Inherent uncertainty of air kerma

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1. Introduction

Air kerma is by definition the kinetic energy released per unit mass of air by indirect ionizing radiation. Air is a mixture of gases. The amount of each gas constituting air is not exact but has been obtained experimentally. Thus air kerma has an inherent uncertainty in measurement because the kerma in air depends on the radiation interactions with the gases in air and the amounts of the constituents of air have some uncertainties.

2. Method

In theory, air kerma of a photon beam at a given energy can be written as:

$$K = \Psi \left(\frac{1}{1 - g_{air}} \right) \left(\frac{\mu_{en}}{\rho} \right)_{air} \tag{1}$$

where K is air kerma, g_{air} the fraction of energy liberated into bremsstrahlung by the secondary electron produced by the incident photon, Ψ the energy fluence of the photon and $(\mu_{en} / \rho)_{air}$ the mass energy absorption coefficient of air. The mass energy absorption coefficient depends on the elemental composition of air.

With Eq. (1), it can be estimated how air kerma is inherently dependent on the composition of air. The mass energy absorption coefficient of air can be deduced using the relation:

$$\left(\frac{\mu_{en}}{\rho} \right)_{air} = \sum_i w_i \left(\frac{\mu_{en}}{\rho} \right)_i \tag{2}$$

where w_i is the weight fraction of the *i*-th constituent in air and $(\mu_{en} / \rho)_i$ the mass energy absorption coefficient of the *i*-th constituent. The weight fraction is given by:

$$w_i = \frac{x_i M_i}{M_a} \tag{3}$$

where x_i is mole fraction of the *i*-th constituent in air, M_i molar mass of the *i*-th constituent and M_a molar mass of air. The molar mass of air is given by:

$$M_a = \frac{\sum_i x_i M_i}{\sum_i x_i} \tag{4}$$

where the denominator is used to normalize the mole fraction. It is assumed that $\sum_i x_i = 1$ within an accuracy of 3 parts in 10^6. Using the law of propagation of the uncertainty, the following relationship is obtained:

$$\frac{u_{in}^2(K)}{K^2} = \frac{1}{(\mu_{en}/\rho)_{air}^2}\left\{\begin{array}{l} \sum_i \left[\frac{\partial}{\partial M_i}\left(\frac{x_i M_i}{\sum_i x_i M_i}\right)\right]^2 \left(\frac{\mu_{en}}{\rho}\right)_i^2 u^2(M_i) + \\ \sum_i \left[\frac{\partial}{\partial x_i}\left(\frac{x_i M_i}{\sum_i x_i M_i}\right)\right]^2 \left(\frac{\mu_{en}}{\rho}\right)_i^2 u^2(x_i) + \\ \sum_i \left[\frac{\partial}{\partial M_i}\left(\frac{\mu_{en}}{\rho}\right)_i\right]^2 \left(\frac{x_i M_i}{\sum_i x_i M_i}\right)^2 u^2(M_i) \end{array}\right\} \quad (5)$$

Or,

$$\frac{u_{in}^2(K)}{K^2} = \frac{1}{(\mu_{en}/\rho)_{air}^2}\left\{\begin{array}{l} \sum_i \left(\frac{x_i}{M_a^2}\right)^2 (M_a - M_i x_i)^2 \left(\frac{\mu_{en}}{\rho}\right)_i^2 u^2(M_i) + \\ \sum_i \left(\frac{M_i}{M_a^2}\right)^2 (M_a - M_i x_i)^2 \left(\frac{\mu_{en}}{\rho}\right)_i^2 u^2(x_i) + \\ \sum_i \left(\frac{x_i}{M_a}\right)^2 \left(\frac{\mu_{en}}{\rho}\right)_i^2 u^2(M_i) \end{array}\right\} \quad (6)$$

where $u_{in}(K)$, $u(M_i)$ and $u(x_i)$ are the inherent uncertainty of air kerma, the uncertainty of the molar mass and the uncertainty of the mole fraction of the *i*-th constituent in air, respectively. The third term in Eq. (6) has been obtained from the relationship [5]:

$$\left(\frac{\mu_{en}}{\rho}\right)_i = \frac{N_A}{M_i}\sum_j f_j (1 - g_j)\sigma_j \quad (7)$$

where N_A is Avogadro's number, f_i the average fraction of the photon energy that is transferred to kinetic energy of charged particles in interaction of type j, g_j the average fraction of the kinetic energy of secondary charged particles produced in an interaction of type j and σ_j the interaction cross section of type j. Note that the interaction types considered here are the atomic interactions, i.e., photoelectric absorption, coherent scattering, incoherent (Compton) scattering, pair and triplet productions.

By rearranging terms in Eq. (6), the relative inherent uncertainty becomes:

$$\frac{u_{in}^2(K)}{K^2} = \frac{1}{(\mu_{en}/\rho)_{air}^2}\left\{\begin{array}{l}\sum_i \left(\frac{M_i x_i}{M_a^2}\right)^2 (M_a - M_i x_i)^2 \left(\frac{\mu_{en}}{\rho}\right)_i^2 \left(\frac{u^2(M_i)}{M_i^2} + \frac{u^2(x_i)}{x_i^2}\right) + \\ \sum_i \left(\frac{M_i x_i}{M_a}\right)^2 \left(\frac{\mu_{en}}{\rho}\right)_i^2 \frac{u^2(M_i)}{M_i^2}\end{array}\right\}. \quad (8)$$

3. Results

The evaluated relative standard uncertainty inherent in air kerma is presented in Fig. 1 for photon beams with energies in the range 1 keV to 20 MeV. The values were distributed from 9 parts in 10^5 for the energies less than 100 keV to 5 parts in 10^5 for the energy range 100 keV to 20 MeV. The inherent standard uncertainty of air kerma remained almost the same within 1 part in 10^8 when other composition data of air was used.

From Eqs. (5), (6) and (8), it is evident that all kerma have their own inherent uncertainties propagated from the uncertainties of the atomic weights. The inherent uncertainties of kermas defined in the several frequently-encountered materials in dosimetry were calculated and values are illustrated in Fig. 2. The inherent standard uncertainties were evaluated to be around 3 parts in 10^5 for graphite kerma, 1 part in 10^5 for water kerma and 2 parts in 10^5 for polymethylmethacrylate (PMMA) and polyethyleneterephthalate (PET) kermas. For a monatomic substance, the inherent uncertainty is the same as the uncertainty of atomic weight regardless of photon energy and it is at a the similar level to the uncertainties of the constituent atomic weights for poly-atomic material. Likewise, the absorbed dose may have the inherent uncertainty component since the energy deposition in matter is due principally to kerma.

4. Concluding remarks

The inherent uncertainty of air kerma due to the air composition is less than 1 part in 10^4 but as a conservative estimate, it would be reasonably acceptable to set the inherent uncertainty equal to 1 part in 10^4. This uncertainty should be included in the uncertainty budget of the air kerma measurement. However, its influence is essentially negligible as the uncertainty of air kerma primary standards can be determined at around 2 parts in 10^3 that includes air density and attenuation uncertainties of around 1 part in 10^4.

It is also noticeable that all kerma have their own inherent uncertainties due to the propagation of uncertainty from the uncertainty of the atomic weight regardless of the material for which kerma is defined. The magnitude is at a similar level to the uncertainty of the atomic weight. The inherent standard uncertainties were around 3 parts in 10^5 for graphite kerma and about 1 part in 10^5 for water kerma. This inherent uncertainty always exists regardless of how accurately air kerma is measured.

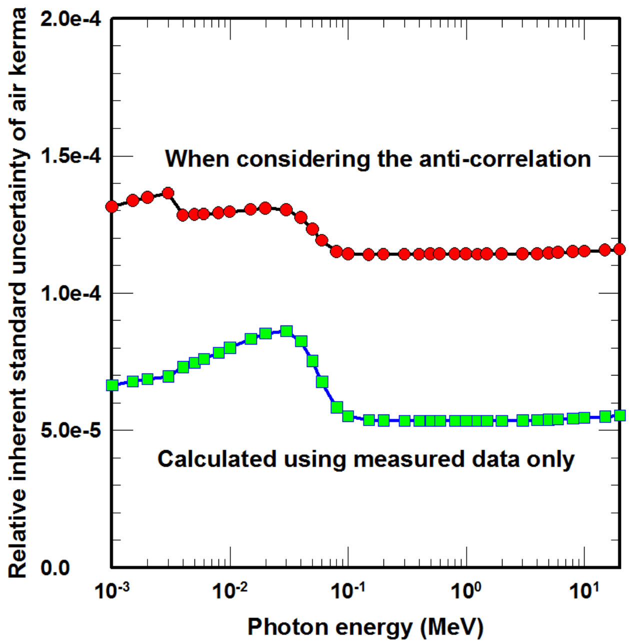

Fig. 1. The relative inherent standard uncertainty of air kerma of photon beams stemming from the uncertainties of atomic weight and air composition data.

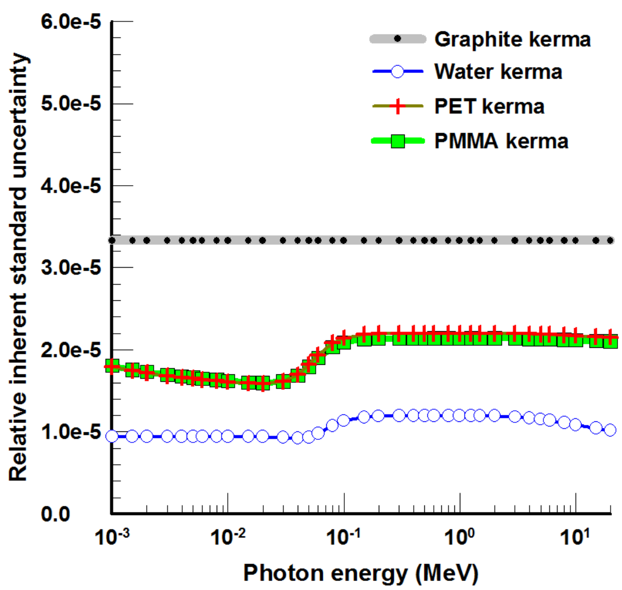

Fig. 2. The relative inherent standard uncertainties of different kermas of photon beams stemming from the uncertainties of atomic weights.