Report on the Calculation of the CCM Consensus Value for the Kilogram 2020

9 December 2020

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Abstract

1 Introduction

In 2017 the CODATA Task Group on Fundamental Constants reviewed the available experimental data which served as input for the least squares adjustment of the numerical values of the defining constants for the new definitions of the kilogram, the ampere, the kelvin and the mole. The set of eight results for the Planck constant was not statistically consistent with differences between determinations as large as four standard uncertainties. The Task Group followed its usual practice of multiplying the standard uncertainties of the input data for the Planck constant by an expansion factor, which in this case was 1.7 [1]. This approach allowed the determination of the most likely value of the Planck constant and its uncertainty, but did not solve the problem that kilogram realizations from the different realization experiments were not in agreement within the combined uncertainties.

The Consultative Committee for Mass and Related Quantities (CCM) requested therefore in its meeting in 2017 that NMIs with a realization of the kilogram should avail themselves of an agreed consensus value, which would be determined from the reference values of comparisons, when disseminating the mass unit [2]. This international coordination of the dissemination of the kilogram should be continued until the dispersion in values becomes compatible with the individual realization uncertainties.

The details of this approach are described in the "CCM detailed note on the dissemination process after the redefinition of the kilogram" [3]. The initial consensus value will be determined based on an arithmetic mean of three sets of data:

1. data directly traceable to the International Prototype of the Kilogram (IPK) used in 2014 (taking into account the additional uncertainty of 10 µg with respect to the new definition based on the Planck constant and a contribution for the temporal stability of the BIPM working standards);
2. extant data from the 2016 CCM Pilot Study of realization experiments (corrected for the shift of 17 parts in 10^9 in h introduced by the CODATA 2017 adjustment) and considering the temporal stability of the BIPM working standards;
3. the KCRV of the first CCM Key Comparison CCM.M-K8.2019 (after removal of outliers).

It was decided that the uncertainty of the consensus value should be 20 µg, unless a statistical analysis showed that this value should be increased. This uncertainty corresponds to the typical uncertainty of a "mature" realization experiment and sets the expectation on future uncertainties from individual realization experiments.

The BIPM Pt-Ir working standards were involved in all three campaigns. Therefore the three data sets can be linked together based on the assumption that the BIPM as-maintained mass unit has been stable, within some uncertainty. Contrary to its name, the consensus value is not an absolute value, but will be expressed in terms of an offset from the BIPM as-maintained mass unit, which represents the mass of the IPK. Since all NMIs have mass standards traceable to the BIPM, this consensus value will be accessible to all of them, without the need of further exchanges of mass standards.

2 The long-term stability of the BIPM working standards

For the combination of the results of the three data sets which contribute to the consensus value, it is necessary to have a common, stable reference which allows to link them. This reference is provided by the BIPM working standards which were used during all three campaigns. It is not necessary that the absolute mass of the working standards is stable. What needs to remain stable is the mass unit maintained by the standards. If the absolute mass of the standards changes, this change needs to be detected and the mass values attributed to the standards corrected accordingly. Since the stability of the BIPM as-maintained mass unit is essential for the calculation of the consensus value, we describe the approach used by the BIPM in some detail.

Following the Extraordinary Calibrations using the IPK in 2014, the BIPM has put in place a new, hierarchical scheme of usage of its 12 Pt-Ir working standards. Three of them form the set of “standards for exceptional use”. They are only used once every five years and they are cleaned and washed before use, to re-establish the mass they had during the Extraordinary Calibration campaign. They have been used for the first time since the Extraordinary Calibrations in 2019. Three other standards form the set of “standards for limited use”. They are used once a year and they are recalibrated every five years using the standards for exceptional use. Finally, six standards form the set of “standards for current use”. They are calibrated once a year using the standards for limited use and are used throughout the year to provide calibrations to NMIs. The different levels of usage allow to detect mass changes which are caused by the manipulations. From 2014 to 2018 this scheme did not allow to detect a potential common contamination rate from the surrounding air because none of the standards for current and limited use were cleaned and washed. This was taken into account by increasing the related uncertainty contribution every year. The recalibrations in 2019 against the cleaned and washed standards for exceptional use showed that the working standards for limited use and for current use had accumulated about 9 µg of contamination (with an uncertainty of about 4 µg). In the future we will take this effect into account by increasing the mass values of the standards for limited use every year by 1.8 µg.

For combining the results of the Extraordinary Calibrations of 2014 and the Pilot Study of 2016 with that of CCM.M-K8.2019, we need to determine the uncertainty related to the stability of the BIPM as-maintained mass unit between 2014 and 2020 and between 2016 and 2020. The uncertainty for both periods is estimated as 5 µg, based on an analysis of the observed mass changes of the working standards in the application of the hierarchical scheme of weighings described above.

3 Data contributing to the consensus value of 2020

3.1 Data directly traceable to the International Prototype of the Kilogram

In 2014 the BIPM working standards were all calibrated against the IPK [4]. The BIPM working standards are steered as described in section 2 in such a way that the as-maintained BIPM mass unit does not change. Therefore, the as-maintained mass unit still represents the mass of the IPK. The related uncertainty has two components:

Calibration of the BIPM working standards against the IPK in 2014:	3 µg
Stability of the BIPM as-maintained mass unit from 2014 to 2020:	5 µg

The combined standard uncertainty is 6 µg.

Since 2019, the kilogram is defined based on the numerical value of the Planck constant. Its value has been determined using the previous definition of the kilogram, the mass of the IPK. Therefore the value of the Planck constant was consistent with the mass of the IPK *m*(IPK) = 1 kg , with an uncertainty of 10 µg [5], at the time of the redefinition. Since the redefinition, which fixed the value of the Planck constant, the uncertainty of 10 µg now applies to the mass of the IPK. By adding in quadrature both uncertainty components, one obtains 11.7 µg for the particular representation of the new definition through the IPK, maintained on the BIPM working standards.

3.2 Extant data from the 2016 CCM Pilot Study of realization experiments

The "CCM Pilot Study of future realizations of the kilogram" [5] compared the realizations of the Kibble balances from LNE, NIST and NRC and the application of the XRCD method by NMIJ and PTB. The comparison consisted of two parts: one set of standards was calibrated under vacuum, another set in air. The results for both sets of standards were very similar. For the present analysis we consider only the results of the weighings in vacuum.

At the time of the Pilot Study, the latest CODATA value of the Planck constant dated from 2014:

$$h_{2014} = 6.626\ 070\ 040\ \times 10^{-34}\ \mathrm{J\ s}$$

For CCM.M-K8.2019 the 2017 CODATA value, which served for the redefinition of the kilogram, was used:

$$h_{2017} = 6.626\ 070\ 15\ \times 10^{-34}\ \mathrm{J\ s}$$

The relative difference between both values is (h_{2017}/h_{2014} – 1) = 16.6 x 10^{-9}. To bring the results of both comparisons to a common basis, the NMIs' results in the Pilot Study, expressed in terms of mass, have to be corrected by +16.6 µg.

The recalibration of the BIPM working standards in 2019 with respect to the working standards of exceptional use led to the conclusion of an annual contamination rate of about 1.8 µg. Since the working standards had been calibrated using the IPK in 2014, the results of the Pilot Study in 2016 were corrected for a mass increase of the BIPM working standards of 3.6 µg, with an uncertainty of 2 µg, taken as 50 % of the correction.

Because of this mass increase of the standards, the results of the participants with respect to the BIPM as-maintained mass unit in the Pilot Study were increased by only (16.6 – 3.6) µg = 13 µg.

The differences between mass values attributed by the participants to a 1 kg mass standard and the values attributed using the BIPM working standards, corrected as explained above, are shown in table 1.

The difference between mass determinations for a 1 kg standard based on the reference value (the weighted mean of the five realizations) and on the BIPM as-maintained mass unit of 2016 is 12.4 µg with a standard uncertainty of 10.2 µg. Combined with the uncertainty of 5 µg for the stability of the as-maintained mass unit from 2016 to 2020, the uncertainty of the reference value, as maintained in 2020, is 11.4 µg.

Table 1: Differences between mass values attributed by the participants of the CCM Pilot Study in 2016 to a 1 kg mass standard and the values attributed using the BIPM working standards, and related standard uncertainty (Table 9 of [5]), corrected for the change from h_{2014} to h_{2017} and for the adjustment of the BIPM as-maintained mass unit of 3.6 µg in 2016, as described in section 3.2.

Institute	$\boldsymbol{m_i^{\mathrm{NMI}} - m_i^{\mathrm{BIPM}}}$ **/ mg**	$\boldsymbol{u(\Delta m_i)}$ **/ mg**
LNE	-0.1913	0.1400
NIST	0.0420	0.0293
NMIJ	0.0113	0.0241
NRC	0.0109	0.0158
PTB	0.0064	0.0195
Reference value (Weighted mean)	0.0124	0.0102

3.3 The KCRV of the first CCM Key Comparison, CCM.M-K8.2019

This comparison compared the realizations of five Kibble balances from BIPM, KRISS, NIM, NIST and NRC and two applications of the XRCD method by NMIJ and PTB. The protocol of this comparison required to determine the mass of the travelling standards in vacuum.

The differences between mass values attributed by the participants of CCM.M-K8.2019 to a 1 kg mass standard and the values attributed using the BIPM working standards are shown in table 2.

The difference between mass determinations for a 1 kg standard based on the key comparison reference value (weighted mean of the seven realizations) and on the BIPM as-maintained mass unit is -18.8 µg with a standard uncertainty of 7.5 µg.

4 Calculation of the consensus value 2020

Figure 1 shows the results of the Extraordinary Calibrations, the 2016 Pilot Study (table 1) and CCM.M-K8.2019 (table 2). The markers show the differences between mass values attributed by the participants of the comparisons to a 1 kg mass standard and the values attributed using the BIPM working standards.

Table 2: The differences between mass values attributed by the participants of CCM.M-K8.2019 to a 1 kg mass standard and the values attributed using the BIPM working standards, and related standard uncertainty (Table 7 of [6]).

Institute	$\boldsymbol{m_i^{\mathrm{NMI}} - m_i^{\mathrm{BIPM}}}$ **/ mg**	$\boldsymbol{u(\Delta m_i)}$ **/ mg**
BIPM	0.0064	0.0491
KRISS	0.0536	0.1072
NIM	-0.0305	0.0456
NIST	-0.0185	0.0270
NMIJ	-0.0166	0.0214
NRC	-0.0034	0.0118
PTB	-0.0399	0.0128
KCRV (Weighted mean)	-0.0188	0.0075

The most significant feature of this graph is that the KCRV of CCM.M-K8.2019 lies 31 µg below the reference value of 2016. The principal reason for this is that the PTB and the NIST results in 2020 are significantly lower as in 2016 (figure 2). Compared to the BIPM as-maintained mass unit, the PTB result has changed by -46 µg. Compared to the NRC result which has been stable within the uncertainties, it has changed by -32 µg. This is related to the use of a new single crystal from isotopically enriched silicon as well as a new value for the lattice parameter and a new method for the determination of the surface layers which led in 2017 to a change in the experimental value of the Avogadro constant of +3.9 x 10^{-8}. If the XRCD method is used to determine a mass, this change translates into a change of -39 µg for a 1 kg mass. This is consistent with the change of the PTB result from the 2016 to the 2019 comparison (-46 µg). The situation is similar for the NIST result: with respect to the BIPM mass unit it has changed by -60 µg. In the Pilot Study the NIST result was significantly higher than the other results, so it could have been expected that it would be lower in future comparisons. However, the reason for this change is unknown. The realizations of NMIJ and the NRC lie also lower than before, but still agree within the uncertainties. In conclusion, the results of the four institutes which participated in both comparisons, are lower in 2019 than in 2016, as compared to the BIPM as-maintained mass unit. The changes range from -14 µg to -60 µg.

The three contributions to the consensus value of 2020: IPK in 2014, reference value of the 2016 Pilot Study and KCRV of CCM.M-K8.2019 are shown in table 3 and on fig. 3. The arithmetic mean of the three results is -2.1 µg with a standard uncertainty of 6.0 µg. The weighted mean of the three results is -7.3 µg with a standard uncertainty of 5.5 µg. These small numbers mask the problem that between 2016 and 2019 the weighted mean of the realization experiments has changed by -31 µg with respect to the BIPM as-maintained mass unit. The principal reasons for this are the changes of the mass unit realized by the PTB and the NIST.

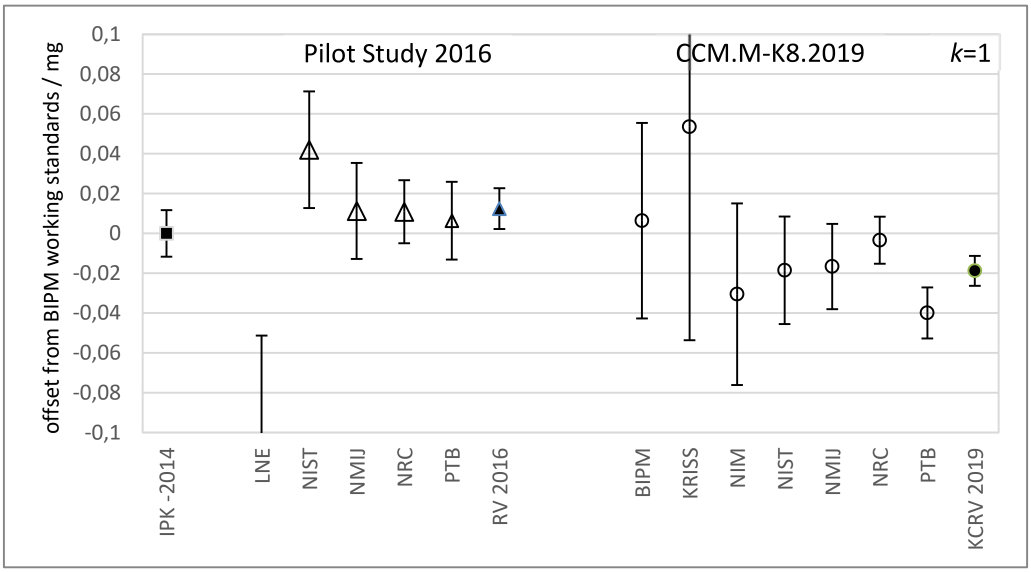

Fig 1.: Differences between kilogram realizations and the as-maintained BIPM mass unit.

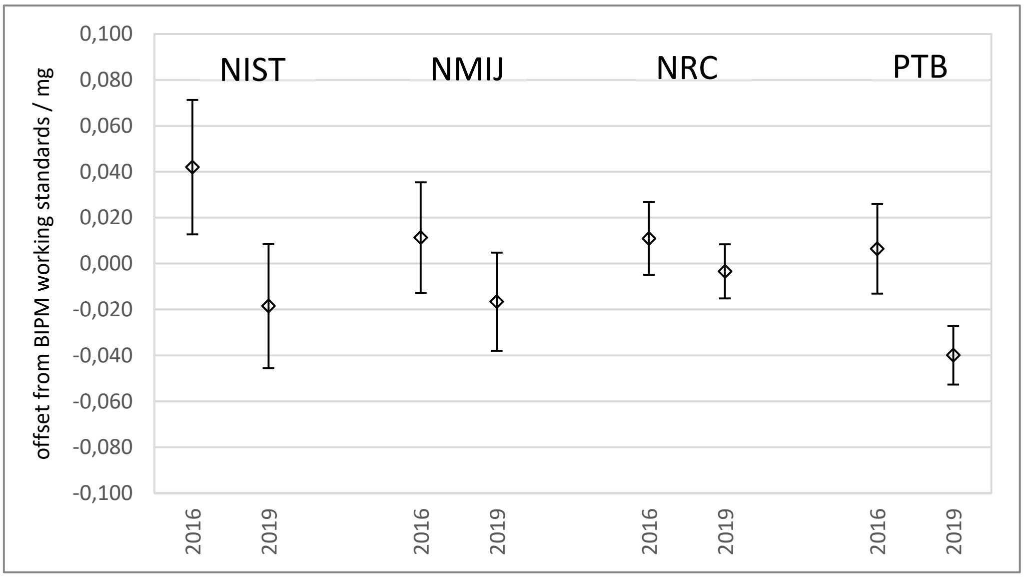

Fig 2.: Comparison of the results of NIST, NMIJ, NRC and PTB in the 2016 Pilot Study and the 2019 Key Comparison. The changes vary from -14 µg (NRC) to -60 µg (NIST).

The standard chi-squared test [7] of the data of table 3 with the 95 % cutoff criterion is just passed. A more stringent criterion is to require that the observed chi-squared lies within the expectation value of the chi-squared distribution plus its standard deviation [7]. For ν degrees of freedom this value is $\nu + \sqrt{2\,\nu}$, for 2 degrees of freedom one obtains 4.0. This test is not passed, since the observed chi-squared is 5.7.

Table 3: Values and uncertainties of the three contributions to the determination of the consensus value of 2020.

Contribution to consensus value 2020	deviation from BIPM as-maintained mass unit	unc. / µg
IPK 2014	0.0	11.7
RV Pilot Study 2016	12.4	11.4
KCRV CCM.M-K8.2019	-18.8	7.5

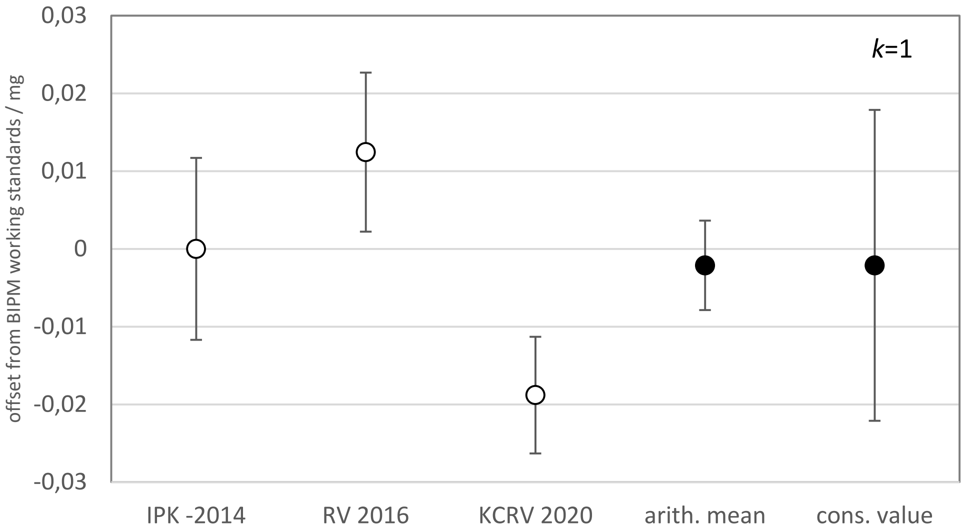

Fig 3.: The three values contributing to the first consensus value: traceability to the IPK using in 2014, reference value of the Pilot Study in 2016 and KCRV of CCM.M-K8.2019.

As decided by the CCM task group on the phases of the dissemination of the kilogram, the consensus value will be the arithmetic mean of the three contributions, -2 µg, with an uncertainty of 20 µg.

This means that mass values of 1 kg standards based on the consensus value will be 2 µg lower than those based on the BIPM as-maintained mass unit.

To achieve consistency with the consensus value of 2020, all NMIs would need to reduce the mass value of their national prototype by 2 µg. Since the uncertainty of the consensus value is 20 µg, in practice no correction of the mass of the national prototype is necessary. Only the uncertainty needs to be increased.

5 References

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[4] M. Stock *et al*., “Calibration campaign against the international prototype of the kilogram in anticipation of the redefinition of the kilogram part I: comparison of the international prototype with its official copies”, *Metrologia* **52** (2015) 310-316

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6 List of Acronyms

BIPM - Bureau International des Poids et Mesures (International Bureau of Weights and Measures)

CCM - Consultative Committee for Mass and Related Quantities

CODATA - Committee on Data for Science and Technology

IPK - International Prototype of the Kilogram

KC - Key Comparison

KCRV - Key Comparison Reference Value

KRISS - Korea Research Institute of Standards and Science (NMI of the Republic of Korea)

NIM - National Institute of Metrology (NMI of China)

NIST - National Institute of Standards and Technology (NMI of the United States of America)

NMI - National Measurement Institute

NMIJ - National Metrology Institute of Japan (NMI of Japan)

NRC - National Research Council Canada (NMI of Canada)

PTB - Physikalisch-Technische Bundesanstalt (NMI of Germany)

XRCD - X-ray Crystal Density