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The Role of Bilateral Comparisons in Evaluating the Consistency of Key Comparisons

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Most international standards comparisons are made with the International Bureau of Weights and Measures (BIPM) in Paris. In principle, this reduces the number of comparisons needed to one per country, with the BIPM. The results of these Key Comparisons are tabulated in the newly established Mutual Recognition Arrangement (MRA) [1]. Trade agreements are facilitated by comparing the entries of two countries.

Bilateral comparisons can also be made directly between two National Measurement Institutes. Until now, this has been done on an ad hoc basis. When two laboratories, A and B, have also made recent comparisons with a third laboratory, C, it is possible to compare the comparison result with a prediction based on the recent results of each laboratory with the third laboratory. We refer to these comparisons as “trilaterals”. This “trilateral” comparison can be represented as a triangular structure. If the primary standards are equivalent and invariant, all three ratios R_{ij} should be close to unity and the “triangle” of comparisons is closed. The relationship between the ratios can be expressed as:

$$R_{AB} \times R_{BC} \times R_{CA} = 1.0000 \qquad (1)$$

In practice, the “triangle” of comparisons will fail to close because of random uncertainties in the transfer chamber measurements. The assumption of invariance during the period of a trilateral comparison requires that each of the three laboratories should use the same primary standard determination for each of the two bilateral comparisons in which it participates. In practice, the three laboratories will be engaged in a process of continual improvements and the value of each primary standard will change slightly in the process. The effect of any such changes should be removed for the purpose of a trilateral comparison, which then represents a snapshot of the relationships between the three standards.

The “triangle” is difficult to show diagrammatically. Instead we have developed the following linear model. Each ratio R_{ij} is close to unity and can be represented in the form $1+\delta_{ij}$. If equation (1) is expanded and terms higher than first order neglected, we expect:

$$\Delta = \delta_{AB} + \delta_{BC} + \delta_{CA} \qquad (2)$$

where we define the “gap” Δ as a measure of the inconsistency of the three bilateral comparisons or the “failure to close the triangle”.

Equation (2) is illustrated in figure 1 for the air kerma comparisons that have been obtained from a recent bilateral comparison between and Australia and Canada [2], and the Key Comparison of each country with the BIPM. Figure 2 illustrates the relationship between the three laboratories for absorbed dose to water.

The "precision" bars on the right hand side of each figure correspond to the random uncertainties of the transfer chamber measurements at each laboratory as discussed above – the "precision" contributes to the size of Δ.

The "accuracy" bars on the left-hand side of each figure correspond to the stated uncertainties of the calibration coefficients as determined at each laboratory. These uncertainties are dominated by the systematic uncertainties, which are mostly of type B – The "accuracy" relates to the individual bilateral comparison results (the δ values).

The relative standard uncertainty S in Δ is the quadrature sum of the relative standard uncertainties in the R_{ij},

$$S^2(\Delta) = S^2(\delta_{AB}) + S^2(\delta_{BC}) + S^2(\delta_{CA}) = 2[(\sigma_A/A)^2 + (\sigma_B/B)^2 + (\sigma_C/C)^2] \qquad (3)$$

In the recent comparison between ARPANSA, Australia (A) and NRC, Canada (C) [2], both laboratories having made recent comparisons with the BIPM (B) [3-6], the relative standard uncertainties σ_A/A, etc are the type A uncertainties shown for I_K and I_{Dw} in table 1. Equation (4) gives S=0.0009 (0.09%) for both air kerma and absorbed dose to water. This is consistent with the observations of Δ.

Table 1. Percentage relative standard uncertainties at three laboratories from [2]

	ARP		BIPM		NRC	
	type A	type B	type A	type B	type A	type B
K	0.076	0.278	0.03	0.17	0.07	0.31
I_K	0.03	0.07	0.02	0.03	0.05	0.08
$\boldsymbol{N_K}$	**0.08**	**0.29**	**0.04**	**0.17**	**0.09**	**0.32**
D_w	0.08	0.18	0.20	0.21	0.21	0.35
I_{Dw}	0.03	0.075	0.02	0.06	0.05	0.082
$\boldsymbol{N_{Dw}}$	**0.09**	**0.19**	**0.20**	**0.22**	**0.22**	**0.36**

Analysis of another recent comparison between NRC and LNHB (France) [7], from which a trilateral result can be constructed with the BIPM [4,6,8], leads to similar results. The observed gaps are commensurate with their uncertainties. The random uncertainties include those due to the transfer chamber set up and current measurements (distance, depth in water, ambient corrections, charge measurements, source decay). The results are summarised for both examples in table 2.

Table 2. Summary of closure of “trilateral” comparisons

Ratio	NRC/BIPM/LNHB			ARP/BIPM/NRC		
	Uncertainties (95% CL)		Gap	Uncertainties (95% CL)		Gap
	Random	Systematic	Δ	Random	Systematic	Δ
N_k	0.0024	0.015	0.0009	0.0015	0.011	+0.0002
$N_{D,w}$	0.0028	0.016	0.0008	0.0015	0.017	-0.0004

The results support recent suggestions that uncertainties are being significantly overstated in international radiation standards comparisons. The systematic uncertainties of the various standards are seriously overestimated as shown by the agreement between the standards, which in each case is less than the stated 67% CL comparison uncertainty (accuracy).

The random uncertainties are apparently also overestimated, as shown by the size of the “gap” which is less than any one of the 67% CL random uncertainties (precision). It is considerably less than the combined random uncertainty of the three comparisons. Bilateral comparison uncertainties would be better expressed as the quadrature sum of the *random* components of the two artefact calibrations (precision). These uncertainties are primarily due to the transfer standard measurements. They could be reduced by appropriate artefact drift interpolations, which may also reduce the “gaps”. A more conservative position, which would allow for any inconsistencies revealed by the trilateral analysis, would be to use the uncertainty $S(\Delta)$ for all three bilateral results.

A recent “trilateral” comparison between ARPANSA, METAS (Switzerland) and BIPM will be able to be analysed soon. It is suggested that a similar analysis be made for any bilateral comparison for which Key Comparisons exist with the BIPM. Consistent participation in several “trilaterals” will provide confidence in the primary standards involved and may also help to resolve difficulties in the interpretation of the degrees of equivalence assigned to these standards.

References

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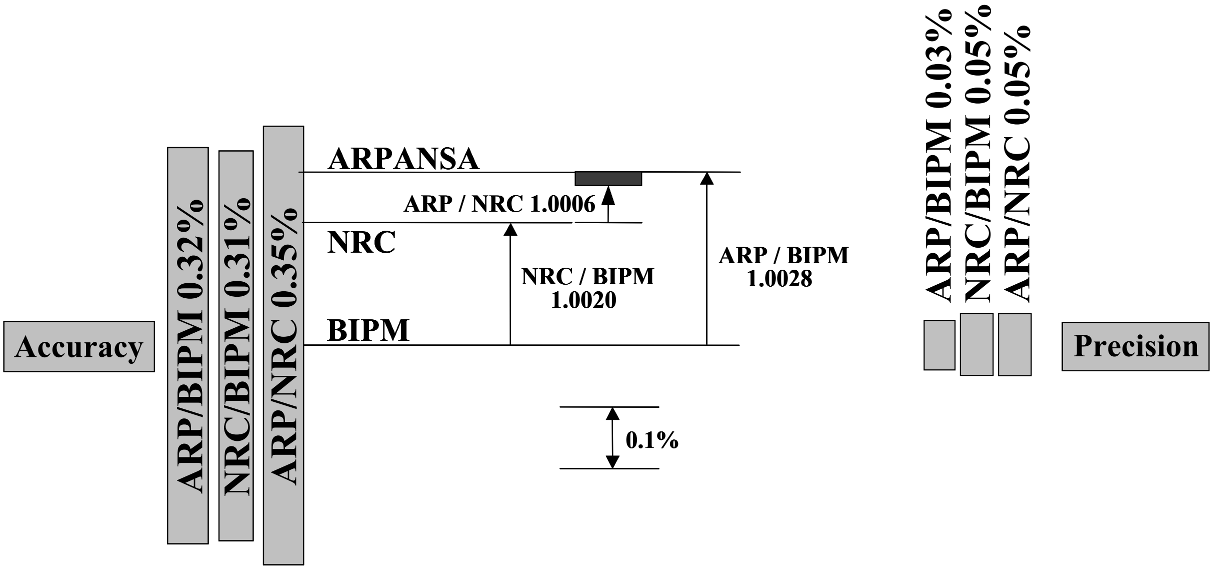

Figure 1. ARP-BIPM-NRC trilateral results for N_K

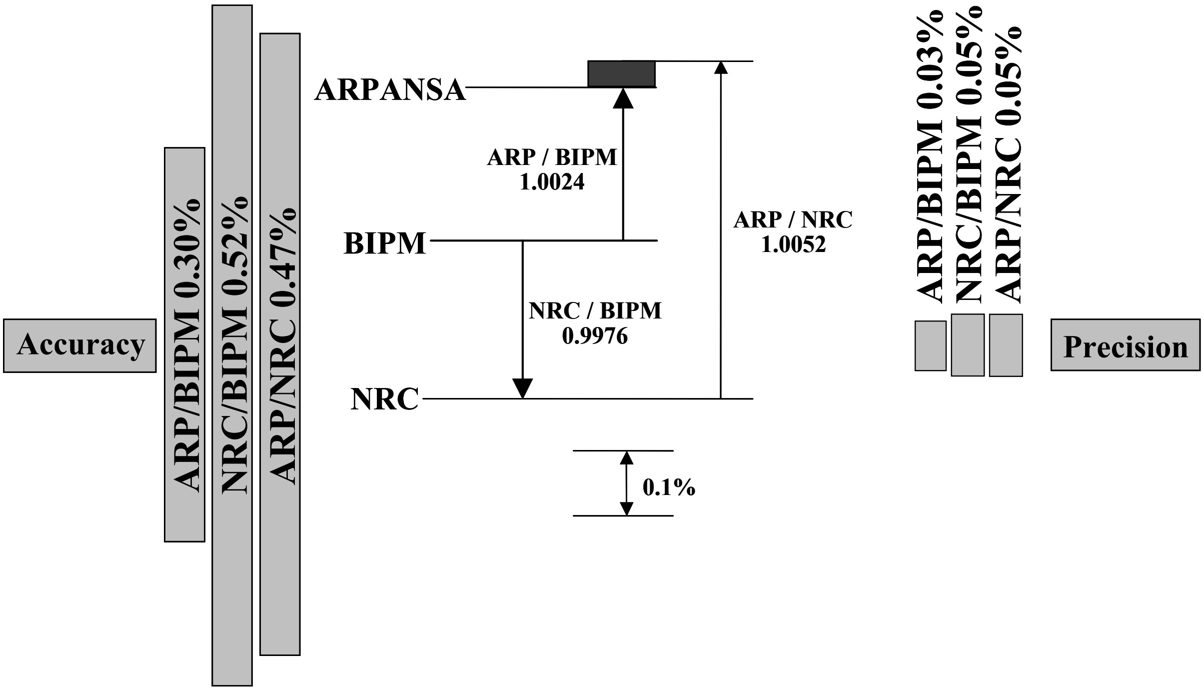

Figure 2. ARP-BIPM-NRC trilateral results for N_{Dw}