The Asia-Pacific Metrology Programme (APMP)
200 MPa HYDRAULIC PRESSURE INTERLABORATORY COMPARISON

Final Report on Supplementary Comparison APMP.M.P-S7.TRI in Hydraulic Gauge Pressure from 40 MPa to 200 MPa

November 2016

Tokihiko Kobata[1], Douglas A Olson[2], and Alaaeldin A Eltawil[3]

Abstract

This report describes the results of a supplementary comparison of hydraulic high-pressure standards at three National Metrology Institutes (NMIs), National Metrology Institute of Japan, AIST (NMIJ/AIST), National Institute of Standards and Technology (NIST), USA and National Institute for Standards (NIS), Egypt, which was carried out at NIST during the period May 2001 to September 2001 within the framework of the Asia-Pacific Metrology Programme (APMP) in order to evaluate their degrees of equivalence at pressures in the range 40 MPa to 200 MPa for gauge mode. The pilot institute was NMIJ/AIST. Three working pressure standards from the institutes, in the form of piston-cylinder assemblies, were used for the comparison. The comparison and calculation methods used are discussed in this report. From the cross-float measurements, the differences between the working pressure standards of each institute were examined through an evaluation of the effective area of each piston-cylinder assembly with its uncertainty. From the comparison results, it was revealed that the values claimed by the participating institutes, NMIJ, NIST, and NIS, agree within the expanded (k = 2) uncertainties. The hydraulic pressure standards in the range 40 MPa to 200 MPa for gauge mode of the three participating NMIs were found to be equivalent within their claimed uncertainties.

[1] NMIJ/AIST: National Metrology Institute of Japan, AIST, AIST Tsukuba, Ibaraki, Japan.

[2] NIST: National Institute of Standards and Technology, Gaithersburg, MD, USA

[3] NIS: National Institute for Standards, El-Haram, Giza, Egypt

Contents:

1. Introduction

This report describes a pressure comparison in the range from 40 MPa to 200 MPa, which was performed at the National Institute of Standards and Technology (NIST). The participants were the National Metrology Institute of Japan (NMIJ), NIST, and the National Institute for Standards (NIS). The comparison has been identified as **APMP.M.P-S7.TRI** by the Consultative Committee for Mass and Related Quantities (CCM) of the International Committee for Weights and Measures (CIPM), the International Bureau of Weights and Measures (BIPM).

In a conventional comparison, at least one complete transfer standard, which usually consists of a piston-cylinder assembly, mass set and base, is circulated among the participating institutes and is compared with the pressure standard at each institute [e.g., 1, 2]. In the present comparison, such a complete transfer standard was not used. Instead, working pressure standards from two institutes, NMIJ and NIS, in the form of piston-cylinder assemblies, were brought and used for the present comparison at NIST. This method with the compact and lightweight transfer standard has advantages in terms of both transportation and cost. Each participating piston-cylinder assembly is the working standard at its respective institute. The effective areas and the uncertainties of each participating piston -cylinder assembly were supplied as a function of pressure by the respective institute. However, it should be noted that the uncertainty claimed for the assembly does not indicate the uncertainty of the primary pressure standards at each institute. The uncertainty claimed for the assembly includes the additional uncertainty of the particular assembly and also depends on the calibration method used at the respective institute. In this report, the claimed values for effective areas and uncertainties are used without modification. The method used at each institute to determine these values is not described.

In this report, the piston-cylinder assemblies and apparatus used are described first. Next, the comparison and calculation methods are described. From the results of the cross-float measurements, the differences between the claimed values are examined through the evaluation of the measured effective areas of the piston-cylinder assemblies. Finally, the results are summarized.

2. Participating institutes and their piston-cylinder assemblies

All of the piston-cylinder assemblies used in this comparison are listed in Table 1. In this table, the names of institutes, the identifications in this report, the piston gauge types, the maximum operating pressures, the nominal areas, the thermal expansion coefficients, the reference temperatures, the relative uncertainty of the claimed effective area for each assembly, and the working fluid used for the calibration at each institute are listed.

Table 1: Piston-cylinder assemblies.

Institute	I.D.	Type	Max. Pres.	Nom. Area	α	Ref. T.	$u(A_p)/A_p$	Working
			[MPa]	[m^2]	[^{o}C^{-1}]	[^{o}C]	(k=1)	Fluid
NIST	PCA_1	Reentrant [Ruska]	276	8.4×10^{-6}	8.22×10^{-6}	23	1.6×10^{-5}	Spinesstic #22
NMIJ	PCA_2	Simple [DH]	200	4.9×10^{-6}	9.00×10^{-6}	23	2.1×10^{-5}	Dioctyl Sebacate
NIS	PCA_3	Simple [DH]	200	4.9×10^{-6}	9.00×10^{-6}	20	1.1×10^{-5}	Dioctyl Sebacate
NIST	PCA_C	Simple [DH]	200	4.9×10^{-6}	9.00×10^{-6}	-	-	Spinesstic #22

Although two assemblies PCA_1 and PCA_C are listed from NIST, only the former one was calibrated formally by the NIST primary standards, and it has been used as the NIST working standard in the pressure range of this comparison. PCA_1 is a re-entrant type piston-cylinder assembly and was used with its base and dedicated mass set in this comparison. It has a maximum operating pressure of 276 MPa.

For the NMIJ and NIS working standards, only the piston-cylinder assemblies, identified as PCA_2 and PCA_3, were brought from each institute. These assemblies are of the simple type, and have a maximum operating pressure of 200 MPa. Since only the piston-cylinder assembly was brought from these two laboratories, a base and dedicated mass set possessed by NIST for PCA_C were used to generate the pressure for these three assemblies.

The piston-cylinder assembly, PCA_C, which has the same shape, type and pressure range as those labeled PCA_2 or PCA_3, was used to investigate the repeatability, reproducibility, and linearity of the experimental system.

Figure 1 shows a schematic drawing of the apparatus used for the comparison. A constant volume valve was used to connect and disconnect the pressures generated by two piston gauges. The temperatures of the piston-cylinder assemblies and the floating positions of the pistons were measured using platinum resistance thermometers and non-contact position sensors, respectively. The equilibrium state of cross-float was determined by observing the change in the fall rate, which is obtained from the relation between the piston position and time. All the acquired data were collected by a computer through a GPIB and recorded in a disk using special software written for this comparison. In this comparison, Spinesstic #22 was used as the working fluid.

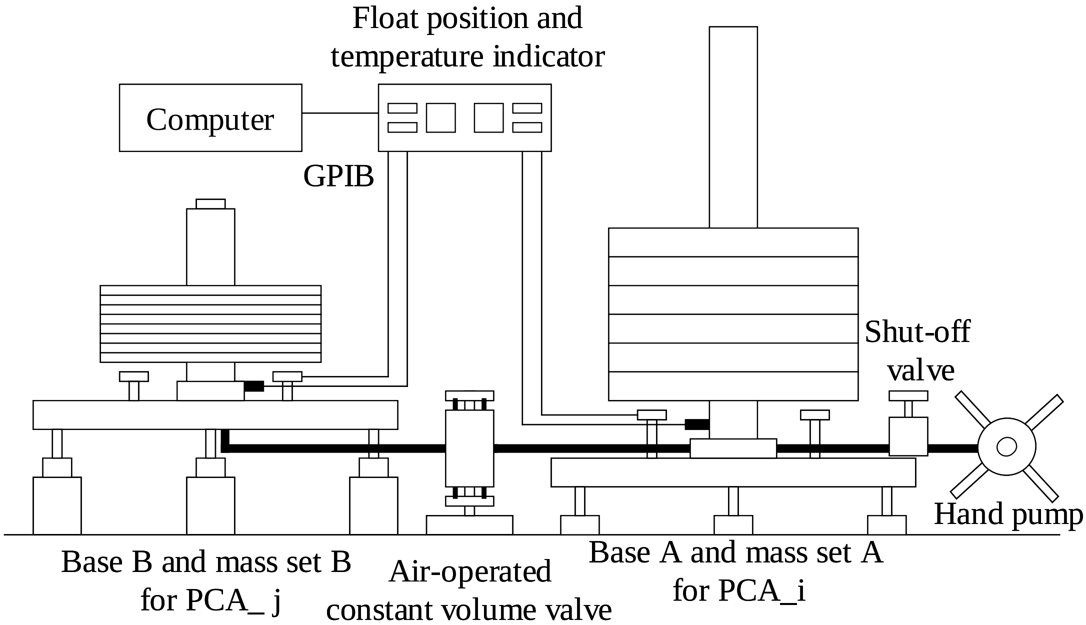

Fig. 1: Schematic drawing of apparatus.

3. Comparison Method

Table 2 shows the five combinations of the piston-cylinder assemblies used in the cross-float measurements. For each combination, two piston-cylinder assemblies PCA_a and PCA_b were compared, where PCA_a and PCA_b are the piston-cylinder assemblies installed in base A and B and used with mass set A and B, respectively. Although two different bases and mass sets were used for this comparison, base A and mass set A were used for PCA_1 exclusively. As shown in the table, PCA_1 was used as PCA_a in all the combinations and forms the basis of comparison for the other three assemblies, PCA_2, PCA_3, and PCA_C, listed as PCA_ b. After each switching of the PCA_b assemblies, the verticality and the output of the floating position of the piston were re-adjusted and re-calibrated.

Table 2: Combinations of piston-cylinder assemblies.

No.	PCA_a	PCA_b	Measurement Period
1	1	C	May 4 2001 – May 11 2001
2	1	2	Jun 27 2001 – Jun 29 2001
3	1	C	Jul 2 2001 – Jul 12 2001
4	1	3	Jul 16 2001 – Jul 23 2001
5	1	C	Jul 24 2001 – Jul 26 2001

In this comparison, the pressure ranged from 40 MPa to 200 MPa. The minimum pressure, p_{min} = 40 MPa, was selected to satisfy the relation, $(p_{min} / p_{max}) \geq 0.1$ for all assemblies including PCA_1, which has a maximum operating pressure, p_{max}, of 276 MPa.

For each combination, the cross-float measurements were performed at nine system pressures ranging from 40 MPa to 200 MPa in steps of 20 MPa. The ascending and descending measurements were repeated 3 times. Therefore, 6 data points for each nominal pressure, or 54 data points in all, were obtained for each combination of piston-cylinder assemblies.

The comparison between the assemblies of the same type, for example, PCA_2 and PCA_C, were not performed because two separate bases for those assemblies were not available. Prior commitments for the assemblies from each of the participating institutes limited the duration of their comparison with PCA_1. The piston-cylinder assemblies were returned to each institute at the completion of the measurements.

4. Calculation method for comparison

4.1 Equations for effective area and generated pressure

Each institute represents the effective area of the piston-cylinder assembly of their working standard by the following equation, which assumes a linear pressure dependence:

$$A(p,t) = A(0,t_r)\cdot(1+b\cdot p)\cdot\{1+\alpha_s\cdot(t-t_r)\} \qquad (1)$$

where $A(p,\ t)$ is the effective area of the piston-cylinder assembly at pressure p and temperature t, $A(\ 0,\ t_r)$ is the effective area of the assembly at 0 Pa and at reference temperature, t_r, in degrees Celsius, b is the pressure distortion coefficient of the effective area, and α_s is the sum of α_p and α_c, which are the linear thermal expansion coefficients for the piston and the cylinder respectively [3].

Using the effective area defined as in equation (1), the pressure generated by the assembly at the reference level is

$$p = \{F/A(p,t)\} + (\rho_f - \rho_a)\cdot g\cdot h \qquad (2)$$

where ρ_f is the density of the working fluid, ρ_a is the air density, g is the local acceleration due to gravity, h is the vertical distance between the bottom of the piston and the reference level, and F is the applied force on the piston including the force exerted on the piston by the surface tension of the working fluid and expressed as follows:

$$F = M\cdot g\cdot(1-\rho_a/\rho_m) + \gamma\cdot C \qquad (3)$$

where M is the sum of the total mass of the weights including the piston mass and tare mass, ρ_m is the average density of the loaded weights, γ is the surface tension of the working fluid, and C is the circumference of the piston where it emerges from the pressurizing fluid.

4.2 Equations for ratio of effective areas

From a cross-float measurement using two deadweight piston gauges, the ratio of the effective areas of the assemblies used can be calculated. When two gauges are cross-floated, the generated pressure of both gauges are equal, that is, $p_{\_\mathrm{i}} = p_{\_\mathrm{j}}$, and so the following relation is obtained using equation (2):

$$\left\{\frac{A_{\_\mathrm{j}}(p,t_{\_\mathrm{j}})}{A_{\_\mathrm{i}}(p,t_{\_\mathrm{i}})}\right\}_{\mathrm{CF}} = \frac{F_{\_\mathrm{j}}}{\left\{F_{\_\mathrm{i}} + \left(\rho_f - \rho_a\right)\cdot g \cdot H \cdot A_{\_\mathrm{i}}(p,t_{\_\mathrm{i}})\right\}} \qquad (4)$$

where $H = h_{\_\mathrm{i}} - h_{\_\mathrm{j}}$, the suffix, i or j shows the values related to the i-th or j-th assembly and the suffix CF means the value determined by the cross-float measurement. The ratio corrected by the reference temperatures, $(R_{\_\mathrm{j/i}})_{\mathrm{CF}}$, is obtained from the following equation:

$$\left(R_{\_\mathrm{j/i}}\right)_{\mathrm{CF}} = \left\{\frac{A_{\_\mathrm{j}}(p,t_{\_\mathrm{j}})}{A_{\_\mathrm{i}}(p,t_{\_\mathrm{i}})}\right\}_{\mathrm{CF}} \cdot \frac{\left\{1+\alpha_{s\_\mathrm{i}}\cdot\left(t_{\_\mathrm{i}} - t_{r\_\mathrm{i}}\right)\right\}}{\left\{1+\alpha_{s\_\mathrm{j}}\cdot\left(t_{\_\mathrm{j}} - t_{r\_\mathrm{j}}\right)\right\}} \qquad (5)$$

where α_{s_i} and α_{s_j} are the linear thermal expansion coefficients and t_{r_i} and t_{r_j} are the reference temperatures of the piston-cylinder assemblies i and j respectively.

4.3 Evaluation of differences between the values claimed and obtained from cross-float measurement

Against the experimental value, R_{CF}, obtained from equation (5), the ratio of the effective areas of the assemblies can be obtained from the claimed values by each institute, as R_{CLAIM} using equation (1).

$$\left(R_{\_\mathrm{j/i}}\right)_{\mathrm{CLAIM}} = \frac{A_{\_\mathrm{j}}(0,t_{r\_\mathrm{j}})\cdot\left(1+b_{\_\mathrm{j}}\cdot p\right)}{A_{\_\mathrm{i}}(0,t_{r\_\mathrm{i}})\cdot\left(1+b_{\_\mathrm{i}}\cdot p\right)} \qquad (6)$$

The differences between the ratios obtained from the cross-float measurement, R_{CF}, and the claimed values, R_{CLAIM}, were evaluated by the following equation:

$$D = \frac{R_{\mathrm{CF}}}{R_{\mathrm{CLAIM}}} - 1 \tag{7}$$

where D is the difference as a function of pressure. The j/i subscript used up to equation 6 is omitted for clarity in this and the following equations. The combined uncertainty of the difference, $u_c(D)$, was calculated by the next equation:

$$u_c(D) = \left(\frac{R_{\mathrm{CF}}}{R_{\mathrm{CLAIM}}} \right) \cdot \sqrt{\left\{ \frac{u(R_{\mathrm{CF}})}{R_{\mathrm{CF}}} \right\}^2 + \left\{ \frac{u(R_{\mathrm{CLAIM}})}{R_{\mathrm{CLAIM}}} \right\}^2} \tag{8}$$

where $u(R_{\mathrm{CF}})$ is the uncertainty evaluated from the cross-float measurement and $u(R_{\mathrm{CLAIM}})$ is the uncertainty obtained from the claimed values. The expanded (k = 2) uncertainty of the difference, $U(D)$, was calculated by the equation, $U(D) = 2 \cdot u_c(D)$.

The uncertainty for the cross-float measurement, $u(R_{\mathrm{CF}})$, was evaluated as the root sum of squares of the Type A and Type B uncertainties:

$$u(R_{\mathrm{CF}}) = \sqrt{(u_a)^2 + (u_b)^2} \tag{9}$$

where u_a and u_b are the uncertainties obtained by Type A and Type B evaluations [4]. The Type A uncertainty, u_a was obtained from the standard deviation of the mean obtained from repeated measurements at the same pressure. The Type B uncertainty, u_b, was estimated by combining the Type B uncertainties of all parameters used according to the root sum of squares method [4]. A detailed explanation of the calculation processes for the estimation of Type B uncertainty of each parameter is not given here.

The relative uncertainty for the claimed values, $u(R_{\mathrm{CLAIM}})/R_{\mathrm{CLAIM}}$, was calculated by taking the root sum of squares of the claimed uncertainties for the effective areas of the two assemblies used in the comparison by the following equation:

$$\frac{u(R_{\mathrm{CLAIM}})}{R_{\mathrm{CLAIM}}} = \sqrt{\left[\frac{u\{A_{\_\mathrm{i}}(t_{r\_\mathrm{i}})\}}{A_{\_\mathrm{i}}(t_{r\_\mathrm{i}})} \right]^2 + \left[\frac{u\{A_{\_\mathrm{j}}(t_{r\_\mathrm{j}})\}}{A_{\_\mathrm{j}}(t_{r\_\mathrm{j}})} \right]^2} \tag{10}$$

5. Results and discussion

As shown in Table 2, the PCA_C was re-installed and cross-floated against PCA_1 three times, and each piston-cylinder assembly, PCA_2 or PCA_3, was cross-floated against PCA_1 directly only once. From these results, the characteristics of the experimental system are evaluated in section 5.1 and 5.2. The results obtained from the cross-float measurements, PCA_2 vs. PCA_1 and PCA_3 vs. PCA_1, are described in the section 5.3. The relation between PCA_3 and PCA_2 was examined using the results of two cross-float measurements, PCA_2 vs. PCA_1 and PCA_3 vs. PCA_1, although they were not cross-floated directly. The result is described in the section 5.4.

5.1 Estimation of uncertainties

After each installation of assembly PCA_b, cross-float measurements between PCA_b and PCA_a were repeated during three cycles of ascending and descending pressures, yielding six values of R_{CF} at each pressure. The repeatability of an individual measurement at a given pressure was evaluated as the standard deviation of the six values about their mean. The relative values for repeatability were generally less than 2×10^{-6} at all pressure points. The reproducibility between measurement sets arising from disassembly then reassembly of the piston-cylinder assembly within the base was evaluated as the standard deviation of three mean values of R_{CF} obtained from the three measurement sets between PCA_C and PCA_1. The relative values for reproducibility were less than 3×10^{-6} for all pressure points for this combination. In the following calculations, this value for reproducibility is included as a part of the Type B relative uncertainty in R_{CF}.

Table 3 shows the relative uncertainties for the cross-float measurement obtained from Type A and Type B evaluations, u_a and u_b, and the combined relative uncertainties (k = 1) calculated by equation (9), $u(R_{\mathrm{CF}})/R_{\mathrm{CF}}$. Values for the Type A uncertainty were calculated as the standard deviation of the mean, $s/6^{1/2}$, where s was the standard deviation of the six values of R_{CF} at each pressure point. The variation of these values as a function of pressure was relatively small and so u_a was taken as the maximum value. The average temperature during these cross-float measurements was 23.5 $^{\mathrm{o}}$C, yielding values for $(t - t_r)$ of 0.5, 0.5, and 3.5 $^{\mathrm{o}}$C for PCA_1, PCA_2, and PCA_3, respectively. The uncertainty for this term was included in u_b. In the table, the relative uncertainties (k = 1) for the claimed values, $u(R_{\mathrm{CLAIM}})/R_{\mathrm{CLAIM}}$, and the combined uncertainties (k = 1) calculated by equation (8), $u_c(D)$, are also shown.

The Type A uncertainty for the combination between PCA_3 and PCA_2, $u_{a\_3/2}$, was calculated by taking the root sum of squares of $u_{a\_2/1}$ and $u_{a\_3/1}$. The Type B uncertainty, $u_{b\_3/2}$, was smaller than the uncertainty, $u_{b\_2/1}$ or $u_{b\_3/1}$, since both assemblies, PCA_2 and PCA_3, used the same mass set, and the level difference in equation (4), H, was zero for this combination.

Table 3: Uncertainty evaluation (k = 1).

PCA_i	PCA_ j	u_a/R_{CF}	u_b/R_{CF}	$u(R_{\mathrm{CF}})/R_{\mathrm{CF}}$	$u(A_{\_\mathrm{i}})/A_{\_\mathrm{i}}$	$u(A_{\_\mathrm{j}})/A_{\_\mathrm{j}}$	$u(R_{\mathrm{CLAIM}})/R_{\mathrm{CLAIM}}$	$u_c(D)$
1	2	1.2	5.4	5.6	16.0	21.0	26.4	27.0
1	3	1.8	5.4	5.7	16.0	10.6	19.2	20.0
2	3	2.1	4.0	4.5	21.0	10.6	23.5	24.0

All the uncertainties are expressed as the standard ones [$\times10^{-6}$].

5.2 Linearity of the ratio of effective areas

The linearity of the ratio obtained from equation (5) was examined for measurement results of PCA_2 vs. PCA_1, PCA_3 vs. PCA_1, and PCA_C vs. PCA_1. Since the characteristics of the effective areas of all participating assemblies were nominally linear functions of pressure, it was expected that the relations between them should also be linear. Figure 2 shows the results obtained. The deviations show the differences between the experimental values and the least-squares-best-fitting straight line as a function of pressure. As shown in the figure, the deviations were below 6×10^{-6} for all pressure points of all combinations in the pressure range between 40 MPa and 200 MPa.

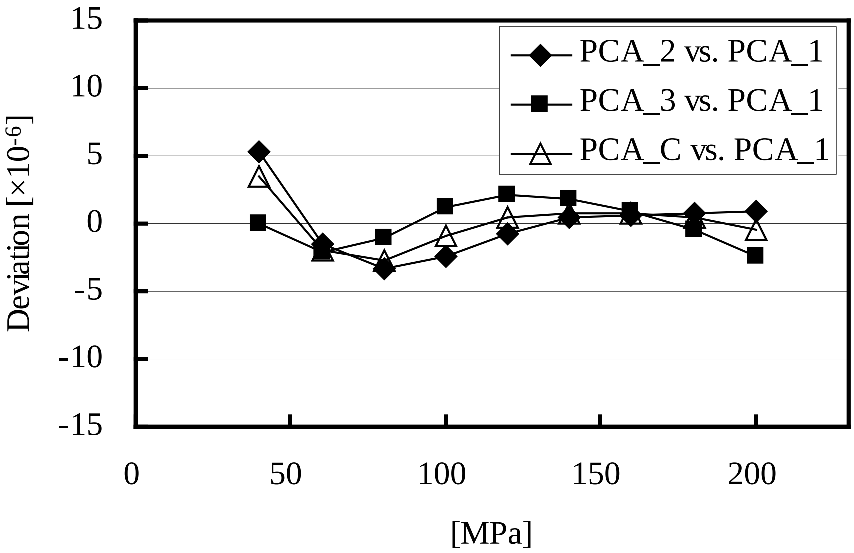

Fig. 2: Linearity of the ratio of effective areas.

5.3 Results from direct comparison

5.3.1 Cross-float between PCA_2 and PCA_1

Figure 3 represents the results obtained from the cross-float between PCA_2 and PCA_1. In the figure, the vertical axis shows the difference *D* in parts per million. The differences were calculated by equation (7) for nine system pressures *p* from 40 MPa to 200 MPa. The error bars show the expanded (k = 2) uncertainty, $U(D)$. For this combination, the results show that the values claimed for the two assemblies, PCA_2 and PCA_1, agree within the expanded (k = 2) uncertainties.

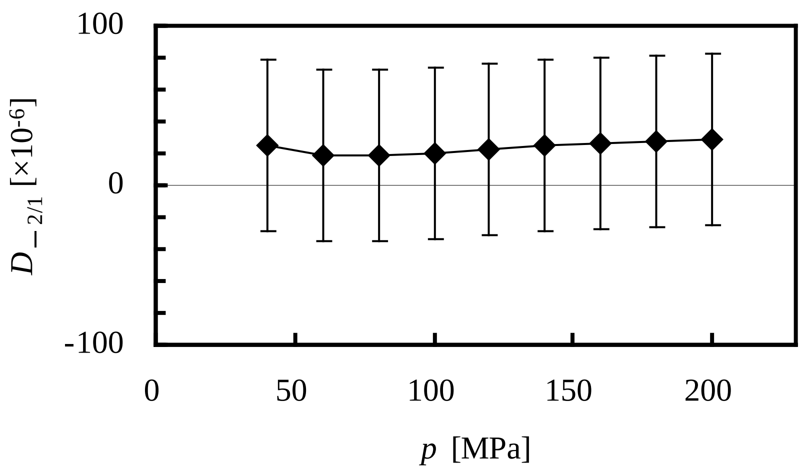

Fig. 3: Difference between PCA_2 and PCA_1.

5.3.2 Cross-float between PCA_3 and PCA_1

Figure 4 represents the results obtained from the cross-float between PCA_3 and PCA_1. The method of calculation and the expression in the graph are the same as those described above in section 5.3.1. The results for this combination show that the values claimed for the two assemblies, PCA_3 and PCA_1, agree within the expanded (k = 2) uncertainties.

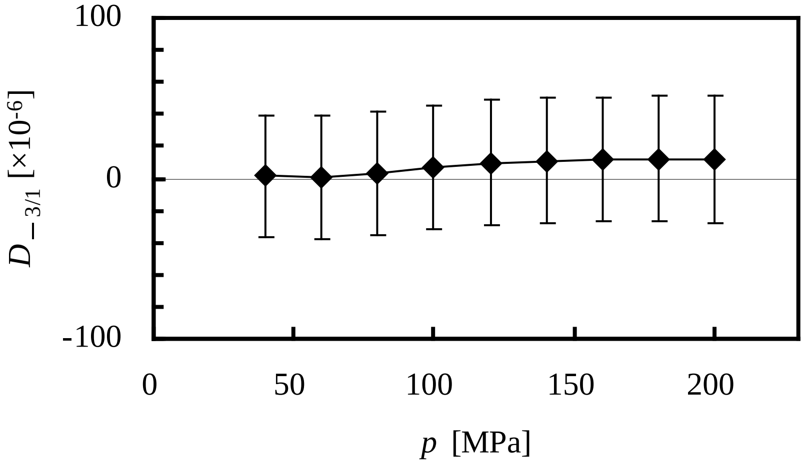

Fig. 4: Difference between PCA_3 and PCA_1.

5.4 Results from indirect comparison

5.4.1 Difference between PCA_3 and PCA_2

The ratios $(R_{\_3/2})_{CF}$ were calculated by taking the ratios of $(R_{\_3/1})_{CF}$ to $(R_{\_2/1})_{CF}$, which were obtained from the results in sections 5.3.1 and 5.3.2.

Figure 5 represents the results calculated from the combination between PCA_3 and PCA_2. The method of calculation and the expression in the graph are the same as those described above in section 5.3.1 or 5.3.2. The results for this combination also show that the values claimed for the two assemblies, PCA_3 and PCA_2, agree within the expanded (k = 2) uncertainties.

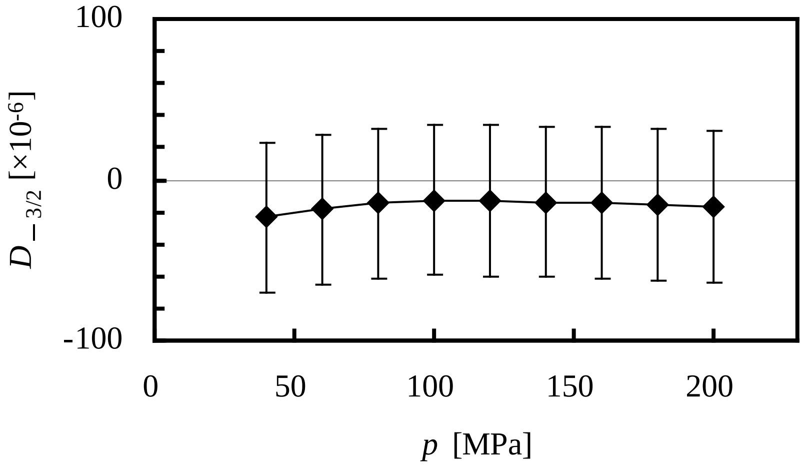

Fig. 5: Difference between PCA_3 and PCA_2.

6. Summary

A hydraulic pressure comparison in the range from 40 MPa to 200 MPa was performed at NIST by the participating institutes, NMIJ, NIST, and NIS. The comparison and calculation methods used were discussed in this report. From the cross-float measurements, the differences between the working pressure standards of each institute were determined through an evaluation of the effective area of the piston-cylinder assembly. From the comparison results, it was shown that the values claimed by the participating institutes, NMIJ, NIST, and NIS, agree within the expanded (k = 2) uncertainties.

Acknowledgements

The authors would like to thank all members of the pressure and vacuum group at NIST, especially, A. Lee, J. W. Schmidt, and A. P. Miiller for their encouragements and suggestions, and R. G. Driver and W. J. Bowers Jr. for their cooperation. We also would like to express our gratitude to the staffs of NMIJ and NIS who supported us to carry out this comparison, especially Mr. Akihiko Yonenaga, NMIJ. One of the authors (Eltawil) appreciates the institutional support of the U.S.- Egypt Science and Technology Joint Fund.

References

[1] Legras J. C., Lewis S. L., Molinar G. F., *Metrologia*, 1988, **25**, 21-28.

[2] Legras J. C., Molinar G. F., *Metrologia*, 1991, **28**, 419-424.

[3] NCSL RISP-4, *Deadweight Pressure Gauges*, July 1998.

[4] Barry N. Taylor and Chris E. Kuyatt, *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note **1297**, 1994 Edition.