	1,959251	2,08	1,959243	2,05
		Tr2/10 kN	5	1,009946	7,5	1,009953	1,4	1,009973	2,07	1,009972	2,09
		Tr2/10kN	10	2,019942	11,5	2,019952	1,4	2,019983	2,11	2,019977	2,08
France	11	Tr1/5 kN	5	1,882003	8,0	1,882011	1,2	1,881990	2,01	1,882003	2,01
		Tr2/5 kN	5	2,027790	2,5	2,027793	1,2	2,027795	2,08	2,027797	2,10
Korea	12	Tr1/5 kN	5	1,8820780	-7,0	1,882077	1,6	1,881976	2,01	1,882062	2,01
		Tr2/5 kN	5	2,0278097	-7,5	2,027802	1,6	2,027794	2,05	2,027807	2,06
Netherlands	13	Tr1/5 kN	5	1,881925	-8,5	1,881917	10,2	1,882006	2,01	1,882009	2,01
		Tr2/5 kN	5	2,027855	-5,0	2,027850	10,0	2,027824	2,02	2,027822	2,03
Singapore	14	Tr1/5 kN	5	1,8820726	2,0	1,882075	10,4	1,882062	2,01	1,882071	2,01
		Tr2/5 kN	5	2,0277859	2,0	2,027788	10,5	2,027807	2,06	2,027801	2,05
Australia	15	Tr1/5 kN	5	1,882019	-2,0	1,882017	3,1	1,881981	2,01	1,881976	2,01
		Tr2/5 kN	5	2,027819	2,0	2,027821	2,9	2,027792	2,05	2,027794	2,08

Table 1.5b. Summary of the measured data, part II.

1.3.2 Relative deviation of the measured values between participant laboratories and pilot

The following figures give, for each transducer, the relative deviations between each participating laboratory and the Pilot, compared against the mean pilot value. Laboratories are numbered from 1 to 15 (see Table 1.1).

The reference value for each loop is defined as the mean value of the A and B measurements;

$$X_P = \frac{X_{PA} + X_{PB}}{2} \qquad (1.5)$$

The relative deviation is calculated for each individual loop using the following equation;

$$d = (X_L - X_P)/X_P \qquad (1.6)$$

The participating laboratory value incorporates the BN100 correction. The uncertainty value includes the uncertainty associated with the mean pilot value but does not include the drift.

Transducer TR1 10 kN

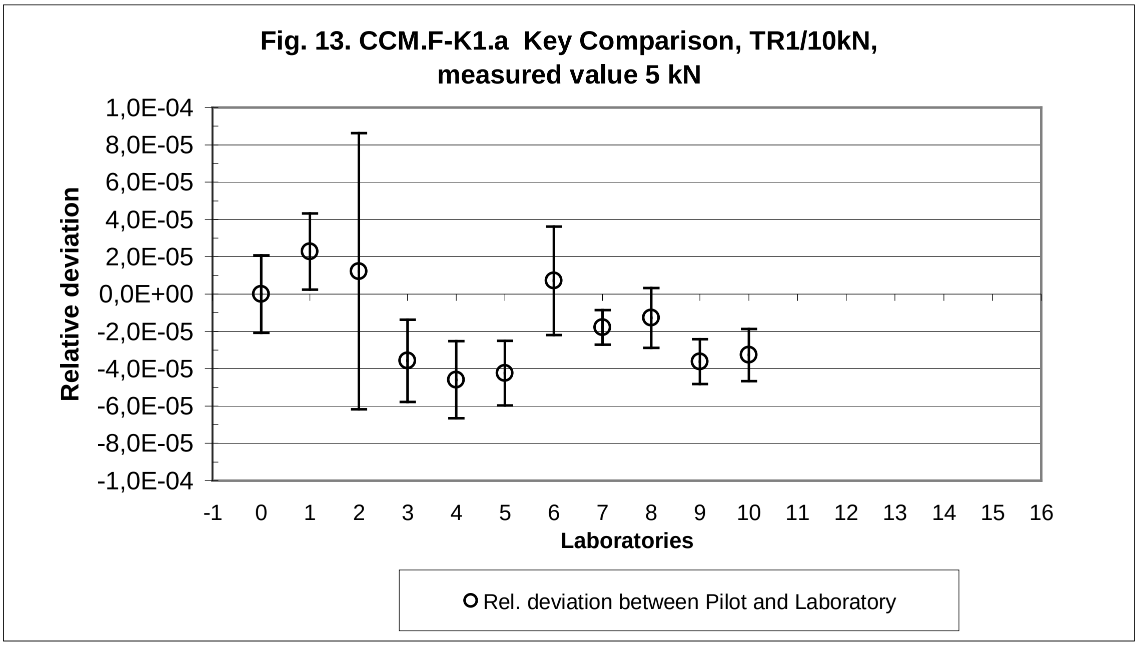


Figure 1.13. Transducer TR1/10kN, 5 kN load, relative deviations between laboratory and pilot and the relative expanded uncertainty of the measurement by the participating laboratory.

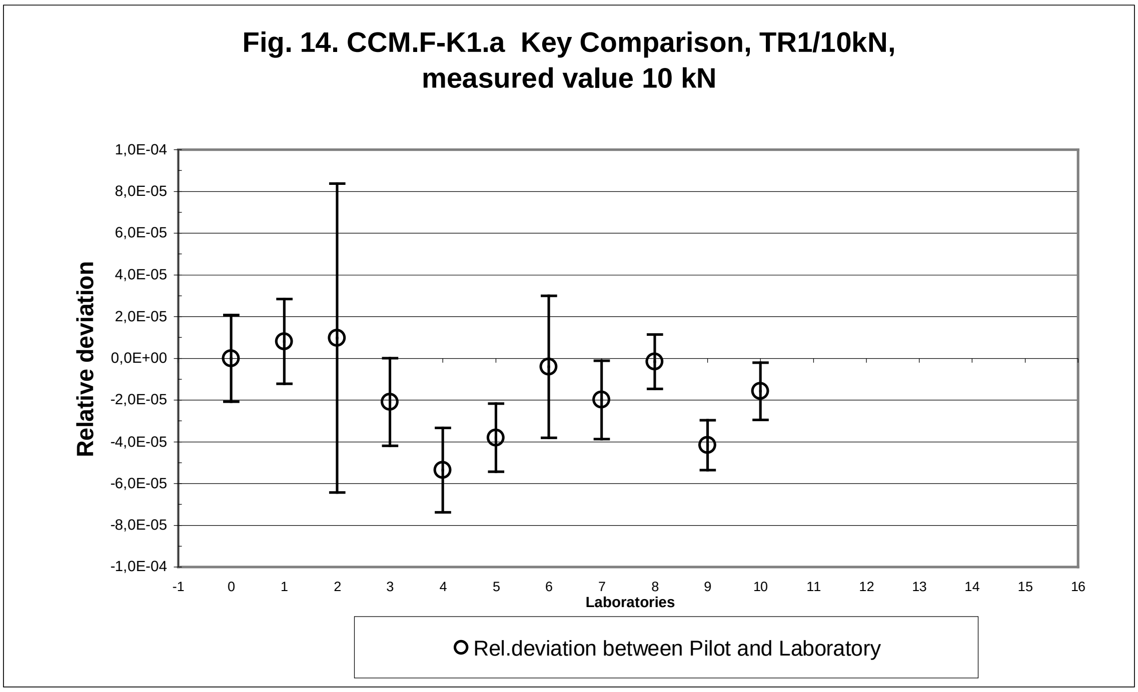


Figure 1.14. Transducer TR1/10kN, 10 kN load, relative deviations between laboratory and pilot and the relative expanded uncertainty of the measurement by the participating laboratory.

TR1/10 kN	5 kN		10 kN	
Laboratory	Rel. difference between Laboratory and Pilot	Rel. expanded uncertainty of the measurement	Rel. difference between Laboratory and Pilot	Rel. expanded uncertainty of the measurement
0	0,0E+00	2,08E-05	0,0E+00	2,08E-05
1	2,3E-05	2,04E-05	8,1E-06	2,03E-05
2	1,2E-05	7,40E-05	9,7E-06	7,40E-05
3	-3,6E-05	2,20E-05	-2,1E-05	2,10E-05
4	-4,6E-05	2,07E-05	-5,4E-05	2,02E-05
5	-4,2E-05	1,74E-05	-3,8E-05	1,63E-05
6	7,1E-06	2,91E-05	-4,1E-06	3,40E-05
7	-1,8E-05	9,30E-06	-2,0E-05	1,87E-05
8	-1,3E-05	1,60E-05	-1,6E-06	1,30E-05
9	-3,6E-05	1,20E-05	-4,2E-05	1,20E-05
10	-3,3E-05	1,40E-05	-1,6E-05	1,37E-05

Table 1.8. Relative differences between pilot and participating laboratories in each loop and the relative expanded uncertainty of the measurements by the laboratories for transducer “TR1/10kN”.

Transducer TR2 10 kN

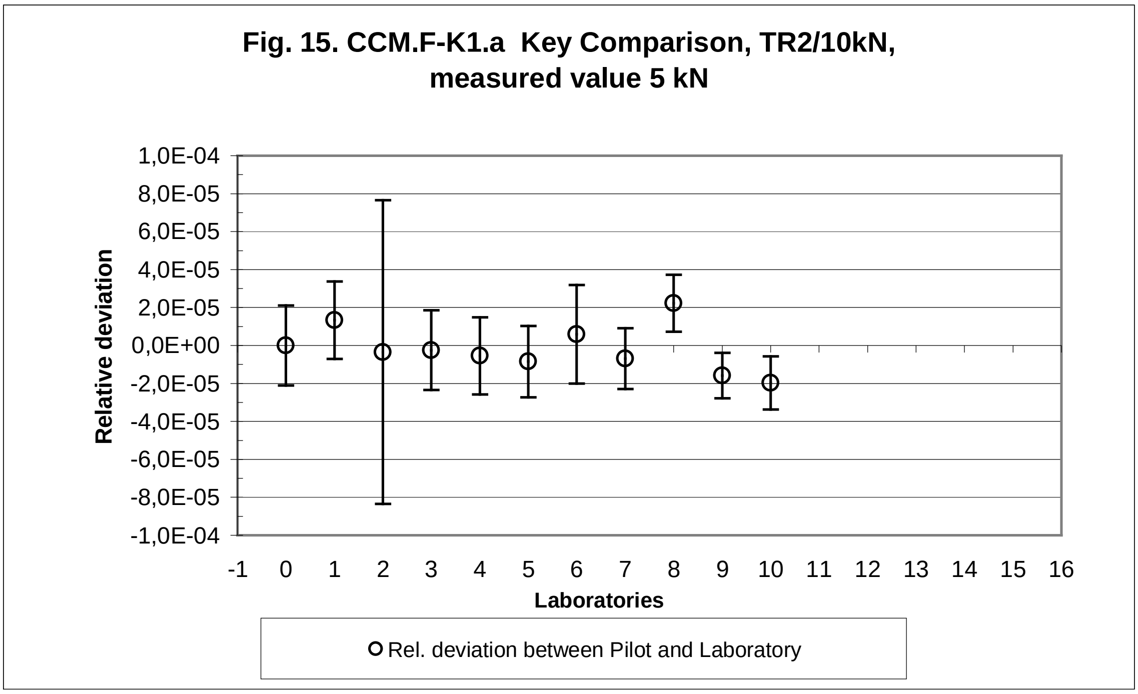


Figure 1.15. Transducer ”TR2/10 kN”, 5 kN load, relative deviations between laboratory and pilot and the relative expanded uncertainty of the measurement by the participating laboratory.

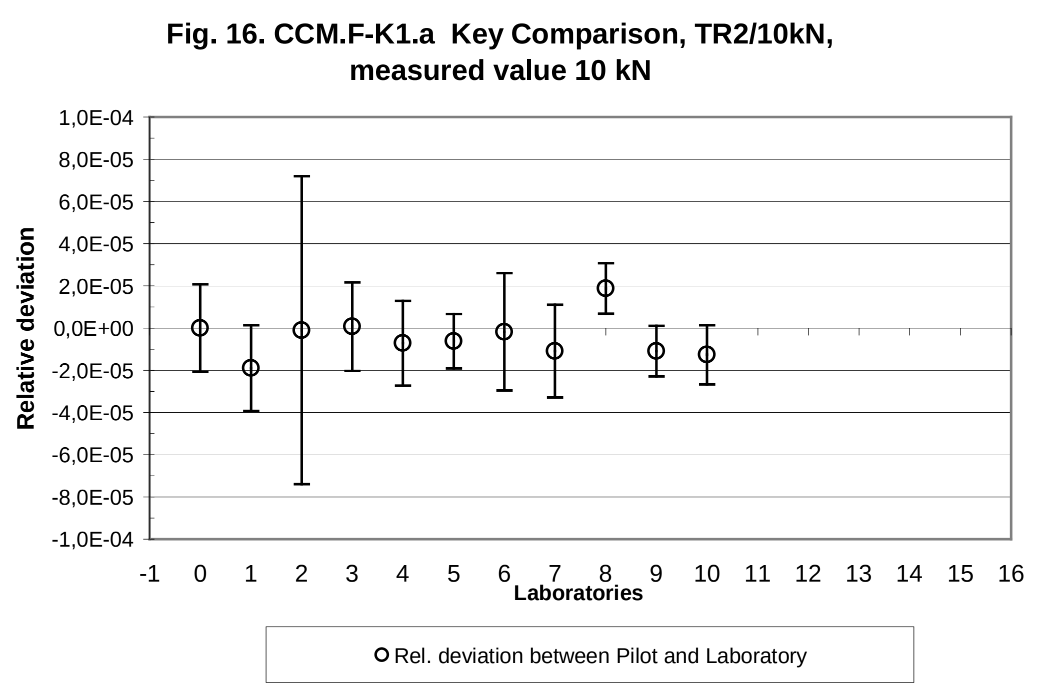


Figure 1.16. Transducer TR2/10kN, 10 kN load, relative deviations between laboratory and pilot and the relative expanded uncertainty of the measurement by the participating laboratory.

TR2/10 kN	5 kN		10 kN	
Laboratory	Rel. difference between Laboratory and Pilot	Rel. expanded uncertainty of the measurement	Rel. difference between Laboratory and Pilot	Rel. expanded uncertainty of the measurement
0	0,0E+00	2,10E-05	0,0E+00	2,08E-05
1	1,3E-05	2,04E-05	-1,9E-05	2,03E-05
2	-3,5E-06	8,00E-05	-1,0E-06	7,30E-05
3	-2,5E-06	2,10E-05	7,0E-07	2,10E-05
4	-5,4E-06	2,03E-05	-7,2E-06	2,01E-05
5	-8,5E-06	1,88E-05	-6,2E-06	1,29E-05
6	5,9E-06	2,60E-05	-1,8E-06	2,78E-05
7	-6,9E-06	1,60E-05	-1,1E-05	2,20E-05
8	2,2E-05	1,50E-05	1,9E-05	1,20E-05
9	-1,6E-05	1,20E-05	-1,1E-05	1,20E-05
10	-2,0E-05	1,40E-05	-1,3E-05	1,40E-05

Table 1.9. Relative differences between pilot and participating laboratories in each loop and the relative expanded uncertainty of the measurements by the laboratories for transducer TR2/10kN.

Transducer TR1/5 kN

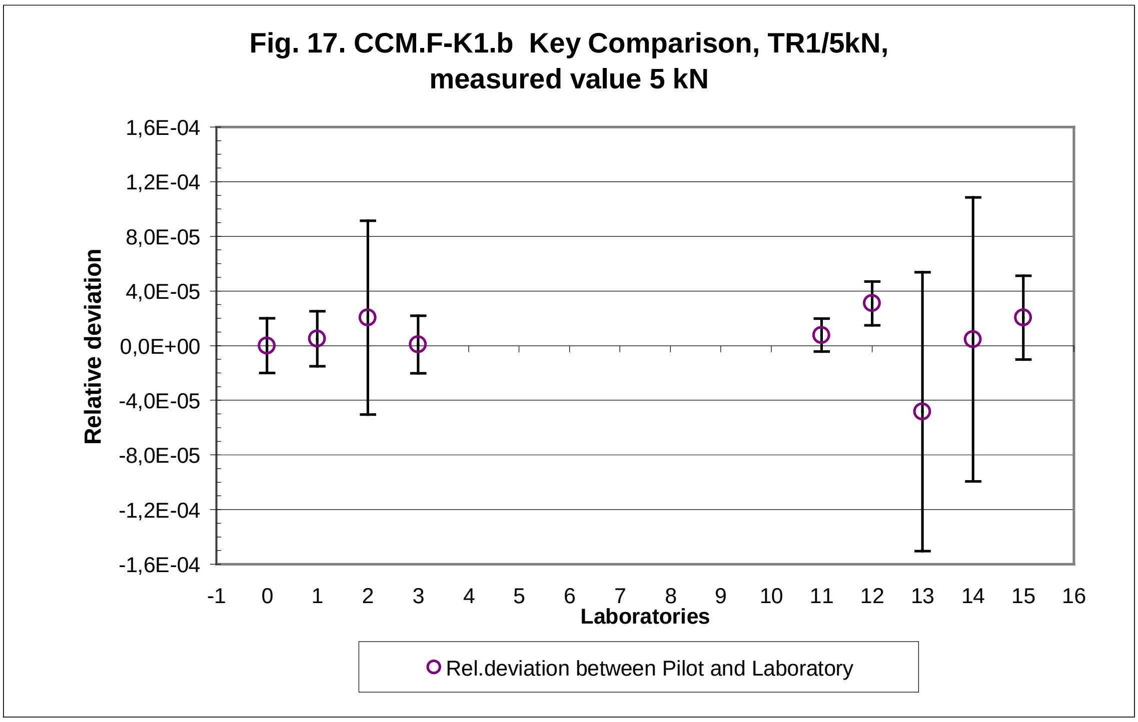


Figure 1.17. Transducer TR1/5kN, 5 kN load, relative deviations between laboratory and pilot and the relative expanded uncertainty of the measurement by the participating laboratory.

TR1/5 kN	5 kN	
Laboratory	Rel. difference between Laboratory and Pilot	Rel. expanded uncertainty of the measurement
0	0,0E+00	2,01E-05
I1	5,0E-06	2,01E-05
2	2,0E-05	7,10E-05
3	8,0E-07	2,10E-05
11	7,7E-06	1,20E-05
12	3,1E-05	1,60E-05
13	-4,8E-05	1,02E-04
14	4,5E-06	1,04E-04
15	2,0E-05	3,07E-05

Table 1.10. Relative differences between Pilot and participating laboratories in each loop and the relative expanded uncertainty of the measurements by the laboratories for transducer TR1/5kN.

Transducer TR2/5 kN

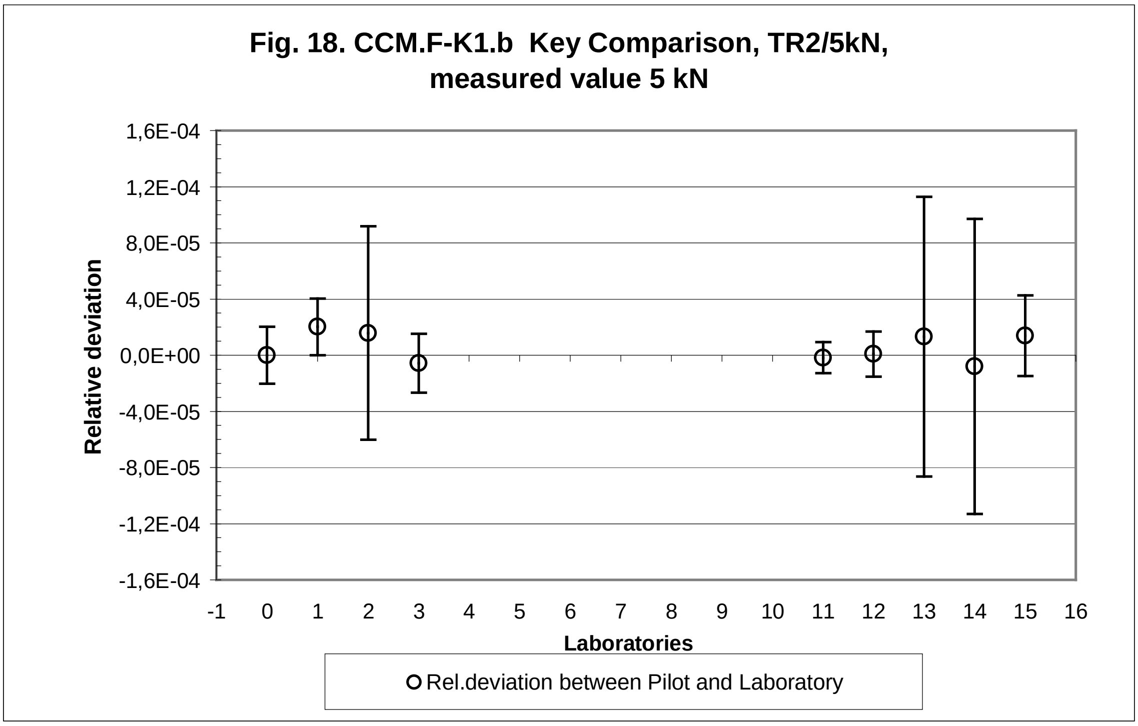


Figure 1.18. Transducer TR2/5kN, 5 kN load, relative deviations between laboratory and pilot and the relative expanded uncertainty of the measurement by the participating laboratory.

TR2/5 kN	5 kN	
Laboratory	Rel. difference between Laboratory and Pilot	Rel. expanded uncertainty of the measurement
0	0,0E+00	2,04E-05
1	2,0E-05	2,02E-05
2	1,6E-05	7,60E-05
3	-5,7E-06	2,10E-05
11	-1,7E-06	1,10E-05
12	8,4E-07	1,60E-05
13	1,3E-05	9,96E-05
14	-7,9E-06	1,05E-04
15	1,4E-05	2,87E-05

Table 1.9. Relative differences between Pilot and participating laboratories in each loop and the relative expanded uncertainty of the measurements by the laboratories for transducer Tr2/5kN.

Chapter 2,
Values for single transducers

2.1 Used symbols

D = abs. difference (laboratory - ref. value)
X_{PA} = Measured deflection by pilot for A measurement
X_{B} = Measured deflection by pilot for B measurement
Δ_{corr} = Correction of the mean value to the true measurement day
X_{O} = Measured deflection by pilot, without correction of drift
X_{P} = Used deflection value for pilot for loop n
X'_{L} = Measured deflection value of the laboratory
X''_{L} = Deflection value with BN100 correction for the laboratory
X_{L} = Used deflection value with correction of the laboratory (BN 100 and possible other corrections etc. temperature, extrapolation aso.)
u_{PA} = Standard uncertainty of the X_{PA}
u_{PB} = Standard uncertainty of the X_{PB}
u_{P} = Standard uncertainty of the X_{P}
u'_{L} = Standard uncertainty of the X'_{L}
u_{L} = Standard uncertainty of the X_{L}
w_{PA} = Relative standard uncertainty of X_{PA}
w_{PB} = Relative standard uncertainty of X_{PB}
w_{P} = Relative standard uncertainty of X_{P}
w_{L} = Relative standard uncertainty of X_{L}
w_{drift} = Relative standard uncertainty by the estimation of the drift of the transducers between A and B measurement by pilot
$w_{\mathrm{L,tot}}$ = Relative standard uncertainty of X_{L} with the uncertainty of drift
d_{L} = Relative measured deviation between the laboratory and the pilot
d_{ref} = Relative reference deviation value
$d_{L,\mathrm{ref}}$ = Relative deviation between the laboratory and the reference values
d_{pairs} = Relative deviation between two laboratories
$w_{d_{\mathrm{L,ref}}}$ = Relative standard uncertainty of the $d_{L,\mathrm{ref}}$
w_{d} = Relative standard uncertainty of the d
w_{dref} = Relative standard uncertainty of the d_{ref}
w_{pairs} = Relative standard uncertainty of the d_{pairs}

Used terminology:

Reference value	calculated reference value for one transducer
Deflection	Indication of the DMP 40 by the measurement

2.2 The deviation between Pilot and Laboratory

For each Laboratory and each transducer will be calculated a single value as deviation. This value is based on results of differences between Pilot and Laboratories. The model of the deviation is:

$$D = X_L - X_P \qquad (2.1)$$

$$X_P = X_0 + \Delta_{corr} \qquad (2.2)$$

The correction is used for the pilot measurement. It is assumed that the drift is linear.

2.2.1 The used deflection value by Pilot for loop n

The used deflection for loop n is calculated from A and B measurements by Pilot with the correction of the drift. The assumption for the drift is a linear drift between A and B measurements, which are made as close as possible before and after the measurement of the participating laboratory. The needed time for transport in one direction has not been equal for all participants, variation from four days up to 2 weeks. The correction of the drift is made as function of the time.

$$\Delta_{corr} = \frac{(X_{PB} - X_{PA})}{t_{total}} \cdot t_1 \qquad (2.3)$$

Where:
t_1 = time between pilot A and calibration by laboratory
t_2 = time between calibration by laboratory and pilot B
$t_{total} = t_1 + t_2$

The value X_P has been used as reference for loop *n*.

$$X_P = X_{PA} + \Delta_{corr} \qquad (2.4)$$

Reference values from Pilot with correction of the non-symmetry in time						
Transducer	TR1/10 kN	TR1/10 kN	TR2/10 kN	TR2/10 kN	TR1/5 kN	TR2/5 kN
Loading value	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
Laboratory	mV/V	mV/V	mV/V	mV/V	mV/V	mV/V
1			1,009997	2,020022	1,881994	2,027821
2	0,979683	1,959176	1,010011	2,020045	1,882008	2,027815
3	0,979722	1,959248	1,009986	2,019999	1,881986	2,027798
4	0,979720	1,959243	1,009980	2,019990		
5	0,979718	1,959243	1,009968	2,019970		
6	0,979694	1,959201	1,009999	2,020025		
7	0,979708	1,959224	1,009960	2,019953		
8	0,979722	1,959245	1,009963	2,019956		
9	0,979720	1,959247	1,009992	2,020012		
10	0,979719	1,959247	1,009972	2,019980		
11					1,881995	2,027796
12					1,882016	2,027801
13					1,882008	2,027823
14					1,882066	2,027805
15					1,881979	2,027793

Table 2.1. Used reference values for each loop after the correction of non-symmetric timing of measurement by participating laboratory.

The correction, Δ_{corr}, is mainly relative small, however the maximal change is $8{,}6\cdot10^{-5}$ mV/V, which is from one single transducer and the reason is unknown, and it seems to be an external influence. The standard deviation for change is $2{,}4\cdot10^{-5}$ mV/V. The most changes happens with transducers TR1/10 kN.

2.3 Principle to use the pilot FSM as link in the comparison

The comparison has been made as a star form; the transducers came back to the pilot after every measurement by the participating laboratory. One complete measurement, Pilot – Participating Laboratory – Pilot is called a loop.

Pilot's measurements have been made always on the same deadweight machine for the whole key-comparison. This pilot's machine is a link between all participants. The pilot's link machine is noticed as PLM on this paper.

The pilot laboratory is also a participating laboratory (lab0). Pilot laboratory, as participant, make also a comparison between PLM and its own reference calibration machine (PM).

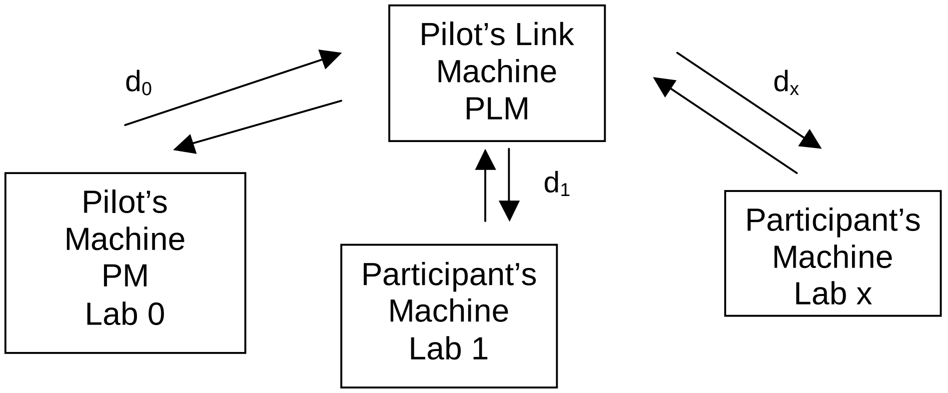


Figure 2.1: Principe of the star type comparison with the link machine.

This graph can be applied even if the PLM and the PM are the same machine.

For each loop, ie for each laboratory, the relative deviation (d) from the PLM is computed according to: $d_L = (X_L - X_P) / X_P$

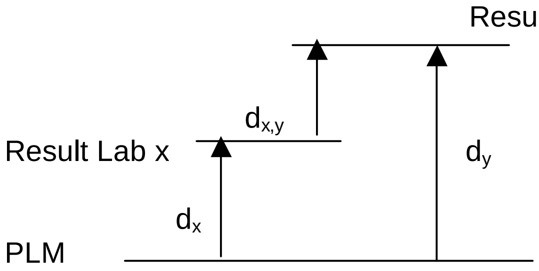


Figure 2.2

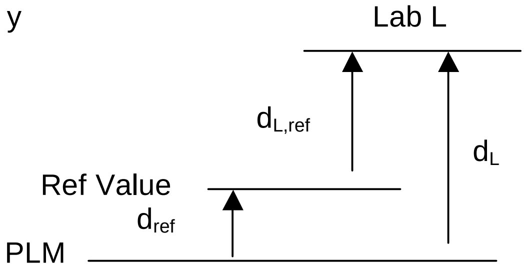


Figure 2.3

Figure 2.2 : Using the pilot as a link, the deviation between 2 laboratories is equal to $d_{x,y} = d_y - d_x$. So to compute the deviation between 2 laboratories, it is not necessary to use a traceable PLM but a machine enough stable between the 2 loops.

Figure 2.3 : The same approach is made for the deviation from the reference value and a laboratory. Using the pilot as a link, the deviation between a laboratory and the reference value is equal to $d_{L,ref} = d_L - d_{ref}$. To compute the deviation between a laboratory and the reference value, it is not necessary to use a traceable PLM but a machine enough stable during all the key-comparison.

2.3.2 Relative standard uncertainty due to the stability of the pilot link machine : w_{PLM}

For the two reasons given above, the uncertainty of the PLM is computed taking into account only components of stability. Following components are given in relative value with k =1

- stability of masses : $1,0\cdot 0^{-6}$
- stability of gravity : $0,2\cdot 10^{-6}$
- stability of the air buoyancy : $3,5\cdot 10^{-6}$

The combined relative standard uncertainty due to the stability of pilot FSM can be estimated w'_{PLM} equal to :

$$w_{PLM} = 0,35\cdot 10^{-5} \qquad (2.5)$$

2.3.3 Relative standard uncertainty of deflection obtained at pilot laboratory : w_P

The reference value X_P for each loop is based on A and B measurements according to equation (2.4). The pilot had a variation of relative reproducibility with rotation $w_{PR} = 0,2\cdot 10^{-5} \ldots 1,0\cdot 10^{-5}$, based on the information from pilots A and B measurements. By discrimination the pure stability of FSM from instability of force transducers the value of $0,6\cdot 10^{-5}$ can be assumed as minimum value of stability of FSM by pilot. The uncertainty of the deflection by pilot (w_{PA} or w_{PB}) includes the pilot reproducibility uncertainty (w_{PR}) and the stability of PLM (w_{PLM}):

$$w_{PA} = w_{PB} = \sqrt{(w_{PR}{}^2 + w_{PLM}{}^2)} = 0,6 \cdot 10^{-5} \qquad (2.6)$$

The mean value of relative standard uncertainty is assumed as well :

$$w_P' = \sqrt{\frac{w_{PA}{}^2 + w_{PB}{}^2}{2}} = 0,6 \cdot 10^{-5} \qquad (2.7)$$

However to get the consistency for all measurements, which are influenced with the uncertainty of pilot FSM we have to use a value for standard uncertainty:

$$\mathbf{w_P = 0,75\cdot 10^{-5}} \qquad (2.8)$$

2.4 Calculation of the reference value for each transducer

2.4.1 Relative deviation between the laboratory and pilot

The relative deviation between laboratory and pilot value has been calculated using equation (2.9) for each transducer and for each laboratory:

$$d_L = (X_L - X_P) / X_P \qquad (2.9)$$

2.4.2 The relative standard uncertainty w_d of the deviation between pilot and laboratory by using the pilot as link

The standard uncertainty w_d of the relative deviation between laboratory and pilot is calculated using the equation (2.10).

Relative uncertainty of the deviation is taking in to account:

- the relative uncertainty of the calibration laboratory,
- the relative uncertainty of the pilot
- the relative uncertainty of the drift.

$$w_d = \sqrt{w_L{}^2 + w_P{}^2 + w_{\Delta,corr}} \qquad (2.10)$$

Where for the relative standard uncertainty of the pilot machine has been used the value from equation calculated by using the equation (2.8) and the value is:

$$w_P = 0{,}75 \cdot 10^{-5}$$

The drift has been assumed as a linear function between A and B measurement by Pilot. The time between these measurements is t_{total} and the participating laboratory has measured it in time t_{total} /2 with deviation ± *n* days. As basement for the evaluation of the uncertainty for drift the following tables show the mean values and standard deviations for the drift as total drifts between Pilot and Laboratories calculated for one day and for four days.

Transducer	TR1/10 kN		TR2/10 kN		TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
$\bar{x}$	3,47E-06	3,52E-06	-9,90E-07	-9,90E-07	8,44E-06	4,93E-07
s	2,24E-05	1,92E-05	1,43E-05	1,19E-05	1,57E-05	3,49E-06

Table 2.2 Mean values and standard deviations of the all drift for transducers between Pilot and Laboratory

The drift as total value are very similar, only for transducer TR2/5kN the drift is closely one decade lower. Closer examination shows that the change for TR1/10kN is less systematic but in same order than drift for other transducers.

Transducer	TR1/10 kN		TR2/10 kN		TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
$\bar{x}$	2,40E-07	2,77E-07	-2,53E-07	-2,24E-07	1,23E-06	1,15E-07
s	3,48E-06	2,96E-06	1,67E-06	1,39E-06	2,22E-06	4,52E-07

Table 2.3. Mean values and standard deviations for drift of one day, calculated individually for each laboratory

Transducer	TR1/10 kN		TR2/10 kN		TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
$\bar{x}$	1,29E-06	1,10E-06	2,19E-07	1,63E-07	2,08E-06	2,61E-07
s	4,61E-06	4,00E-06	2,32E-06	1,98E-06	2,83E-06	6,17E-07

Table 2.4. Mean values and standard deviations for drift of four days, calculated individually for each laboratory

As conclusion the uncertainty of the drift is selected as four days drift of the total change for each laboratory. It is about 1/10 part of the total drift. It gives also for the transducer TR1/10kN closely twice larger uncertainty as more unstable drift.

The relative uncertainty of the drift is based on the variation width (*a*) during four days and assumed to have rectangular distribution. The uncertainty is connected individual to the drift of each transducer.

$$a_{drift} = \frac{drift_{total}}{t_{total}} * 4 \qquad (2.11)$$

The relative uncertainty of the drift is:

$$w_{\Delta,corr} = \frac{a_{drift}}{2 \cdot \sqrt{3} \cdot X_P} \qquad (2.12)$$

2.4.3 Reference value as weighted mean

The weighted mean has been calculated for each transducer separately, TR1/10 kN and TR2/10kN for K1a and TR1/5kN and TR2/5kN for K1b. The weighted value has been calculated by weighting the mean value with the uncertainty of the deviation as follow:

For the calculation the uncertainties of the calibration laboratory and the relative measured deviation between the laboratory and the pilot have been used.

Reference value as relative deviation for each transducer:

$$d_{ref} = \frac{\sum_{L=1...n} p_L d_L}{\sum_{L=1...1n} p_L} \qquad n = 11 \text{ or } 9 \qquad (2.13)$$

where the weighing factor is $$p_L = \frac{1}{w_{dL}{}^2}$$

The standard uncertainty of the reference value, weighted by its uncertainties:

$$w_{d\,\mathrm{Re}\,f} = \sqrt{\frac{1}{\sum_{L=1...n} \frac{1}{w_{dL}{}^2}}} \qquad n = 11 \text{ or } 9 \qquad (2.14)$$

2.5 The degree of equivalence

The degree of equivalence of each participating laboratory's is expressed by (according to M.G. Cox, Metrologia 2002,39):

- its deviation from the key comparison reference value, equation (2.15)
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (2.16).

$$d_{L,ref} = d - d_{ref} \qquad (2.15)$$

The uncertainty of the relative deviation between laboratory and reference value is calculated using the equation (2.13) and standard uncertainty of the deviation is taking in to account:

- the uncertainty between the calibration laboratory and the pilot, caused by the deviation and including the uncertainty of laboratory,
- the uncertainty of the reference value.

$$w_{d,\ ref}^{2} = (w_d^2 - w_{ref}^2\ ) \qquad (2.16$$

The values are given on table 2.19.

The degree of equivalence between pairs of laboratories is expressed by:

- its difference of their deviations from each other, equation (2.17),
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (2.18).

$$d_{pairs} = d_{L,n} - d_{L,n+1} \qquad (2.17)$$

$$w_{pairs} = \sqrt{w_{L,tot,n}^{\ \ 2} + w_{L,tot,n+1}^{\ \ 2}} \qquad (2.18)$$

The values of pairs and their uncertainties are given on table 2.15 …2.26.

2.6 Consistency check

According the proposal of M.G. Cox “The evaluation of key comparison data” the consistency should be checked with X^2-test. The results are following:

Transducer	TR1/10 kN		TR2/10 kN		TR1/5kN	TR1/5 kN
Force	5 kN	10 KN	5 kN	10 kN	5 kN	5 kN
X^2_{obs}	22,2	16,6	13,8	6,4	8,2	3,5
$X^2(v)$	18,3	18,3	18,3	18,3	15,5	15,5

Table 2.5. Result of the consistency check

According this result the load 5 kN transducer TR1/10 kN does not have required consistency but it is very closely for that. It depends mainly from relative high deviation from reference value with a small measurement uncertainty. The using pilot as link reduces the uncertainty and the deviation are still same which lead to the fact that consistency is less and the results is less good. To get the consistency on that level the results for transducer TR1/10 kN by laboratory no 1 are not included in to evaluation.

2.7 Tables and figures

Transducer TR1/10 kN

Reference values for transducer TR1/10 kN, values in 10^{-5}			
Load 5 kN		Load 10 kN	
Ref. Value	Uncertainty W_{ref} (k=2)	Ref. Value	Uncertainty W_{ref} (k=2)
-2,15	0,70	-1,62	0,72

Table 2.6. Reference values for transducer “TR1/10 kN” with corresponding expanded uncertainties

	Transducer TR 1/10 kN, values in 10^{-5}			
Laboratory	Rel. deviation to reference value	Expanded uncertainty of laboratory, $W_{L,\ ref}$	Rel. deviation to reference value	Expanded uncertainty of laboratory, $W_{L,\ ref}$
	5 kN	*U (k=2)*	10 kN	*U (k=2)*
0	2,14	1,66	1,60	1,66
1				
2	3,37	7,29	2,42	7,29
3	-1,30	2,81	-1,88	2,73
4	-2,24	2,71	-2,26	2,66
5	-2,13	2,40	-2,20	2,33
6	2,69	3,37	2,33	3,63
7	0,35	1,92	-0,33	2,70
8	0,49	2,37	0,23	2,16
9	-1,22	2,13	-0,88	2,11
10	-1,10	2,17	-0,17	2,17

Table 2.7. Results, transducer TR1/10 kN as relative deviation to the reference value and with expanded uncertainty of the calibration laboratory.

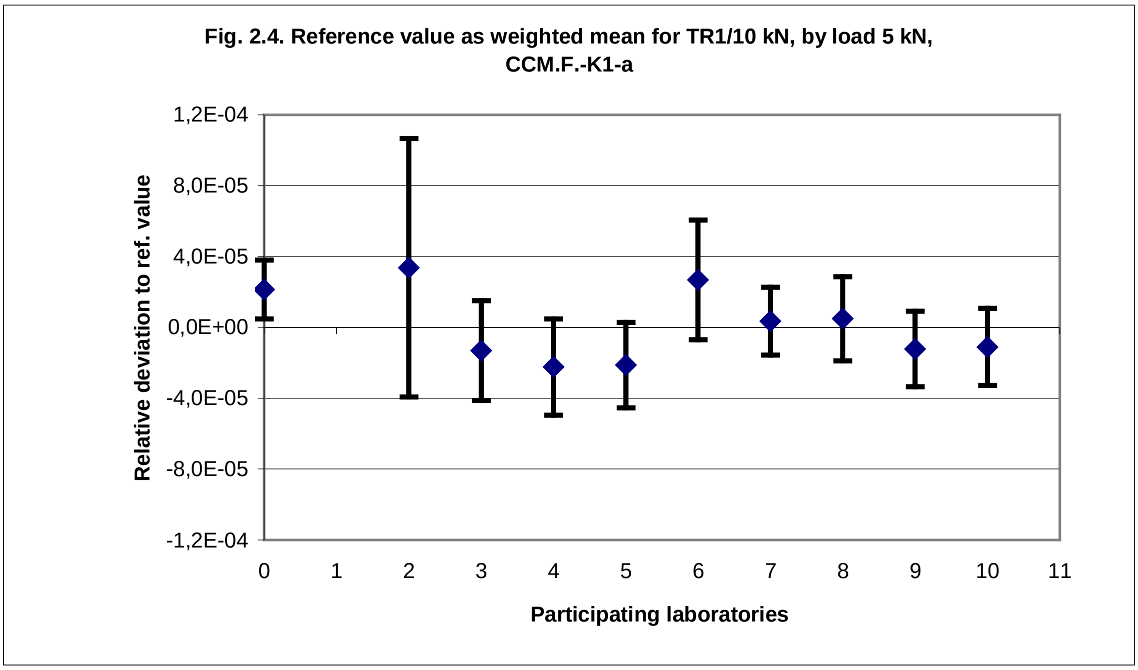


Figure 2.4. Transducer TR1/10 kN; 5 kN loading point: relative deviation to the reference value and expanded uncertainty of the calibration laboratory

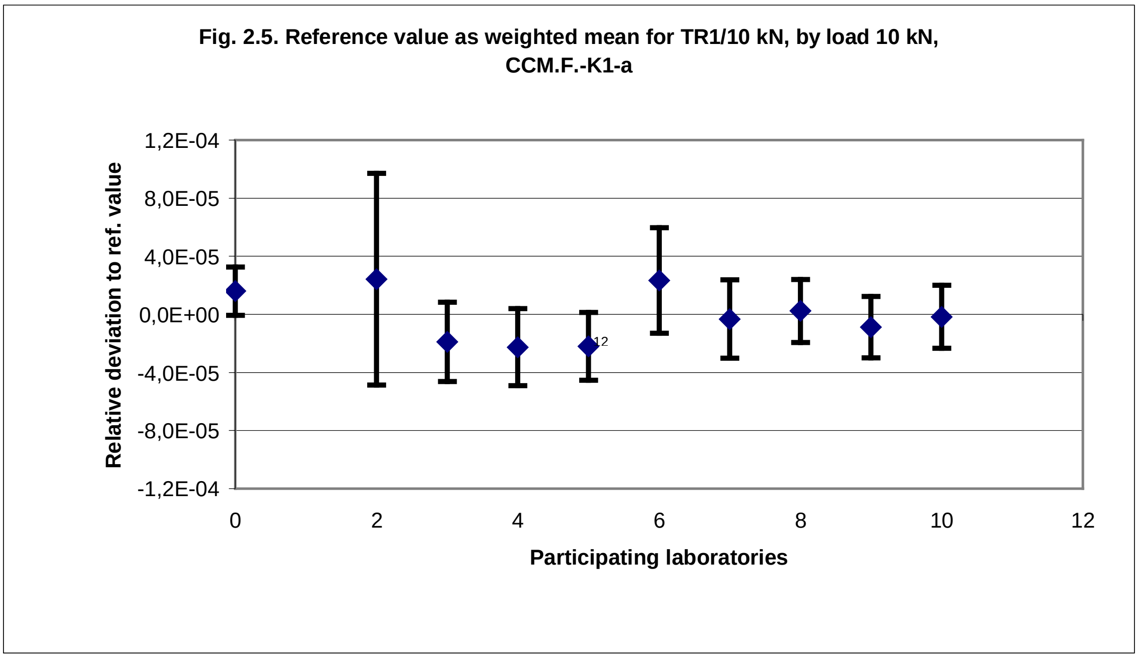


Figure 2.5. Transducer TR1/10 kN; 10 kN loading point: relative deviation to the reference value and expanded uncertainty of the calibration laboratory

Transducer TR2/10 kN

Reference values for transducer TR2/10 kN, values in 10^{-5}			
Load 5 kN		Load 10 kN	
Ref. Value	Uncertainty W_{ref} $(k=2)$	Ref. Value	Uncertainty W_{ref} $(k=2)$
-0,20	0,68	-0,44	0,68

Table 2.8. Reference values for transducer TR2/10 kN with corresponding expanded uncertainties

	Transducer TR2/10 kN, values in 10^{-5}			
Laboratory	Rel. deviation to reference value	Expanded uncertainty of laboratory, $W_{L,\ ref}$	Rel. deviation to reference value	Expanded uncertainty of laboratory, $W_{L,\ ref}$
	5 kN	*U (k=2)*	10 kN	*U (k=2)*
0	0,19	1,65	0,44	1,65
1	1,74	2,67	0,21	2,65
2	-0,18	7,29	0,23	7,29
3	-0,04	2,67	0,06	2,67
4	-0,38	2,62	-0,31	2,61
5	-0,61	2,50	-0,43	2,09
6	0,99	2,99	-0,26	3,24
7	-0,50	2,32	-0,62	2,72
8	2,38	2,10	2,12	2,23
9	-1,39	2,10	-0,77	2,04
10	-1,78	2,18	-0,96	2,04

Table 2.9. Results for transducer TR2/10 kN as relative deviation to the reference value and expanded uncertainty of the calibration laboratory.

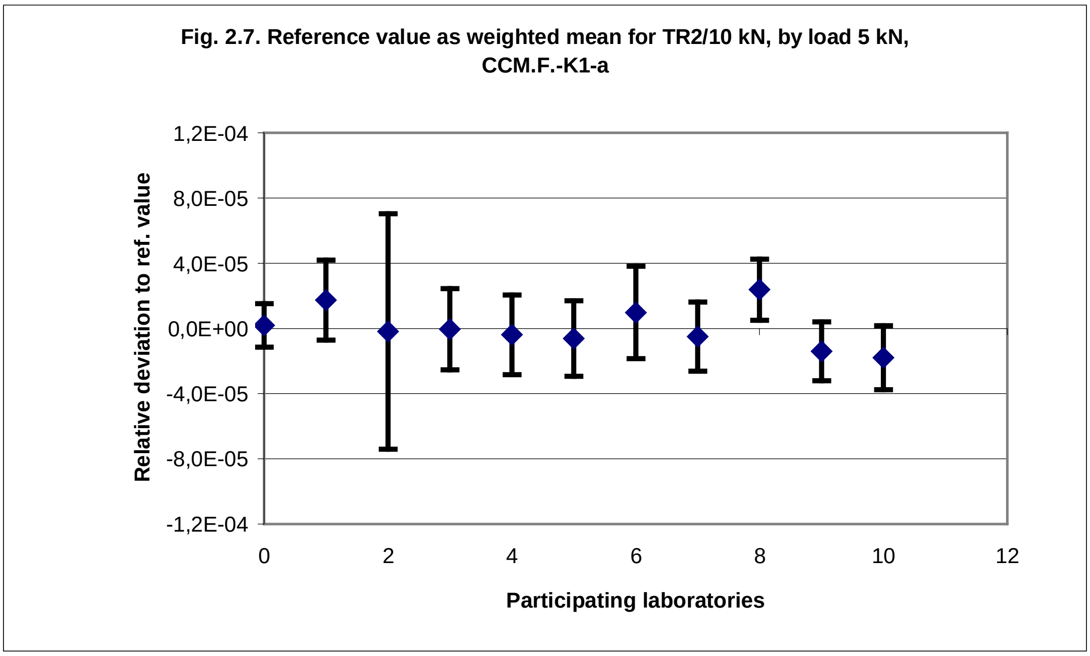


Figure 2.6. Transducer TR2/10 kN; 5 kN loading point: relative deviation to the reference value and expanded uncertainty of the calibration laboratory

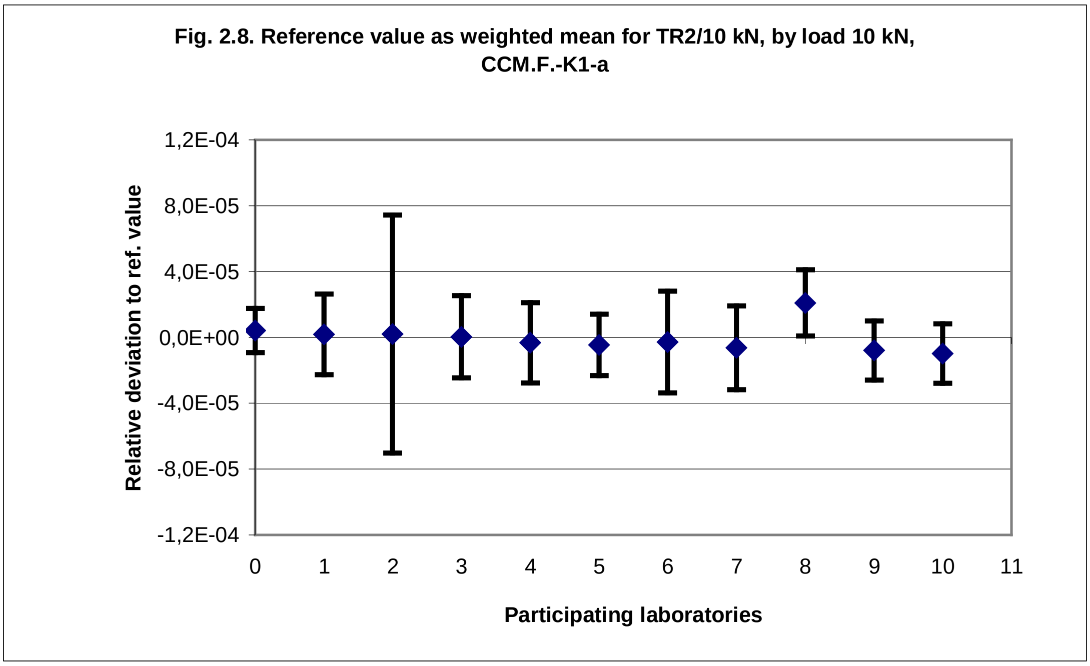


Figure 2.7. Transducer TR2/10 kN; 10 kN loading point: relative deviation to the reference value and expanded uncertainty of the calibration laboratory

Transducer TR1/5 kN

Reference values for transducer TR1/5 kN, values in 10^{-5}	
Load 5 kN	
Ref. Value	Uncertainty W_{ref} $(k=2)$
0,83	0,87

Table 2.10. Reference values for transducer TR1/5 kN with corresponding expanded uncertainty

	Transducer TR1/5 kN, values in 10^{-5}	
Laboratory	Rel. deviation to reference value	Expanded uncertainty of laboratory, $W_{L,\ ref}$
	5 kN	U $(k=2)$
0	-0,83	1,73
1	-0,23	2,67
2	1,21	7,31
3	-0,79	2,72
11	0,01	2,11
12	2,43	2,48
13	-5,69	10,35
14	-0,33	10,54
15	1,19	3,53

Table 2.11. Results for transducer TR1/5 kN as relative deviation to the reference value and expanded uncertainty of the calibration laboratories

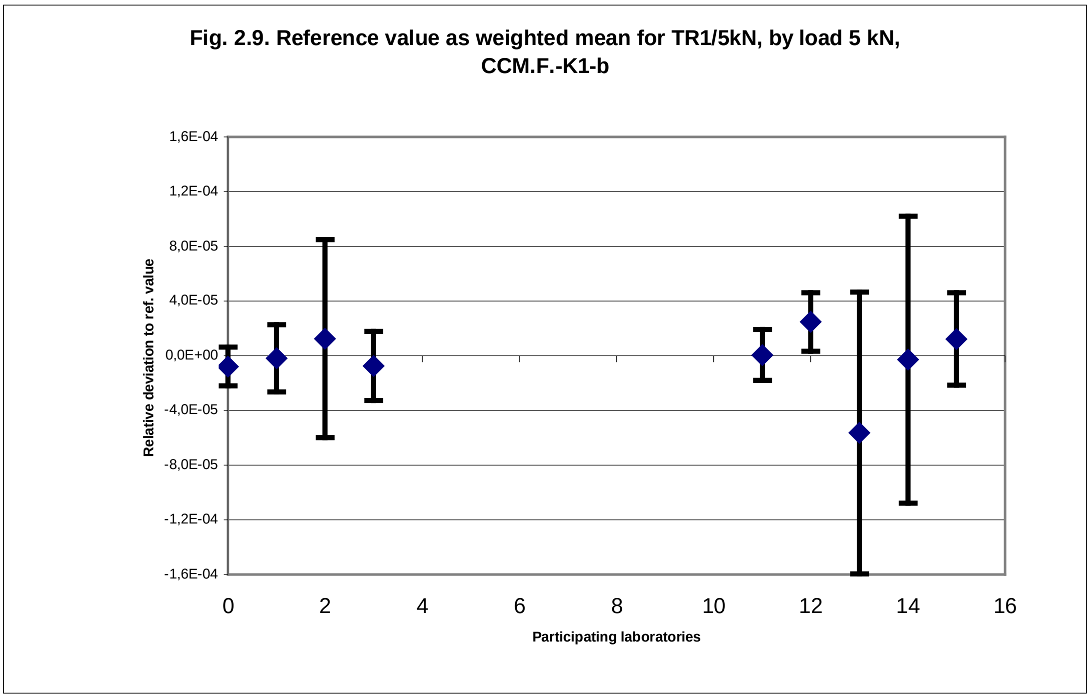


Figure 2.8. Transducer TR1/5 kN; 5 kN loading point: relative deviation to the reference value and expanded uncertainty of the calibration laboratory

Transducer TR2/5 kN

Reference values for transducer TR2/5 kN, values in 10^{-5}	
Load 5 kN	
Ref. Value	Uncertainty W_{ref} $(k=2)$
0,27	0,85

Table 2.12 Reference values for transducer TR2/5 kN with corresponding expanded uncertainty

	Transducer TR2/5 kN, values in 10^{-5}	
Laboratory	Rel. deviation to reference value	Expanded uncertainty of laboratory, $W_{L,\ ref}$
	5 kN	U $(k=2)$
0	-0,27	1,72
1	1,79	2,66
2	1,39	7,31
3	-0,85	2,72
11	-0,44	2,05
12	-0,19	2,36
13	1,06	10,11
14	-1,11	10,65
15	1,11	3,35

Table 2.13. Results for transducer TR2/5 kN as relative deviation to the reference value and expanded uncertainty of the calibration laboratory

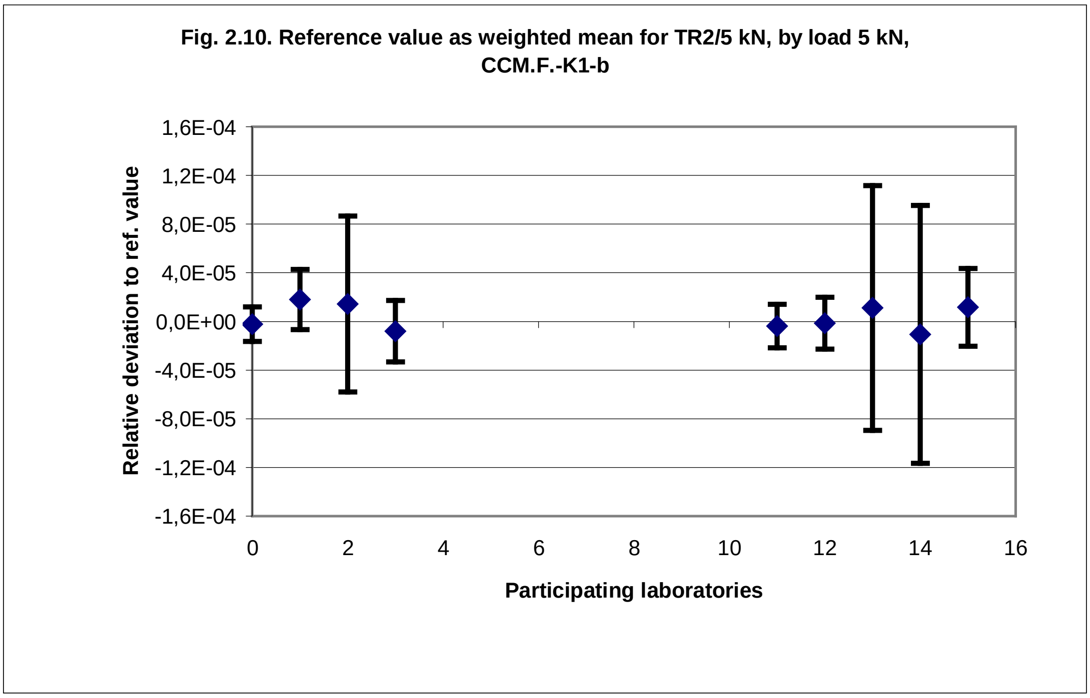


Figure 2.9. Transducer TR2/5 kN; 5 kN loading point: relative deviation to the reference value and expanded uncertainty of the calibration laboratory

Degree of equivalence for each participant to reference value, values in 10^{-5}												
	Tr1/10 kN				Tr2/10kN				Tr1/5kN		Tr2/5kN	
	5 kN		10 kN		5 kN		10 kN		5 kN		5 kN	
Lab	$d_{L,ref}$	$W_{d,L,ref}$	$d_{L,ref}$	$W_{d,L,ref}$	$d_{L,ref}$	$W_{d,L,ref}$	$d_{L,ref}$	$W_{d,L,ref}$	$d_{L,ref}$	$W_{d,L,ref}$	$d_{L,ref}$	$W_{d,L,ref}$
0	2,16	1,95	1,61	1,95	0,19	1,99	0,44	1,97	-0,86	1,81	-0,27	1,72
1					1,74	2,46	0,21	2,45	-0,26	2,37	1,79	2,66
2	3,38	7,22	2,43	7,22	-0,18	7,23	0,23	7,22	1,18	7,21	1,39	7,31
3	-1,29	2,59	-1,87	2,49	-0,04	2,49	0,06	2,49	-0,82	2,43	-0,85	2,72
4	-2,22	2,49	-2,25	2,44	-0,39	2,44	-0,31	2,43				
5	-2,12	2,19	-2,19	2,10	-0,61	2,31	-0,43	1,86				
6	2,70	3,20	2,34	3,48	0,98	2,83	-0,26	3,09				
7	0,36	1,64	-0,32	2,50	-0,50	2,12	-0,62	2,54				
8	0,51	2,10	0,24	1,87	2,38	1,87	2,12	2,01				
9	-1,21	1,81	-0,87	1,80	-1,39	1,82	-0,77	1,80				
10	-1,09	1,93	-0,16	1,92	-1,79	1,96	-0,96	1,80				
11									-0,01	1,72	-0,44	2,05
12									2,40	2,05	-0,19	2,35
13									-5,72	10,27	1,06	10,11
14									-0,36	10,47	-1,11	10,65
15									1,16	3,31	1,11	3,35

Table 2.14. Degree of equivalence for laboratories to reference value

Values in 10^{-5}											
d_{pairs}	0	1	2	3	4	5	6	7	8	9	10
0		0,00	-1,22	3,45	4,38	4,27	-0,54	1,79	1,65	3,36	3,25
1											
2	1,22			4,67	5,61	5,50	0,68	3,02	2,88	4,59	4,47
3	-3,45		-4,67		0,94	0,83	-3,99	-1,65	-1,80	-0,08	-0,20
4	-4,38		-5,61	-0,94		-0,11	-4,92	-2,59	-2,73	-1,02	-1,14
5	-4,27		-5,50	-0,83	0,11		-4,82	-2,48	-2,62	-0,91	-1,03
6	0,54		-0,68	3,99	4,92	4,82		2,34	2,19	3,91	3,79
7	-1,79		-3,02	1,65	2,59	2,48	-2,34		-0,14	1,57	1,45
8	-1,65		-2,88	1,80	2,73	2,62	-2,19	0,14		1,71	1,60
9	-3,36		-4,59	0,08	1,02	0,91	-3,91	-1,57	-1,71		-0,12
10	-3,25		-4,47	0,20	1,14	1,03	-3,79	-1,45	-1,60	0,12	

Table 2.15. Relative difference between laboratory A (left column) and laboratory B (top row) for the transducer TR1/10 kN at loading point 5 kN.

Values in 10^{-5}											
$w_{\mathbf{pairs}}$	0	1	2	3	4	5	6	7	8	9	10
0			7,10	2,22	2,10	1,74	2,92	0,97	1,63	1,24	1,40
1											
2	7,10			7,44	7,41	7,31	7,68	7,17	7,28	7,21	7,24
3	2,22		7,44		3,06	2,82	3,66	2,42	2,75	2,54	2,62
4	2,10		7,41	3,06		2,73	3,60	2,32	2,66	2,44	2,53
5	1,74		7,31	2,82	2,73		3,40	1,99	2,38	2,14	2,23
6	2,92		7,68	3,66	3,60	3,40		3,07	3,34	3,17	3,24
7	0,97		7,17	2,42	2,32	1,99	3,07		1,89	1,57	1,70
8	1,63		7,28	2,75	2,66	2,38	3,34	1,89		2,04	2,15
9	1,24		7,21	2,54	2,44	2,14	3,17	1,57	2,04		1,87
10	1,40		7,24	2,62	2,53	2,23	3,24	1,70	2,15	1,87	

Table 2.16. The combined relative standard uncertainties ($k=2$) for the corresponding values in Table 2.15.

Values in 10^{-5}										
d_{pairs}	0	1	2	3	4	5	6	7	8	9	10
0			-0,82	3,48	3,86	3,80	-0,73	1,93	1,37	2,49	1,77
1											
2	0,82			4,30	4,68	4,62	0,08	2,74	2,19	3,30	2,59
3	-3,48		-4,30		0,37	0,32	-4,22	-1,56	-2,11	-1,00	-1,71
4	-3,86		-4,68	-0,37		-0,06	-4,59	-1,93	-2,49	-1,37	-2,08
5	-3,80		-4,62	-0,32	0,06		-4,54	-1,88	-2,43	-1,32	-2,03
6	0,73		-0,08	4,22	4,59	4,54		2,66	2,10	3,22	2,51
7	-1,93		-2,74	1,56	1,93	1,88	-2,66		-0,56	0,56	-0,15
8	-1,37		-2,19	2,11	2,49	2,43	-2,10	0,56		1,12	0,40
9	-2,49		-3,30	1,00	1,37	1,32	-3,22	-0,56	-1,12		-0,71
10	-1,77		-2,59	1,71	2,08	2,03	-2,51	0,15	-0,40	0,71	

Table 2.17. Relative difference between laboratory A (left column) and laboratory B (top row) for the transducer TR1/10 kN at loading point 10 kN.

Values in 10^{-5}											
$w_{\mathbf{pairs}}$	0	1	2	3	4	5	6	7	8	9	10
0			7,40	2,96	2,92	2,64	3,83	2,97	2,46	2,41	2,50
1											
2	7,40			7,41	7,39	7,28	7,79	7,41	7,22	7,21	7,23
3	2,96		7,41		2,95	2,67	3,85	3,00	2,49	2,45	2,53
4	2,92		7,39	2,95		2,62	3,81	2,95	2,44	2,39	2,48
5	2,64		7,28	2,67	2,62		3,60	2,67	2,10	2,04	2,14
6	3,83		7,79	3,85	3,81	3,60		3,85	3,48	3,44	3,50
7	2,97		7,41	3,00	2,95	2,67	3,85		2,50	2,45	2,54
8	2,46		7,22	2,49	2,44	2,10	3,48	2,50		1,80	1,92
9	2,41		7,21	2,45	2,39	2,04	3,44	2,45	1,80		1,85
10	2,50		7,23	2,53	2,48	2,14	3,50	2,54	1,92	1,85	

Table 2.18. The combined relative standard uncertainties (*k*=2) for the corresponding values in Table 2.17.

Values in 10^{-5}											
d_{pairs}	0	1	2	3	4	5	6	7	8	9	10
0		-1,55	0,37	0,23	0,58	0,80	-0,79	0,69	-2,19	1,58	1,98
1	1,55		1,92	1,78	2,12	2,35	0,75	2,24	-0,64	3,13	3,52
2	-0,37	-1,92		-0,14	0,20	0,43	-1,17	0,32	-2,56	1,21	1,60
3	-0,23	-1,78	0,14		0,35	0,57	-1,02	0,46	-2,42	1,35	1,74
4	-0,58	-2,12	-0,20	-0,35		0,22	-1,37	0,11	-2,77	1,01	1,40
5	-0,80	-2,35	-0,43	-0,57	-0,22		-1,59	-0,11	-2,99	0,78	1,17
6	0,79	-0,75	1,17	1,02	1,37	1,59		1,49	-1,40	2,38	2,77
7	-0,69	-2,24	-0,32	-0,46	-0,11	0,11	-1,49		-2,88	0,89	1,28
8	2,19	0,64	2,56	2,42	2,77	2,99	1,40	2,88		3,77	4,16
9	-1,58	-3,13	-1,21	-1,35	-1,01	-0,78	-2,38	-0,89	-3,77		0,39
10	-1,98	-3,52	-1,60	-1,74	-1,40	-1,17	-2,77	-1,28	-4,16	-0,39	

Table 2.19. Relative difference between laboratory A (left column) and laboratory B (top row) for the transducer TR2/10 kN at loading point 5 kN.

Values in 10^{-5}										
$w_{\mathbf{pairs}}$	0	1	2	3	4	5	6	7	8	9	10
0		2,95	7,40	2,97	2,93	2,82	3,26	2,67	2,47	2,42	2,54
1	2,95		7,39	2,95	2,90	2,79	3,24	2,64	2,44	2,39	2,51
2	7,40	7,39		7,40	7,39	7,34	7,52	7,29	7,22	7,20	7,24
3	2,97	2,95	7,40		2,93	2,82	3,26	2,67	2,47	2,42	2,54
4	2,93	2,90	7,39	2,93		2,77	3,22	2,62	2,42	2,37	2,49
5	2,82	2,79	7,34	2,82	2,77		3,12	2,50	2,29	2,23	2,36
6	3,26	3,24	7,52	3,26	3,22	3,12		2,98	2,81	2,77	2,87
7	2,67	2,64	7,29	2,67	2,62	2,50	2,98		2,09	2,03	2,18
8	2,47	2,44	7,22	2,47	2,42	2,29	2,81	2,09		1,77	1,93
9	2,42	2,39	7,20	2,42	2,37	2,23	2,77	2,03	1,77		1,87
10	2,54	2,51	7,24	2,54	2,49	2,36	2,87	2,18	1,93	1,87	

Table 2.20. The combined relative standard uncertainties (k=2) for the corresponding values in Table 2.19

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Values in 10^{-5}											
d_{pairs}	0	1	2	3	4	5	6	7	8	9	10
0		0,23	0,21	0,38	0,75	0,87	0,70	1,05	-1,68	1,21	1,40
1	-0,23		-0,02	0,15	0,52	0,64	0,47	0,83	-1,91	0,98	1,17
2	-0,21	0,02		0,17	0,54	0,66	0,49	0,84	-1,89	1,00	1,19
3	-0,38	-0,15	-0,17		0,37	0,49	0,32	0,67	-2,06	0,83	1,02
4	-0,75	-0,52	-0,54	-0,37		0,12	-0,05	0,31	-2,43	0,46	0,65
5	-0,87	-0,64	-0,66	-0,49	-0,12		-0,17	0,19	-2,55	0,34	0,53
6	-0,70	-0,47	-0,49	-0,32	0,05	0,17		0,35	-2,38	0,51	0,70
7	-1,05	-0,83	-0,84	-0,67	-0,31	-0,19	-0,35		-2,73	0,16	0,34
8	1,68	1,91	1,89	2,06	2,43	2,55	2,38	2,73		2,89	3,08
9	-1,21	-0,98	-1,00	-0,83	-0,46	-0,34	-0,51	-0,16	-2,89		0,19
10	-1,40	-1,17	-1,19	-1,02	-0,65	-0,53	-0,70	-0,34	-3,08	-0,19	

Table 2.21. Relative difference between laboratory A (left column) and laboratory B (top row) for the transducer TR2/10 kN at loading point 10 kN.

Values in 10^{-5}											
$w_{\mathbf{pairs}}$	0	1	2	3	4	5	6	7	8	9	10
0		2,92	7,40	2,96	2,91	2,45	3,48	3,00	2,57	2,40	2,51
1	2,92		7,39	2,93	2,88	2,42	3,46	2,98	2,54	2,37	2,48
2	7,40	7,39		7,40	7,38	7,22	7,63	7,42	7,26	7,20	7,24
3	2,96	2,93	7,40		2,92	2,47	3,49	3,01	2,58	2,42	2,52
4	2,91	2,88	7,38	2,92		2,41	3,44	2,96	2,52	2,36	2,47
5	2,45	2,42	7,22	2,47	2,41		3,07	2,52	1,98	1,76	1,90
6	3,48	3,46	7,63	3,49	3,44	3,07		3,52	3,16	3,03	3,11
7	3,00	2,98	7,42	3,01	2,96	2,52	3,52		2,63	2,47	2,57
8	2,57	2,54	7,26	2,58	2,52	1,98	3,16	2,63		1,92	2,05
9	2,40	2,37	7,20	2,42	2,36	1,76	3,03	2,47	1,92		1,84
10	2,51	2,48	7,24	2,52	2,47	1,90	3,11	2,57	2,05	1,84	

Table 2.22. The combined relative standard uncertainties (k=2) for the corresponding values in Table 2.21.

Values in 10^{-5}									
d_{pairs}	0	1	2	3	11	12	13	14	15
0		-0,60	-2,04	-0,04	-0,85	-3,26	4,86	-0,50	-2,02
1	0,60		-1,44	0,56	-0,24	-2,66	5,46	0,10	-1,41
2	2,04	1,44		2,00	1,20	-1,22	6,90	1,54	0,02
3	0,04	-0,56	-2,00		-0,80	-3,22	4,90	-0,46	-1,97
11	0,85	0,24	-1,20	0,80		-2,42	5,70	0,34	-1,17
12	3,26	2,66	1,22	3,22	2,42		8,12	2,76	1,24
13	-4,86	-5,46	-6,90	-4,90	-5,70	-8,12		-5,36	-6,88
14	0,50	-0,10	-1,54	0,46	-0,34	-2,76	5,36		-1,52
15	2,02	1,41	-0,02	1,97	1,17	-1,24	6,88	1,52	

Table 2.23. Relative difference between laboratory A (left column) and laboratory B (top row) for the transducer TR1/5 kN at loading point 5 kN.

Values in 10^{-5}									
$w_{\mathbf{pairs}}$	0	1	2	3	11	12	13	14	15
0		2,85	7,38	2,91	2,34	2,68	10,40	10,59	3,67
1	2,85		7,38	2,92	2,36	2,69	10,40	10,60	3,68
2	7,38	7,38		7,40	7,20	7,32	12,43	12,59	7,74
3	2,91	2,92	7,40		2,42	2,75	10,41	10,61	3,72
11	2,34	2,36	7,20	2,42		2,14	10,27	10,47	3,30
12	2,68	2,69	7,32	2,75	2,14		10,35	10,55	3,54
13	10,40	10,40	12,43	10,41	10,27	10,35		14,57	10,65
14	10,59	10,60	12,59	10,61	10,47	10,55	14,57		10,84
15	3,67	3,68	7,74	3,72	3,30	3,54	10,65	10,84	

Table 2.24. The combined relative standard uncertainties (k=2) for the corresponding values in Table 2.23.

Values in 10^{-5}									
d_{pairs}	0	1	2	3	11	12	13	14	15
0		-2,07	-1,66	0,58	0,17	-0,08	-1,33	0,84	-1,38
1	2,07		0,41	2,64	2,24	1,98	0,74	2,91	0,69
2	1,66	-0,41		2,23	1,83	1,57	0,33	2,50	0,28
3	-0,58	-2,64	-2,23		-0,41	-0,66	-1,91	0,26	-1,96
11	-0,17	-2,24	-1,83	0,41		-0,25	-1,50	0,67	-1,55
12	0,08	-1,98	-1,57	0,66	0,25		-1,25	0,92	-1,30
13	1,33	-0,74	-0,33	1,91	1,50	1,25		2,17	-0,05
14	-0,84	-2,91	-2,50	-0,26	-0,67	-0,92	-2,17		-2,22
15	1,38	-0,69	-0,28	1,96	1,55	1,30	0,05	2,22	

Table 2.25. Relative difference between laboratory A (left column) and laboratory B (top row) for the transducer TR2/5 kN at loading point 5 kN.

Values in 10^{-5}									
$w_{\mathbf{pairs}}$	0	1	2	3	11	12	13	14	15
0		2,88	7,39	2,93	2,32	2,59	10,17	10,70	3,52
1	2,88		7,38	2,92	2,31	2,58	10,16	10,70	3,52
2	7,39	7,38		7,40	7,18	7,28	12,23	12,68	7,66
3	2,93	2,92	7,40		2,37	2,64	10,18	10,71	3,56
11	2,32	2,31	7,18	2,37		1,94	10,02	10,56	3,07
12	2,59	2,58	7,28	2,64	1,94		10,09	10,63	3,29
13	10,17	10,16	12,23	10,18	10,02	10,09		14,48	10,37
14	10,70	10,70	12,68	10,71	10,56	10,63	14,48		10,89
15	3,52	3,52	7,66	3,56	3,07	3,29	10,37	10,89	

Table 2.26. The combined relative standard uncertainties (*k*=2) for the corresponding values in Table 2.25.

Chapter 3,
one reference value
for 5 kN and for 10 kN

3.1 Used symbols

D = abs. difference (laboratory - ref. value)
X_{PA} = Measured deflection by pilot for A measurement
X_{PB} = Measured deflection by pilot for B measurement
Δ_{corr} = Correction of the mean value to the true measurement day
X_0 = Measured deflection by pilot, without correction of drift
X_P = Used deflection value for pilot for loop n
X'_L = Measured deflection value of the laboratory
X''_L = Deflection value with BN100 correction for the laboratory
X_L = Used deflection value with correction of the participating laboratory (BN 100 and possible other corrections etc. temperature, extrapolation aso.)
u_{PA} = Standard uncertainty of the X_{PA}
u_{PB} = Standard uncertainty of the X_{PB}
u_{teor} = Standard uncertainty of the X_{PA} and X_{PB} with perfect correlation
u_P = Standard uncertainty of the X_P (from X_{PA} and X_{PB})
u'_L = Standard uncertainty of the X'_L
u_L = Standard uncertainty of the X_L
$u_{L,tot}$ = Standard uncertainty of the X_L with the uncertainty of drift
$u_{L,P}$ = Standard uncertainty of the deviation including $u_{L,tot}$ and uncertainty of participating laboratory
u_{teor} = Standard uncertainty of the measurements by Pilot with perfect correlation
w_{PA} = Relative standard uncertainty of X_{PA}
w_{PB} = Relative standard uncertainty of X_{PB}
w_P = Relative standard uncertainty of X_P
w_L = Relative standard uncertainty of X_L
w_{drift} = Relative standard uncertainty of the drift of the transducers between A and B measurement by pilot
$w_{L,tot}$ = Relative standard uncertainty of X_L with the uncertainty of drift
$w_{L,P}$ = Relative standard uncertainty of the deviation including $w_{L,tot}$ and uncertainty of participating laboratory
d_L = Relative measured deviation between the laboratory and the pilot
$\bar{d}$ = Mean values of deflections measured by the participating laboratory, number of transducers 2 or 4
d_{ref} = Relative reference deviation value
$d_{L,ref}$ = Relative deviation between the laboratory and the reference values
$w_{d_{L,ref}}$ = Relative standard uncertainty of the $d_{L,ref}$
w_d = Relative standard uncertainty of the d
$w_{\bar{d}}$ = Relative standard uncertainty of the $\bar{d}$
$w_{d,ref}$ = Relative standard uncertainty of the d_{ref}

Used terminology:

Reference value	calculated reference value for one transducer
Deflection	Indication of the DMP 40 by the measurement

3.2 The deviation between Pilot and Laboratory

For each Laboratory and each transducer will be calculated a single value as deviation. This value is based on results of differences between Pilot and Laboratories. The model of the deviation is:

$$D = X_L - X_P \tag{3.1}$$

$$X_P = X_0 + \Delta_{corr} \tag{3.2}$$

The correction is used for the pilot measurement. It is assumed that the drift is linear.

3.2.1 The used deflection value by Pilot for loop n

The used deflection for loop n is calculated from A and B measurements by Pilot with the correction of the drift. The assumption for the drift is a linear drift between A and B measurements, which are made as close as possible before and after the measurement of the participating laboratory. The needed time for transport in one direction has not been equal for all participants, variation from four days up to 2 weeks. The correction of the drift is made as function of the time.

$$\Delta_{corr} = \frac{(X_{PB} - X_{PA})}{t_{total}} \cdot t_1 \tag{3.3}$$

Where:
t_1 = time between pilot A and calibration by laboratory
t_2 = time between calibration by laboratory and pilot B
$t_{total} = t_1 + t_2$

The value x_P has been used as reference for loop *n*.

$$X_P = X_{PA} + \Delta_{corr} \tag{3.4}$$

Reference values from Pilot with correction of the non-symmetric in time						
Transducer	TR1 10 kN	TR1 10 kN	TR2 10 kN	TR2 10 kN	TR1 5 kN	TR2 5 kN
Loading value	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
Laboratory	mV/V	mV/V	MV/V	mV/V	mV/V	mV/V
1			1,009997	2,020022	1,881994	2,027821
2	0,979683	1,959176	1,010011	2,020045	1,882008	2,027815
3	0,979722	1,959248	1,009986	2,019999	1,881986	2,027798
4	0,979720	1,959243	1,009980	2,019990		
5	0,979718	1,959243	1,009968	2,019970		
6	0,979694	1,959201	1,009999	2,020025		
7	0,979708	1,959224	1,009960	2,019953		
8	0,979722	1,959245	1,009963	2,019956		
9	0,979720	1,959247	1,009992	2,020012		
10	0,979719	1,959247	1,009972	2,019980		
11					1,881995	2,027796
12					1,882016	2,027801
13					1,882008	2,027823
14					1,882066	2,027805
15					1,881979	2,027793

Table 3.1. Used reference values for each loop after the correction of non-symmetric timing of measurement by participating laboratory. Results of Laboratory 1 for transducer TR1 10kN have been excluded as discussed in section 3.5.

The correction, Δ_{corr}, is mainly relative small, however the maximal change is $8{,}6\cdot10^{-5}$ mV/V, which is by one transducers and the reason is unknown, and it seems to be an external influence. The standard deviation for change is $2{,}4\cdot10^{-5}$ mV/V. The most changes happens with transducers TR1/10 kN. The results of laboratory 1 for TR1 10 kN have been excluded as discussed below in section 3.

3.3 Principle to use the pilot FSM as link in the comparison

The same principle given Chapter 2 is used to handle the results of comparison. The comparison has been made as a star form; the transducers came back to the pilot after every measurement by the participating laboratory. One complete measurement, Pilot – Participating Laboratory – Pilot is called a loop.

Pilot's measurements have been made always on the same deadweight machine for the whole key-comparison. This pilot's machine is a link between all participants. The pilot's link machine is noticed as PLM on this paper.

The pilot laboratory is also a participating laboratory (lab0). Pilot laboratory, as participant, make also a comparison between PLM and its own reference calibration machine (PM).

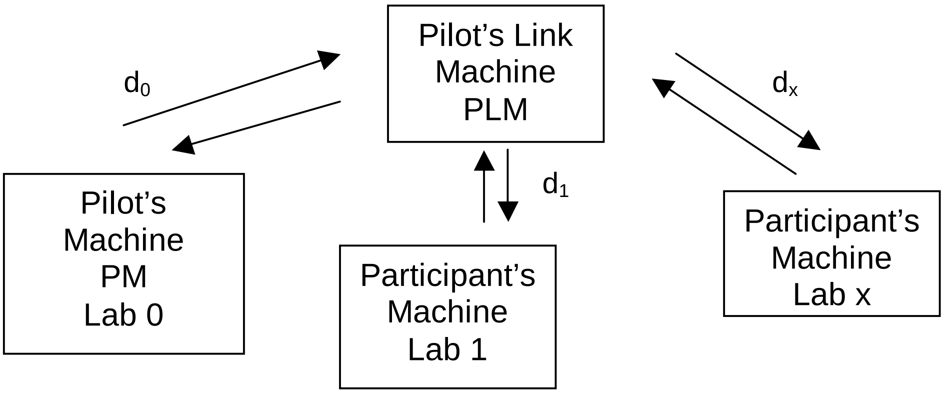

Figure 3.1: Principle of the star type comparison with the link machine.

This graph can be applied even if the PLM and the PM are the same machine.

For each loop, ie for each laboratory, the relative deviation (d) from the PLM is computed according to: $d_L = (X_L - X_P) / X_P$

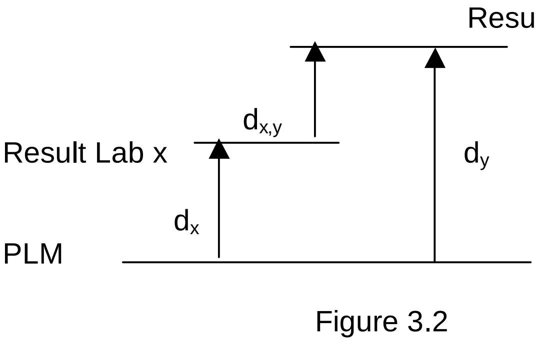

Figure 3.2

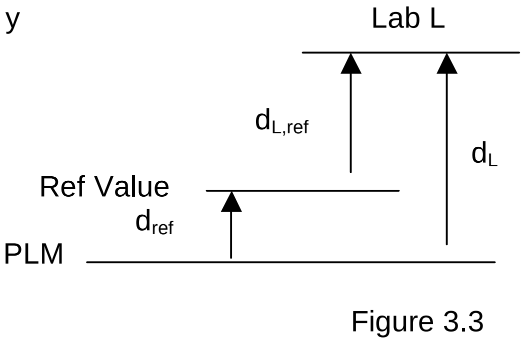

Figure 3.3

Figure 3.2 : Using the pilot as a link, the deviation between 2 laboratories is equal to $d_{x,y} = d_y - d_x$. So to compute the deviation between 2 laboratories, it's not necessary to use a traceable PLM but a machine enough stable between the 2 loops.

Figure 3.3 : The same approach is made for the deviation from the reference value and a laboratory. Using the pilot as a link, the deviation between a laboratory and the reference value is equal to $d_{L,ref} = d_L - d_{ref}$. To compute the deviation between a laboratory and the reference value, it is not necessary to use a traceable PLM but a machine enough stable during all the key-comparison.

3.3.1 Relative standard uncertainty due to the stability of the pilot link machine : w_{PLM}

For the two reasons given above, the uncertainty of the PLM is computed taking into account only components of stability. Following components are given in relative value with k =1

- stability of masses : $1{,}0\cdot 0^{-6}$
- stability of gravity : $0{,}2\cdot 10^{-6}$
- stability of the air buoyancy : $3{,}5\cdot 10^{-6}$

The combined relative standard uncertainty due to the stability of pilot FSM can be estimated w'_{PLM} equal to :

$$w_{PLM} = 0{,}35\cdot 10^{-5} \qquad (3.5)$$

3.3.2 Relative standard uncertainty of deflection obtained at pilot laboratory : w_P

The reference value X_P for each loop is based on A and B measurements according to equation (3.4). The pilot had a variation of relative reproducibility with rotation $w_{PR} = 0{,}2\cdot 10^{-5}\ \ldots 1{,}0\cdot 10^{-5}$, based on the information from pilots A and B measurements. By separating the pure stability of the FSM from the instability of the force transducers, the value $0{,}6\cdot 10^{-5}$ can be assumed by the pilot to be the minimum value of stability of the FSM. The uncertainty of the deflection by pilot (w_{PA} or w_{PB}) includes the pilot reproducibility uncertainty (w_{PR}) and the stability of PLM (w_{PLM}):

$$w_{PA} = w_{PB} = \sqrt{(w_{PR}{}^2 + w_{PLM}{}^2)} = 0{,}6\cdot 10^{-5} \qquad (3.6)$$

The mean value of relative standard uncertainty is assumed as well :

$$w_P{}' = \sqrt{\frac{w_{PA}{}^2 + w_{PB}{}^2}{2}} = 0{,}6\cdot 10^{-5} \qquad (3.7)$$

However to get the consistency (X^2-test) for all measurements, which are influenced by the uncertainty of pilot FSM it is necessary to increase the value for standard uncertainty:

$$\boldsymbol{w_P = 0{,}75\cdot 10^{-5}} \qquad (3.8)$$

3.4 Calculation of the reference values for 5 kN and 10 kN

To get the connection between all laboratories it is necessary to create only one reference value for measured forces, 5 kN and 10 kN. This means that for every laboratory should have only one value for 5 kN and 10 kN force.

3.4.1 Calibration result as relative deviation to pilot and relative uncertainty of the participant laboratory, X_L and w_L

A laboratory result is defined by the mean deflection obtained from each calibrated force transducer. The deflection is noted $\boldsymbol{X}_L$ for the participant laboratory no. L. The relative deviation between laboratory and pilot values has been calculated using equation (3.9).

$$d_L = (X_L - X_P) / X_P \qquad (3.9)$$

The relative uncertainty on this deflection is announced by the participant laboratory. This uncertainty is noted w_L.

3.4.2 The weighted mean relative deviation of the laboratory for each measured force (5kN and 10 kN) and associated relative uncertainties

The mean value has been calculated as the weighted mean of all of the measured relative deviations of 5 kN (K1a and K1b together) and correspondingly of 10 kN values. In any case the deviation with PLM is measured with more than each transducer for one participating laboratory. The deviation value $\bar{d}_L$ used for each participating laboratory is the weighted mean of the deviation obtained for this laboratory :

$$\bar{d}_L = \frac{\sum\limits_{t=1\ldots n} p_{L_t} d_{L_t}}{\sum\limits_{t=1\ldots n} p_{L_t}} \quad \text{;n = 2 or 4 depending of numbers of transducers} \qquad (3.10)$$

where the weighing factor is $p_L = \frac{1}{w_{dL}{}^2}$ (3.11)

Uncertainty of this deviation is calculated according to following equation:

$$w_{\bar{d}L} = \sqrt{\frac{1}{\sum\limits_{n_t=1\ldots n} \frac{1}{w_{dL}{}^2}}} \quad \text{;n = 2 or 4 depending of numbers of transducers} \qquad (3.12)$$

3.4.3 The relative standard uncertainty w_d of the weighted mean deviation between pilot and laboratory by using the pilot as link (presented as well ready in chapter 2.4.2)

The standard uncertainty w_d of the relative deviation between laboratory and pilot is calculated using the equation (3.13).

Relative uncertainty of the deviation is taking in to account:

- the relative uncertainty of the calibration laboratory,
- the relative uncertainty of the pilot
- the relative uncertainty of the drift.

$$w_d = \sqrt{w_L{}^2 + w_P{}^2 + w_{\Delta,corr}} \qquad (3.13)$$

Where for the relative standard uncertainty of the pilot machine is taken from (3.8):

$$w_P = 0{,}75 \cdot 10^{-5}$$

The drift has been assumed as a linear function between A and B measurement by Pilot. The time between these measurements is t_{total} and the participating laboratory has measured it in time t_{total} /2 with deviation ± *n* days. As basement for the evaluation of the uncertainty for drift the following tables show the mean values and standard deviations for the drift as total drifts between Pilot and Laboratories calculated for one day and for four days.

Transducer	TR1/10 kN		TR2/10 kN		TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
$\bar{x}$	3,47E-06	3,52E-06	-9,90E-07	-9,90E-07	8,44E-06	4,93E-07
s	2,24E-05	1,92E-05	1,43E-05	1,19E-05	1,57E-05	3,49E-06

Table 2.2 Mean values and standard deviations of the all drift for transducers between Pilot and Laboratory

The drift as total value are very similar, only for transducer TR2/5kN the drift is closely one decade lower. Closer examination shows that the change for TR1/10kN is less systematic but in same order than drift for other transducers.

Transducer	TR1/10 kN		TR2/10 kN		TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
$\bar{x}$	2,40E-07	2,77E-07	-2,53E-07	-2,24E-07	1,23E-06	1,15E-07
s	3,48E-06	2,96E-06	1,67E-06	1,39E-06	2,22E-06	4,52E-07

Table 2.3. Mean values and standard deviations for drift of one day, calculated individual for each laboratory

Transducer	TR1/10 kN		TR2/10 kN		TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
$\bar{x}$	1,29E-06	1,10E-06	2,19E-07	1,63E-07	2,08E-06	2,61E-07
s	4,61E-06	4,00E-06	2,32E-06	1,98E-06	2,83E-06	6,17E-07

Table 2.4. Mean values and standard deviations for drift of four days, calculated individual for each laboratory

As conclusion the uncertainty of the drift is selected as four days drift of the total change for each laboratory. It is about 1/10 part of the total drift. It gives also for the transducer TR1/10kN closely twice larger uncertainty as more unstable drift.

The relative uncertainty of the drift is based on the variation width (*a*) during four days and assumed to have rectangular distribution. The uncertainty is connected individual to the drift of each transducer.

$$a_{drift} = \frac{drift_{total}}{t_{total}} * 4 \qquad (3.14)$$

The relative uncertainty of the drift is:

$$w_{\Delta,corr} = \frac{a_{drift}}{2 \cdot \sqrt{3} \cdot X_P} \qquad (3.15)$$

3.4.4 Reference value and associated relative uncertainties

The reference value has been calculated from all of the mean relative deviations $\bar{d}_L$ of 5 kN and correspondingly of 10 kN values as the weighted mean value. The pilot is as laboratory 0 included with a deviation from the PLM equal to 0 because the pilot calibration machine is the PLM. The weighting factor is the uncertainty $w_{\bar{d}L}$ of the mean deviation $\bar{d}_L$.

The reference deviation d_{ref} is calculated using equation (3.16) and (3.17)

For 5 kN $$d_{ref,5kN} = \frac{\sum_{L=0...15} p_L d_L}{\sum_{L=0...15} p_L} \tag{3.16}$$

For 10 kN $$d_{ref,10kN} = \frac{\sum_{L=0...10} p_L d_L}{\sum_{L=0...10} p_L} \tag{3.17}$$

where the weighing factor is $$p_L = \frac{1}{w_{dL}^{\;2}} \tag{3.18}$$

The uncertainty of the reference deviation is calculated considering that values are not correlated :

For 5 kN $$w_{d\,\mathrm{Re}\,f5kN} = \sqrt{\frac{1}{\sum_{L=0...15} \frac{1}{w_{dL}^{\;2}}}} \tag{3.19}$$

For 10 kN $$w_{d\,\mathrm{Re}\,f10kN} = \sqrt{\frac{1}{\sum_{L=0...10} \frac{1}{w_{dL}^{\;2}}}} \tag{3.20}$$

3.4.6 The numerical values and associated uncertainties of reference values

Reference values as weighted mean, values in 10^{-5}			
Load 5 kN		Load 10 kN	
Ref. Value	Uncertainty W_{ref} *(k=2)*	Ref. Value	Uncertainty W_{ref} *(k=2)*
-0,59	**0,66**	**-1,05**	**0,77**

Table 3.2 The reference values for forces of 5 kN and 10 kN with expanded uncertainties.

The results and assigned uncertainties, concerning deviation with the reference value, are given for all participants, including the pilot laboratory, in Figures 3.4 and 3.5. The data are given in Tables 3.3 … 3.9.

3.4.7 References and uncertainties between participants

Deviation between Laboratory $\overline{d}_L$ and Pilot d_P

$$d_{L,P} = \overline{d}_L - d_P \tag{3.21}$$

and the uncertainty $W_{L.P} = 2 * \sqrt{w_{\bar{d}}{}^2 + w_{d,P}{}^2 + w_{drift}{}^2}$ (3.22)

Deviation between two participants laboratories using again the pilot laboratory as a link :

$$d_{L,ref} = \overline{d}_L - d_{ref} \tag{3.23}$$

The measurements are considered as uncorrelated, the expanded uncertainty (*k*=2) of this deviation is :

$$W_{d_{L,ref}} = 2 * \sqrt{w_{\bar{d}}{}^2 + w_{d,ref}{}^2} \tag{3.24}$$

Table 3.3 gives numerical values of relatives deviations and assigned uncertainty for a force of 5 and 10 kN between Laboratory and Pilot. Table 3.4 gives corresponding values between Laboratory and reference value. Figures 3.5 and 3.6 shows the values between laboratories and reference value in graphics.

3.4.8 Degree of equivalence

The degree of equivalence of each participating laboratory's is expressed by (according to M.G. Cox, Metrologia 2002,39):

- its deviation from the key comparison reference value, equation (3.25),
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (3.26).

$$d_{L,ref} = d_L - d_{ref} \tag{3.25}$$

The uncertainty of the relative deviation between laboratory and reference value is calculated using the equation (3.13) and standard uncertainty of the deviation is taking in to account:

- the uncertainty between the calibration laboratory and the pilot, caused by the deviation and including the uncertainty of laboratory,
- the expanded uncertainty (*k*=2) of the reference value.

$$W_{d_{L.ref}} = 2 * \sqrt{w_{\bar{d}_L}{}^2 - w_{ref}{}^2} \tag{3.26}$$

The values are given on table 3.5.

The degree of equivalence between pairs of laboratories is expressed by:

- its difference of their deviations from each other, equation (3.27),
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (3.28).

$$d_{pairs} = d_{L,n} - d_{L,n+1} \tag{3.27}$$

$$W_{pairs} = 2 * \sqrt{{w_{L,tot,n}}^2 + {w_{L,tot,n+1}}^2} \tag{3.28}$$

The values of pairs and their uncertainties are given on table 3.6 …3.9.

3.5 Consistency check

According the proposal of M.G. Cox the consistency should be checked with X^2-test. The results are following:

Force	5 kN	10 KN
X^2_{obs}	25,1	7,7
$X^2(v)$	25,0	18,3

Table 3.3. Result of the consistency check

According this result there is the required consistency. However, to achieve consistency for transducer TR1/10 kN it was necessary to exclude the results from Laboratory 1. (Values has been with the laboratory 1 X^2_{obs} for 5 kN 32,2 and for 10 kN 10,9.)

3.5 Tables and figures

	Values in (*k*=2), 10^{-5}			
Laboratory	F = 5 kN		F = 10 kN	
	$\bar{d}$	$\pm W_{\bar{d}}$	$\bar{d}$	$\pm W_{\bar{d}}$
1	1,4	2,5	-0,2	2,5
2	1,1	7,3	0,3	7,3
3	-1,0	2,6	-1,9	2,6
4	-2,4	2,6	-2,3	2,5
5	-2,6	2,4	-2,2	2,1
6	0,7	3,1	-0,1	3,4
7	-1,4	2,0	-1,5	2,6
8	0,5	2,1	0,1	2,1
9	-2,5	1,9	-1,8	1,9
10	-2,6	2,1	-1,6	2,0
11	0,3	1,9		
12	1,6	2,2		
13	-1,7	10,2		
14	-0,2	10,6		
15	1,7	3,3		

Table 3.3 Mean relative deviation between participant laboratory and pilot laboratory : $\bar{d}$ see. (3.10), and expanded relative uncertainty assigned 2 x $w_{L,P}$ see. (3.11).

	Values in 10^{-5}			
Laboratory	F = 5 kN		F= 10 kN	
	$d_{L,Ref}$	$\pm\ w_{d,L,ref}$ (*k*=2)	$d_{L,Ref}$	$\pm\ w_{d,L,ref}$ (*k*=2)
0				
1	1,40	2,54	-0,23	2,55
2	1,14	7,26	0,30	7,26
3	-1,00	2,61	-1,92	2,60
4	-2,43	2,57	-2,29	2,54
5	-2,62	2,35	-2,17	2,10
6	0,68	3,10	-0,07	3,36
7	-1,36	2,00	-1,50	2,61
8	0,49	2,11	0,08	2,07
9	-2,45	1,95	-1,84	1,94
10	-2,62	2,06	-1,59	2,05
11	0,32	1,89		
12	1,62	2,23		
13	-1,69	10,19		
14	-0,16	10,56		
15	1,68	3,33		

Table 3.4 : Relative deviation between participant laboratory and reference value : $d_{L,ref}$ see (3.14, 3.15), expanded relative uncertainty assigned 2 x $w_{d,L,ref}$ see (3.17, 3.18)

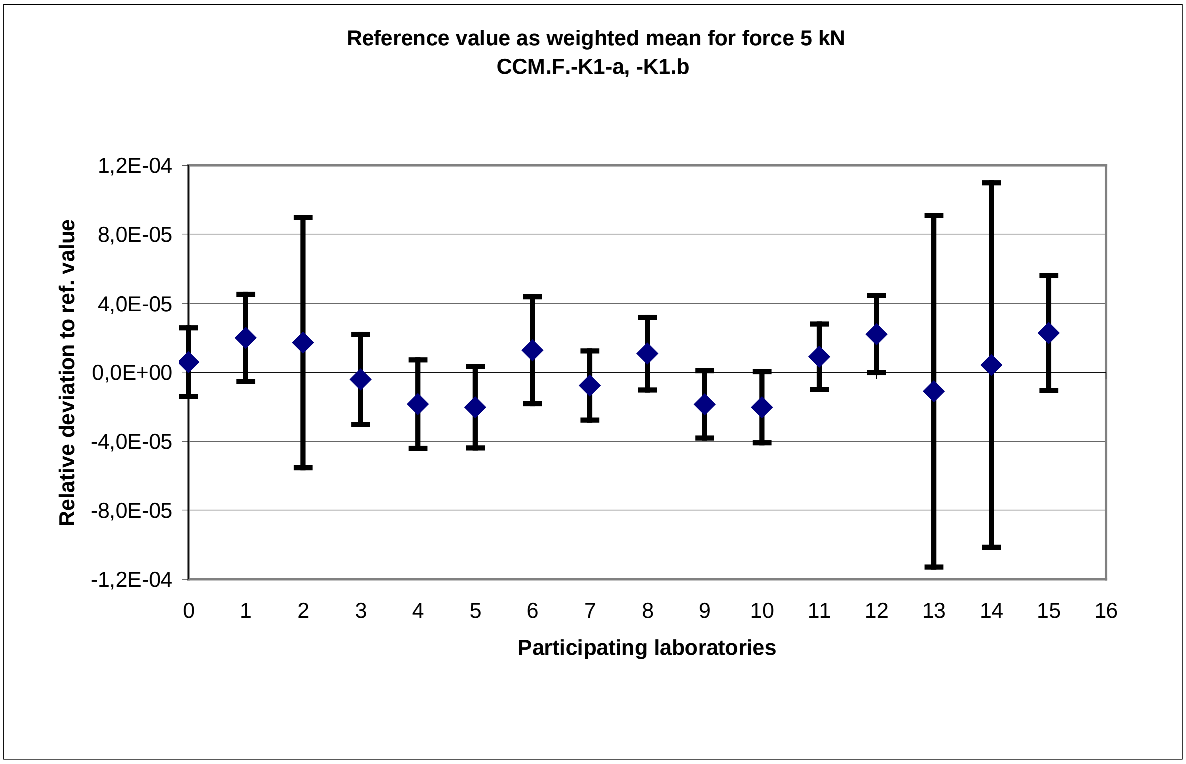

Figure 3.4. For a force of 5 kN

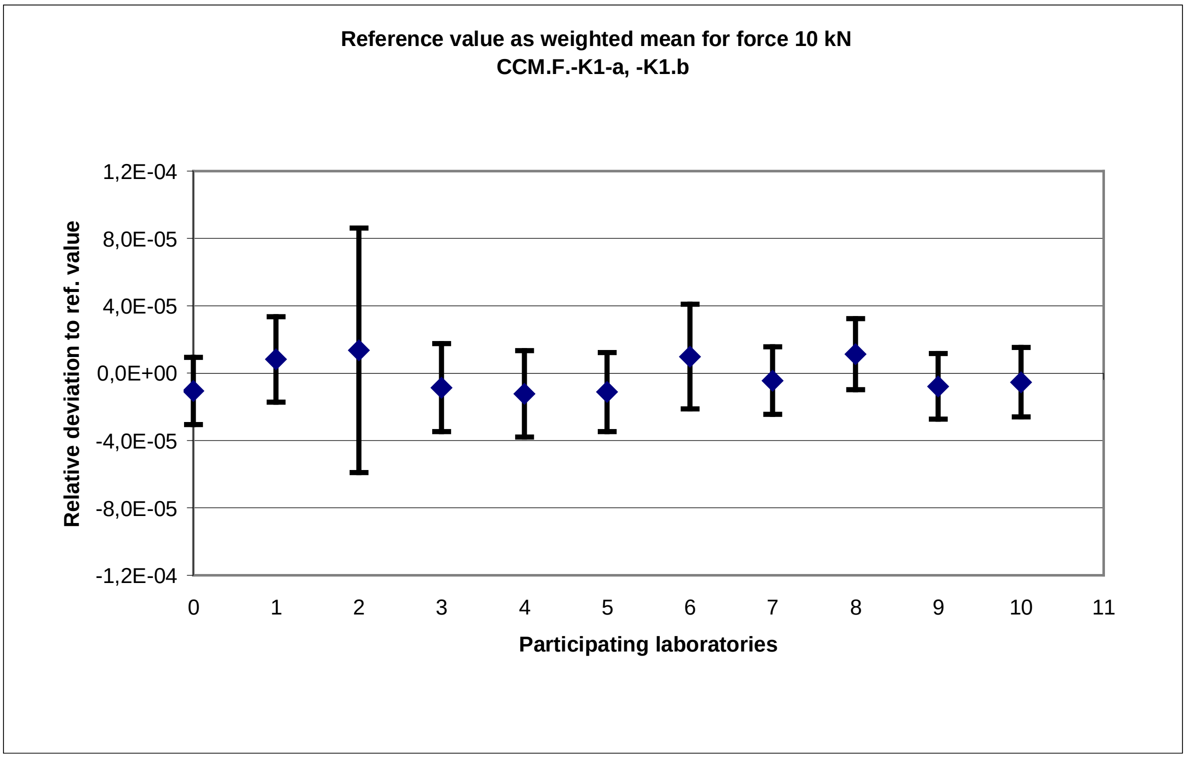

Figure 3.5 . For a force of 10 kN

Laboratory	$d_{L,P} = \bar{d}_L - d_P$, in 10^{-5}			
	5 kN		10 kN	
	$d_{\mathrm{L,P}}$	$W_{\mathbf{L,P}}$	$d_{\mathrm{L,P}}$	$W_{\mathbf{L,P}}$
0	0,6	1,9	1,1	1,9
1	2,0	2,5	0,8	2,4
2	1,7	7,2	0,0	7,2
3	-0,4	2,5	-0,9	2,5
4	-1,8	2,5	-1,2	2,4
5	-2,0	2,3	-1,1	2,0
6	1,3	3,0	1,0	3,3
7	-0,8	1,9	-0,4	2,5
8	1,1	2,0	1,1	1,9
9	-1,9	1,8	-0,8	1,8
10	-2,0	2,0	-0,5	1,9
11	0,9	1,8		
12	2,2	2,1		
13	-1,1	10,2		
14	0,4	10,5		
15	2,3	3,3		

Table 3.5. Degree of equivalence between Laboratory and reference value

	$d_{pairs} = d_{L,n} - d_{L,n+1}$ d_{pairs} in 10^{-5}															
Lab	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0		-1,4	-1,1	1,0	2,4	2,6	-0,7	1,4	-0,5	2,5	2,6	-0,3	-1,6	1,7	0,2	-1,7
1	1,4		0,3	2,4	3,8	4,0	0,7	2,8	0,9	3,9	4,0	1,1	-0,2	3,1	1,6	-0,3
2	1,1	-0,3		2,1	3,6	3,8	0,5	2,5	0,7	3,6	3,8	0,8	-0,5	2,8	1,3	-0,5
3	-1,0	-2,4	-2,1		1,4	1,6	-1,7	0,4	-1,5	1,5	1,6	-1,3	-2,7	0,7	-0,8	-2,7
4	-2,4	-3,8	-3,6	-1,4		0,2	-3,1	-1,1	-2,9	0,0	0,2	-2,8	-4,1	-0,8	-2,3	-4,1
5	-2,6	-4,0	-3,8	-1,6	-0,2		-3,3	-1,3	-3,1	-0,2	0,0	-2,9	-4,3	-0,9	-2,5	-4,3
6	0,7	-0,7	-0,5	1,7	3,1	3,3		2,0	0,2	3,1	3,3	0,4	-1,0	2,4	0,8	-1,0
7	-1,4	-2,8	-2,5	-0,4	1,1	1,3	-2,0		-1,8	1,1	1,3	-1,7	-3,0	0,3	-1,2	-3,0
8	0,5	-0,9	-0,7	1,5	2,9	3,1	-0,2	1,8		2,9	3,1	0,2	-1,2	2,2	0,6	-1,2
9	-2,5	-3,9	-3,6	-1,5	0,0	0,2	-3,1	-1,1	-2,9		0,2	-2,8	-4,1	-0,8	-2,3	-4,1
10	-2,6	-4,0	-3,8	-1,6	-0,2	0,0	-3,3	-1,3	-3,1	-0,2		-2,9	-4,3	-0,9	-2,5	-4,3
11	0,3	-1,1	-0,8	1,3	2,8	2,9	-0,4	1,7	-0,2	2,8	2,9		-1,3	2,0	0,5	-1,4
12	1,6	0,2	0,5	2,7	4,1	4,3	1,0	3,0	1,2	4,1	4,3	1,3		3,3	1,8	0,0
13	-1,7	-3,1	-2,8	-0,7	0,8	0,9	-2,4	-0,3	-2,2	0,8	0,9	-2,0	-3,3		-1,5	-3,4
14	-0,2	-1,6	-1,3	0,8	2,3	2,5	-0,8	1,2	-0,6	2,3	2,5	-0,5	-1,8	1,5		1,5
15	1,7	0,3	0,5	2,7	4,1	4,3	1,0	3,0	1,2	4,1	4,3	1,4	0,0	3,4	1,8	

Table 3.6 Degree of equivalence between two Laboratories for force of 5 kN

	$w_{d,L,ref}$ in 10^{-5}															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0		3,3	7,5	3,3	3,3	3,1	3,7	2,9	2,9	2,8	2,9	2,8	3,0	10,4	10,8	3,9
1	3,3		7,7	3,6	3,6	3,5	4,0	3,2	3,3	3,2	3,3	3,2	3,4	10,5	10,9	4,2
2	7,5	7,7		7,7	7,7	7,6	7,9	7,5	7,6	7,5	7,5	7,5	7,6	12,5	12,8	8,0
3	3,3	3,6	7,7		3,7	3,5	4,0	3,3	3,3	3,2	3,3	3,2	3,4	10,5	10,9	4,2
4	3,3	3,6	7,7	3,7		3,5	4,0	3,2	3,3	3,2	3,3	3,2	3,4	10,5	10,9	4,2
5	3,1	3,5	7,6	3,5	3,5		3,9	3,1	3,2	3,0	3,1	3,0	3,2	10,5	10,8	4,1
6	3,7	4,0	7,9	4,0	4,0	3,9		3,7	3,7	3,6	3,7	3,6	3,8	10,7	11,0	4,5
7	2,9	3,2	7,5	3,3	3,2	3,1	3,7		2,9	2,8	2,9	2,8	3,0	10,4	10,7	3,9
8	2,9	3,3	7,6	3,3	3,3	3,2	3,7	2,9		2,9	2,9	2,8	3,0	10,4	10,8	3,9
9	2,8	3,2	7,5	3,2	3,2	3,0	3,6	2,8	2,9		2,8	2,7	2,9	10,4	10,7	3,8
10	2,9	3,3	7,5	3,3	3,3	3,1	3,7	2,9	2,9	2,8		2,8	3,0	10,4	10,8	3,9
11	2,8	3,2	7,5	3,2	3,2	3,0	3,6	2,8	2,8	2,7	2,8		2,9	10,4	10,7	3,8
12	3,0	3,4	7,6	3,4	3,4	3,2	3,8	3,0	3,0	2,9	3,0	2,9		10,4	10,8	4,0
13	10,4	10,5	12,5	10,5	10,5	10,5	10,7	10,4	10,4	10,4	10,4	10,4	10,4		14,7	10,7
14	10,8	10,9	12,8	10,9	10,9	10,8	11,0	10,7	10,8	10,7	10,8	10,7	10,8	14,7		11,1
15	3,9	4,2	8,0	4,2	4,2	4,1	4,5	3,9	3,9	3,8	3,9	3,8	4,0	10,7	11,1	

Table 3.7 Associated expanded uncertainties for values on table 3.6 for force of 5 kN

	$d_{pairs} = d_{L,n} - d_{L,n+1}$ d_{pairs} in 10^{-5}										
	0	1	2	3	4	5	6	7	8	9	10
0		0,2	-0,3	1,9	2,3	2,2	0,1	1,5	-0,1	1,8	1,6
1	-0,2		-0,5	1,7	2,1	1,9	-0,2	1,3	-0,3	1,6	1,4
2	0,3	0,5		2,2	2,6	2,5	0,4	1,8	0,2	2,1	1,9
3	-1,9	-1,7	-2,2		0,4	0,2	-1,9	-0,4	-2,0	-0,1	-0,3
4	-2,3	-2,1	-2,6	-0,4		-0,1	-2,2	-0,8	-2,4	-0,5	-0,7
5	-2,2	-1,9	-2,5	-0,2	0,1		-2,1	-0,7	-2,2	-0,3	-0,6
6	-0,1	0,2	-0,4	1,9	2,2	2,1		1,4	-0,1	1,8	1,5
7	-1,5	-1,3	-1,8	0,4	0,8	0,7	-1,4		-1,6	0,3	0,1
8	0,1	0,3	-0,2	2,0	2,4	2,2	0,1	1,6		1,9	1,7
9	-1,8	-1,6	-2,1	0,1	0,5	0,3	-1,8	-0,3	-1,9		-3,4
10	-1,6	-1,4	-1,9	0,3	0,7	0,6	-1,5	-0,1	-1,7	0,3	

Table 3.8 Degree of equivalence between two Laboratories for force of 10 kN

	$w_{d,L,ref}$ in 10^{-5}										
	0	1	2	3	4	5	6	7	8	9	10
0		3,3	7,5	3,3	3,3	3,0	3,9	3,3	2,9	2,8	2,9
1	3,3		7,7	3,6	3,6	3,3	4,2	3,6	3,3	3,2	3,3
2	7,5	7,7		7,7	7,7	7,6	8,0	7,7	7,5	7,5	7,5
3	3,3	3,6	7,7		3,6	3,3	4,2	3,7	3,3	3,2	3,3
4	3,3	3,6	7,7	3,6		3,3	4,2	3,6	3,3	3,2	3,3
5	3,0	3,3	7,6	3,3	3,3		4,0	3,4	2,9	2,8	2,9
6	3,9	4,2	8,0	4,2	4,2	4,0		4,3	3,9	3,9	3,9
7	3,3	3,6	7,7	3,7	3,6	3,4	4,3		3,3	3,2	3,3
8	2,9	3,3	7,5	3,3	3,3	2,9	3,9	3,3		2,8	2,9
9	2,8	3,2	7,5	3,2	3,2	2,8	3,9	3,2	2,8		6,1
10	2,9	3,3	7,5	3,3	3,3	2,9	3,9	3,3	2,9	2,8	

Table 3.9 Associated expanded uncertainties for values on table 3.8 for force of 10 kN