

# On Secondary Representations of the Second

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The emergence of microwave and optical frequency standards with accuracies close to those of caesium primary frequency standards has led to the concept of secondary representations of the second. The recently-formed Joint Working Group of the CCL / CCTF has now recommended as secondary representations to the CCTF / CIPM one microwave radiation in  $^{87}\text{Rb}$  and three optical radiations in  $^{199}\text{Hg}^+$ ,  $^{88}\text{Sr}^+$  and  $^{171}\text{Yb}^+$ . This paper describes the rationale, evolution and development of criteria for acceptance of a radiation as a secondary representation, together with a review of the JWG assessment and recommendation of the four radiations already proposed.

## I. INTRODUCTION

The year 2005 marked the 50th anniversary of the atomic clock, originally introduced by Louis Essen. Some twelve years after this introduction, the Conference Generale des Poids et Mesures (CGPM) adopted as a new definition of the SI unit of time, the second, in terms of the transition between hyperfine levels in the ground state of caesium. This definition has proved to be a durable one, with subsequent development of commercial caesium beam clocks for a wide variety of time and frequency applications such as satellite navigation. In addition the primary caesium frequency standards themselves have evolved considerably from Essen's original device. Contemporary primary standards now apply time-separated 9.2 GHz microwave Ramsey interrogation pulses to probe cold Cs atoms launched upwards in a 1-m fountain, and as they fall back under gravity. These cold atom fountain primary standards have demonstrated fractional uncertainties better than 1 part in  $10^{15}$  over a day's averaging, with corresponding timing accuracy of  $\sim 100$  picoseconds per day. There are to date, six Cs fountain primary standards operational at national metrology institutes (SYRTE, PTB, NIST, IEN, NPL and NMIJ) and which make a significant contribution to steering TAI. Additionally, with the emergence of high accuracy optical frequency standards based on cold atoms and cold trapped ions, the fountains currently provide the ultimate reference for highest accuracy femtosecond comb measurements of optical frequencies.

However, the international metrology community has long debated the potential of optical frequency standards to out-perform microwave standards in the long run. The reason for this is the much higher frequency of the optical

standard, with a typical value approaching  $10^{15}$  Hz, is some  $10^5$  higher than the Cs frequency. With achievable linewidths in the 1 Hz range, whether dictated by interrogation time or natural decay time, this gives rise to significantly improved signal to noise, and hence reduced instability ( $\sigma$ ) according to the equation:

$$\sigma(\tau) = \frac{1}{2\pi\nu\sqrt{NT_{int}}\tau}$$

where  $\nu$  is the clock transition frequency,  $N$  is the number of atoms or ions,  $T_{int}$  is the Ramsey interrogation time and  $\tau$  the sampling time. With such improved stability in the optical relative to the microwave standards, this allows quicker and more rigorous assessment of the atomic optical clock's reproducibility with respect to environmental parameters such as electric and magnetic field variations, and hence potentially more accurate sub-division of the second.

## II. IMPROVEMENTS IN OPTICAL FREQUENCY STANDARDS

The recent improvement in optical frequency standards can be contrasted against that for the primary microwave caesium clocks in figure 1. Over the past several years, optical standards based on cold atoms and cold trapped ions have benefited significantly from the advent of the self-referenced femtosecond comb [1, 2], with result that the rate of improvement for the optical standards is now such that comb measurement of optical frequencies will very soon be limited by the Cs microwave standard itself. It is anticipated that optical frequency uncertainties will likely demonstrate reproducibilities below that of caesium, raising the prospect of an optical redefinition of the second in the future.

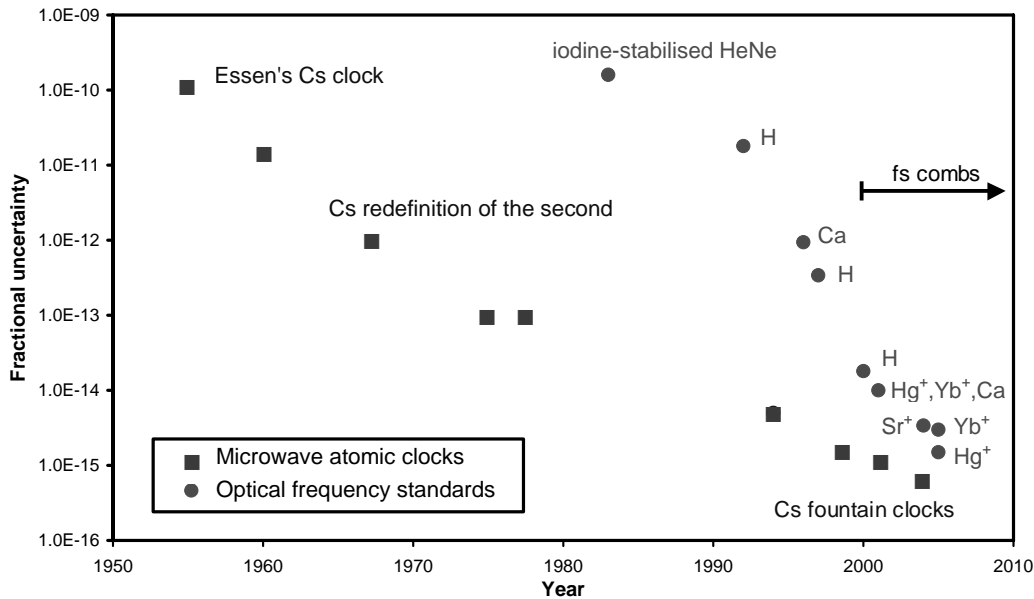


Figure 1: Improvements in atomic frequency standards

However, once the optical uncertainty becomes Cs-limited, the metrology community is faced with a position where optical standards are unable to demonstrate lower uncertainty until such time as a redefinition might occur, even though they may be shown to be more reproducible by eg comparing two optical systems directly. This projected difficulty led to an initial proposal from NIST to the 2001 CCTF to consider the establishment of secondary representations of the second [3], where such representations, whether optical or microwave, could be used to realise the second, provided that their accuracy was close to the Cs uncertainty. Obviously, their uncertainty could be no better than the Cs primary uncertainty. However, it was considered that the establishment of these representations would help with the detailed evaluation of reproducibility at the highest level, and significantly aid the process of comparing different standards in the preparation of a future redefinition.

### III. SECONDARY REPRESENTATION CANDIDATES

The perceived range of alternatives as Secondary Representations included microwave standards such as Rb,  $^{171}\text{Yb}^+$  and  $^{199}\text{Hg}^+$  microwave transitions, and a variety of single cold trapped ion clock transitions in  $^{88}\text{Sr}^+$ ,  $^{199}\text{Hg}^+$ ,  $^{171}\text{Yb}^+$ ,  $\text{Ca}^+$  and cold atom transitions in Sr, Yb, Ca and Hg. All these systems were potentially capable of demonstrating reproducibilities better than Cs, but of course, as Secondary Representations, would remain limited in uncertainty by Cs.

Against this background, a Joint Working group (JWG) of the CCL/CCTF was set up in Sept. 2003, reporting to the Presidents of the CCTF (Prof Leschiutta) and CCL (Dr Chung). The JWG comprised members from BNM-SYRTE, IEN, NMIJ, NIST, NPL, NRC, PTB and VNIIFTRI under the chairmanship of F Riehle and P Gill. The preliminary terms of reference of the JWG were to

- Ask for proposals from the NMIs for microwave and optical frequency standards to adopt as secondary representations.
- Meet to review the associated uncertainty budgets for these proposals.
- Evaluate the validity of the proposal and make recommendation to the CCTF for the addition of particular standards to the list of secondary representations.

### IV. SECONDARY REPRESENTATION / MISE EN PRATIQUE RELATIONSHIP

The Mise en Pratique Working Group of the CCL (MePWG) already maintains a recommended radiation list for the realization of the SI metre and other frequency metrology applications [4]. There are some 22 radiations listed, including cold atom and trapped ion optical standards, gas-cell stabilized lasers and even discharge lamps, and the accuracies associated with this diverse list span several orders of magnitudes. This raised a number of issues, including:

- Relationship between SR and MeP lists?
- Common values and uncertainties?
- Whether / how to integrate the two lists?

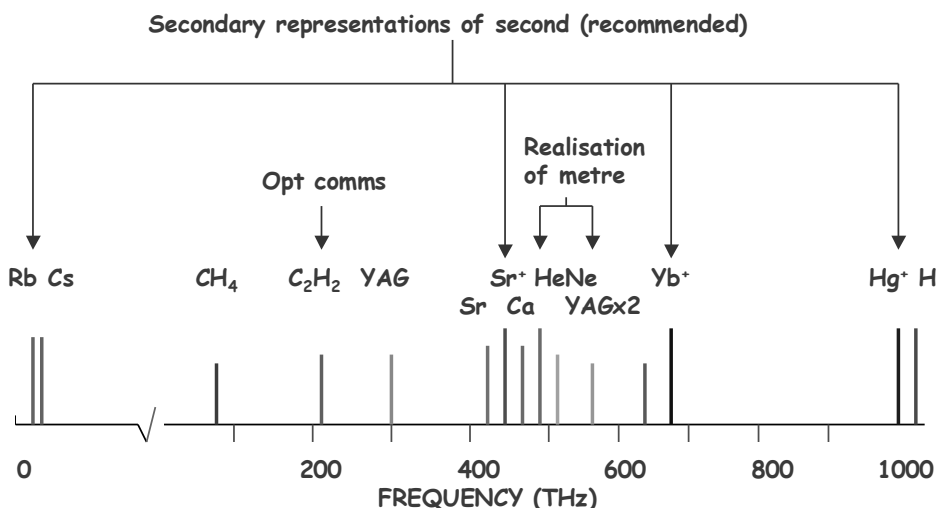


Figure 2: Example of a unified list of recommended radiations

As a result, the CIPM 2003 adopted a single list approach which should be periodically updated, where certain radiations could be

- approved by the CCTF as secondary representations of the second and others
- accepted by the CCL as suitable for realization of the metre.

The consequences of this approach were that there would be no ambiguity in recommended frequency values, and that it strengthened the metre definition linking time and length measurement. However, it was recognized that the particular needs of the length and time communities were orthogonal in the sense that iodine-stabilised lasers, for example, were not well suited to time keeping requirements, and, conversely, microwave clocks were not at present so practical for length metrology.

#### V. CRITERIA FOR ADOPTION AS A SECONDARY REPRESENTATION

On the basis that a future CCTF will look to the list of secondary representations for a new definition of the second, the following criteria for adoption of a proposed radiation were derived:

1. "The SI value of the unperturbed frequency of a quantum transition suitable as a secondary representation of the second must have an uncertainty that is *evaluated* and *documented* so as to meet the requirements adopted for the primary frequency standard for use in TAI"
2. "This uncertainty should be no larger than about a factor of 10 of the primary standards *of that date* that serve as the best realizations of the second"

The implications of these criteria are such that only frequency standards with currently attributed accuracies of around 1 part in  $10^{14}$  or better, should be considered. Thus it can be seen that these requirements are significantly more stringent than those relevant to incorporation into the MeP.

#### VI. RECOMENDED Rb MICROWAVE STANDARD

The 6.8 GHz cold Rb microwave fountain standard was reported to the JWG in 2004 by SYRTE, with a quoted uncertainty of 1.2 parts in  $10^{15}$ , and subsequently confirmed in 2005 [5], with an uncertainty of  $1.6 \times 10^{-15}$ . As a result, the JWG recommended it as a secondary representation with a frequency value of

$$\nu(\text{Rb}) = 6\,834\,682\,610.904\,324 \text{ Hz}$$

with a relative standard uncertainty of  $3 \times 10^{-15}$ . This was accepted by the CCTF and is awaiting ratification by the CIPM.

#### VII. PROPOSED OPTICAL RADIATION CANDIDATES

A number of possible optical frequency standard candidates were considered at the JWG meeting in September 2005. These are listed in Table 1. As can be seen from the Table, only three radiations were considered by the JWG to be appropriate for recommendation to the CCTF as secondary representations at this time. These were the  $^2S_{1/2} F=0 - ^2D_{5/2} F=2$  quadrupole transition in  $^{199}\text{Hg}^+$  at 282 nm, the  $^2S_{1/2} - ^2D_{5/2}$  quadrupole transition in  $^{88}\text{Sr}^+$  at 674 nm and the  $^2S_{1/2} F=0 - ^2D_{3/2} F=2$  quadrupole transition in  $^{171}\text{Yb}^+$  at 436 nm.

System	Studied at	Quoted uncertainty	MeP uncertainty	Conclusion
$^{199}\text{Hg}^+$	NIST	0.94 Hz	3 Hz (3 in $10^{15}$ )	Recommended
$^{88}\text{Sr}^+$	NPL, NRC	1.5 Hz / 15 Hz Difference 0.6 Hz	3 Hz (7 in $10^{15}$ )	Recommended
$^{171}\text{Yb}^+$	PTB	2 Hz	6 Hz (9 in $10^{15}$ )	Recommended
$^{40}\text{Ca}$	PTB, NIST	3.4 Hz / 5.3 Hz Difference 8 Hz	8 Hz (1.6 in $10^{14}$ )	Consider at next JWG meeting
$^{87}\text{Sr}$	Tokyo, JILA	15 Hz / 20 Hz Difference 85 Hz	85 Hz (1.7 in $10^{13}$ )	Consider at next JWG meeting
$^{27}\text{Al}^+$	NIST	1.1 Hz	Not considered due to lack of published data	Presentation only

Table 1. Radiations considered at Sept 2005 JWG meeting

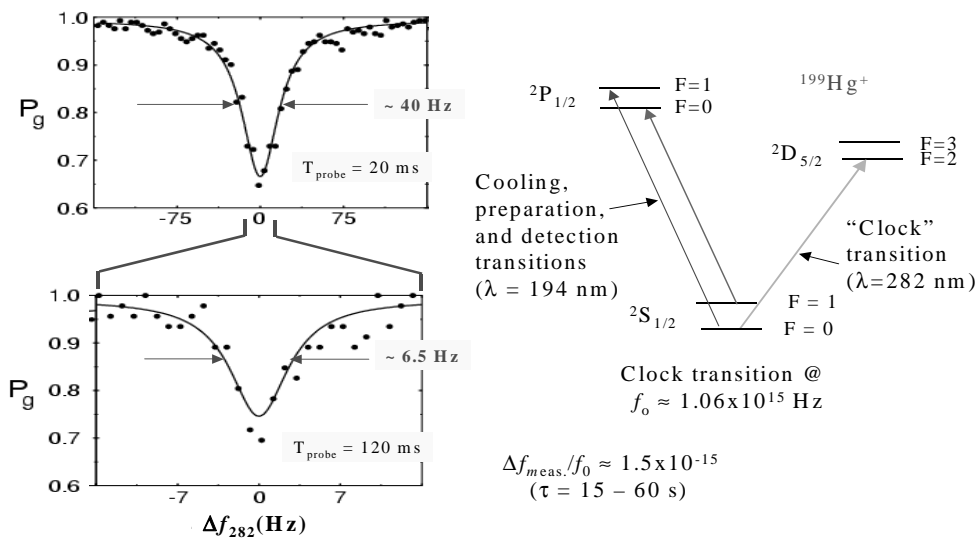


Figure 3:  $^{199}\text{Hg}^+$  282 nm quadrupole clock transition

### VIII. $^{199}\text{Hg}^+$ 1.06 PHz CLOCK TRANSITION

The  $^{199}\text{Hg}^+$  quadrupole transition at 282 nm (1.06 PHz) proposed by NIST has a lifetime of  $\sim 90$  ns and corresponding natural linewidth of  $\sim 1.7$  Hz. Figure 3 shows the partial energy level structure for the  $^{199}\text{Hg}^+$  ion, together with the respective cooling and clock transitions. Also shown are the observed clock transition transform-limited linewidths for 20 ms and 120 ms interrogation pulses.

The most recent absolute frequency measurement of this  $^{199}\text{Hg}^+$  282 nm quadrupole transition [6] gives the value

$$1\ 064\ 71\ 609\ 899\ 144.98\ (0.94)\ \text{Hz}$$

which is in excellent agreement with the weighted mean of all measurements since 2001, which has the value

$$1\ 064\ 721\ 609\ 899\ 145.05\ (0.74)\ \text{Hz}$$

The stability of the transition at the  $10^{-16}$  level for  $\sim 100$  second averaging times has been verified comparison with the 267 nm  $^{27}\text{Al}^+$  clock transition by means of femtosecond comb comparison. Given the fact that this transition was only under study at NIST, the JWG accepted the MePWG recommendation that for its consideration as a secondary representation, the uncertainty should be increased to 3 Hz. As a result, the JWG recommended the  $^{199}\text{Hg}^+$  282 nm quadrupole clock transition to the CCTF as a secondary representation with a value

$$\nu(^{199}\text{Hg}^+) = 1\ 064\ 721\ 609\ 899\ 145\ (3)\ \text{Hz}$$

which applies to the unperturbed quadrupole transition

## IX. $^{88}\text{Sr}^+$ 445 THz CLOCK TRANSITION

The  $^{88}\text{Sr}^+$  quadrupole transition at 674 nm (445 THz) was proposed as a secondary representation by both NPL and NRC. It has a lifetime of  $\sim 0.4$  s and corresponding natural linewidth of  $\sim 0.4$  Hz. Figure 4 shows the partial energy level structure for the  $^{88}\text{Sr}^+$  ion, together with the respective cooling, repumping and clock transitions.

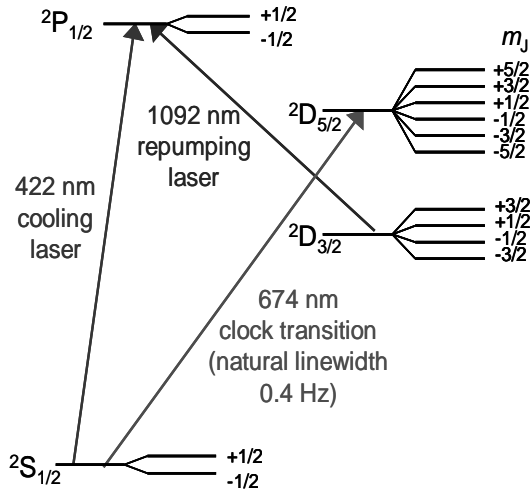


Figure 4:  $^{88}\text{Sr}^+$  674 nm quadrupole clock transition

The most recent absolute frequency measurement [7] at NPL gave the value

$$444\,779\,044\,095\,484.6 \text{ (1.5) Hz}$$

in comparison with the measurement value from NRC [8]

$$444\,779\,095\,484 \text{ (15) Hz.}$$

There is excellent agreement between the measurements made on independent systems at the two laboratories, but recognising the order of magnitude difference in stated uncertainties between the laboratories, the JWG accepted the MePWG recommendation to increase the uncertainty by a factor of 2. As a result, the JWG recommended the  $^{88}\text{Sr}^+$  674 nm quadrupole clock transition to the CCTF as a secondary representation with a value

$$\nu(^{88}\text{Sr}^+) = 444\,779\,044\,095\,484.6 \text{ (3.0) Hz}$$

which applies to the unperturbed quadrupole transition and the centre of the Zeeman multiplet.

## X. $^{171}\text{Yb}^+$ 688 THz CLOCK TRANSITION

The  $^{171}\text{Yb}^+$  quadrupole transition at 436 nm (688 THz) was proposed as a secondary

representation by PTB. It has a lifetime of  $\sim 40$  ms and corresponding natural linewidth of  $\sim 3.1$  Hz. Figure 5 shows the partial energy level structure for the  $^{171}\text{Yb}^+$  ion, together with the respective cooling, re-pumping and clock transitions.

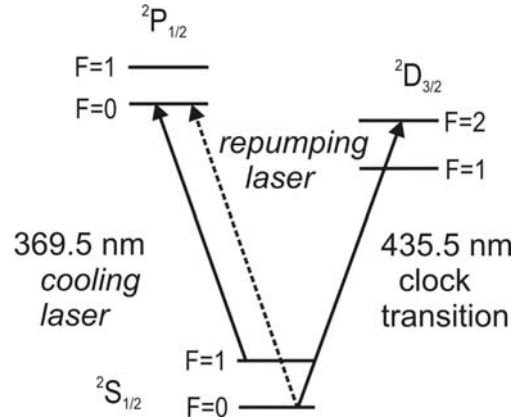


Figure 5:  $^{171}\text{Yb}^+$  436 nm quadrupole clock transition

The most recent absolute frequency measurement at PTB [9] gave the value

$$688\,358\,979\,309\,307.65 \text{ (2.14) Hz.}$$

Combining this value with earlier frequency measurements yielded a mean frequency less than 0.3 Hz different from this value. Two-trap comparisons were carried out immediately prior to the measurement. The results of those comparisons [10] are consistent with a difference of  $3.8 \text{ (6.1)} \times 10^{-16}$  between the two ytterbium standards. However, since the  $^{171}\text{Yb}^+$  quadrupole standard was under investigation at PTB only, the MePWG considered it prudent to increase the uncertainty by a factor of  $\sim 3$  to 6 Hz, and the JWG recommended the  $^{171}\text{Yb}^+$  436 nm quadrupole clock transition to the CCTF as a secondary representation of the second with a value

$$\nu(^{171}\text{Yb}^+) = 688\,358\,979\,309\,308 \text{ (6) Hz}$$

which applies to the unperturbed quadrupole transition.

## XI. POSSIBLE CONTRIBUTION OF SECONDARY REPRESENTATIONS TO TAI

Currently, the CCTF working group on primary frequency standards is considering the standardisation of formats for the reporting of primary frequency measurements to BIPM. This comprises three components:

- Publication of experimental details and full uncertainty budget in peer-reviewed scientific journal.
- A report to BIPM detailing the results obtained in a particular reporting period.
- Results reported as frequency offsets from a clock contributing to TAI, and including uncertainty budget and references.

Data is then published in Circular-T within the section for primary frequency standards. With this protocol in mind, two possible levels of contribution of microwave or optical clock secondary representation data to TAI have been suggested by BIPM as:

1. A report to BIPM of secondary representation clock measurements against clocks already contributing to TAI (eg NPL's  $^{88}\text{Sr}^+$  optical frequency standard against the NPL HM1 hydrogen maser). This data could then be itemised within circular-T as an offset from TAI, for information only.
2. Use of the secondary representation data reported by the NMIs to contribute to the steering of TAI in the same way as for primary frequency standards.

Thus, following CCTF and CIPM acceptance of a particular secondary representation, it is anticipated that the reporting only option itemized in 1 would apply for an initial period, with the introduction of 2 after a suitable period of reporting, as confidence in the reported representation data builds.

## XII. EVOLUTION OF UNIFIED RECOMMENDED RADIATION LIST & CCL/CCTF WORKING GROUP

Mindful of the need for a unified recommended radiation list which caters for

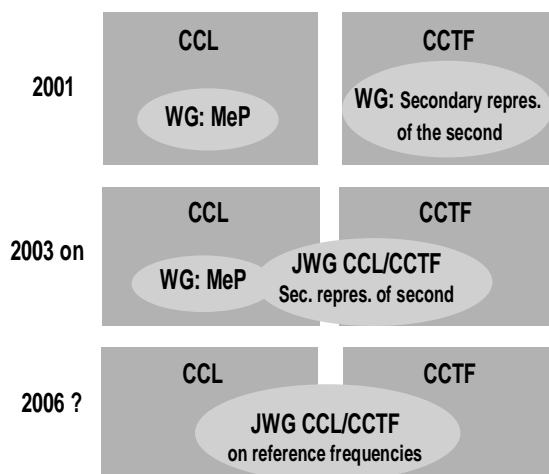


Figure 6: Evolution of respective CC working groups

both requirements of CCTF secondary representations and CCL practical realizations, the 2005 JWG meeting has proposed a revised title for the unified list, and also a consolidation of working groups. This aims to ensure a future co-ordinated approach to the construction and updating of the recommended list, without duplication of effort and potential and potential contradictory decisions regarding frequency values and uncertainties. The name for the recommended radiation list proposed by the JWG to the CCTF is

### “Recommended radiations, including practical realisations of the metre, and secondary representations of the second”

The proposed name for the consolidated working group is

### Joint Working Group of the CCL/CCTF on Reference Frequencies

to replace the MePWG and JWG. The proposed Terms of Reference for the new WG are:

1. To recommend to the CCL, radiations for the practical realisation of the SI metre.
2. To recommend to the CCTF, radiations acceptable as secondary representations of the SI second.
3. To maintain a list of recommended frequency standard values and associated wavelength values for applications including the above.

## XIII. NEXT STEPS AND SUMMARY

These proposals for a new combined list, combined WG and Terms of Reference were accepted by the September 2005 meeting of the CCL, and will be considered by the September 2006 meeting of the CCTF, in parallel with the JWG recommendations for the proposals of the three optical radiations as secondary representations. In addition, the existing JWG will next meet immediately prior to the CCTF 2006 meeting and consider further proposals and updated values for existing proposals. Decisions reached by the CCTF will then form the basis of recommendations to the CIPM 2006.

In the meantime, national measurement laboratories are encouraged to prepare fully documented proposals for secondary representations of the second for consideration by the CCTF, both in 2006 and by future

CCTF meetings. In addition, work is scheduled to start on the preparation of a Metrologia article itemising the full and updated recommended radiations list (rather than an electronic update) for publication after CIPM 2006.

With the approval of these JWG proposals, it is believed this will form an enduring basis for the incorporation of high accuracy optical and microwave clocks into the frequency standards infrastructure of the international metrology community, and establish their capability for the future. It is anticipated that this will provide significant support in the deliberations for a future possible redefinition of the SI second.

### Acknowledgements

The Authors would like to acknowledge the significant contributions of all the members of the CCL/CCTF JWG in the consideration of the requirements of a secondary representation list and the development of strategies to achieve this.

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