Improvements in the algorithms for UTC

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Summary

- Progress on the UTC algorithm: changing of the weighting algorithm (January 2014)

- Analysis and limits of the algorithm for the uncertainty of [*UTC*-*UTC*(*k*)] published on sec. 1 of *Circular T* (with Gerard Petit and Aurelie Harmegnies)

- Future development: Kalman Filter applied to UTC (with Federica Parisi from the University of Turin)

The weighting algorithm

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The weighting algorithm

To improve the reliability and long-term stability of UTC, the maximum number of clocks of different types contributing is required.

With the revision of the weighting algorithm the clocks presenting strong signatures are used in an optimal way.

New Algorithm
Good clock: Predictable Clock
Weight = f(predictability)
Frequency drift considered
All type of clocks used in an optimal way
Wmax=4/N

Weighting procedure

x_i = EAL-H_i
is obtained with the weights ω_i calculated in the previous interval.

The residuals
$\varepsilon_i = y(EAL - h_i) - \hat{y}(EAL - h_i)$
are calculated

Weighted mean of $\varepsilon_i{}^2$
over M months

$$\sigma_i^2 = \frac{\sum_{i=1}^{M}(\varepsilon_i{}^2/(M-i+1))}{\sum_{i=1}^{M}(1/(M-i+1))}$$

The relative weight ω_{iTEMP} of each clock is obtained

$$\omega_{i\,TEMP} = \frac{1/\sigma_i^2}{\sum_{i=1}^{N} 1/\sigma_i^2}$$

1) The weight is bigger than the upper limit : $\omega_i = \omega_{max}$
2) Abnormal behaviour : $\omega_i = 0$

$|y(EAL - hi) - \hat{y}(EAL - hi)| > 5\ ns/day$

4
iterations

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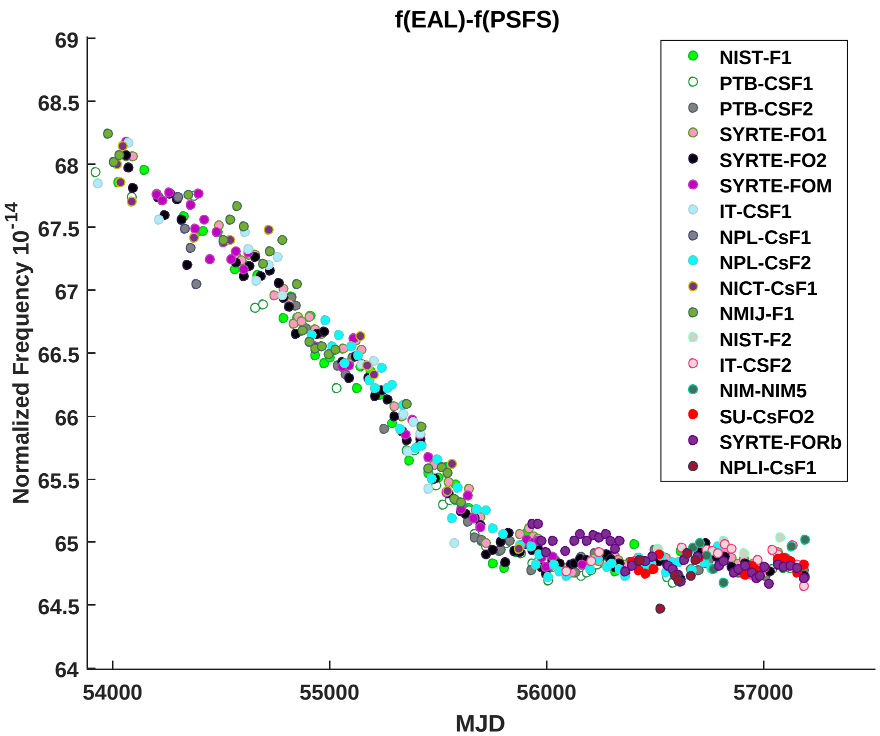
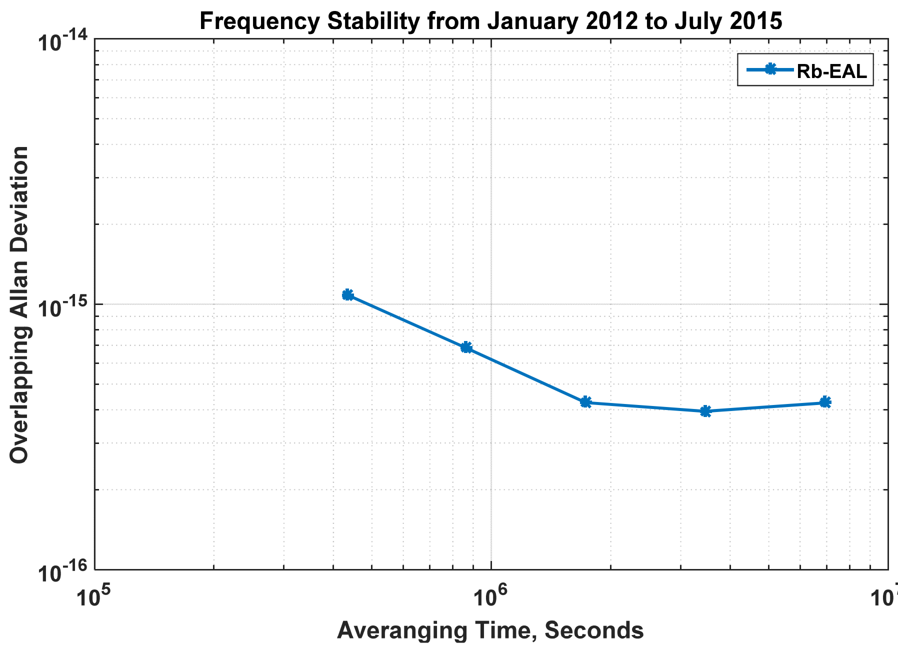
Perfomance of EAL with repect to each PSFS and to the Rubidium Fountains



The data coming from USNO Rubidium fountain (2012-July 2015) are used to evaluate EAL short and medium term stability.



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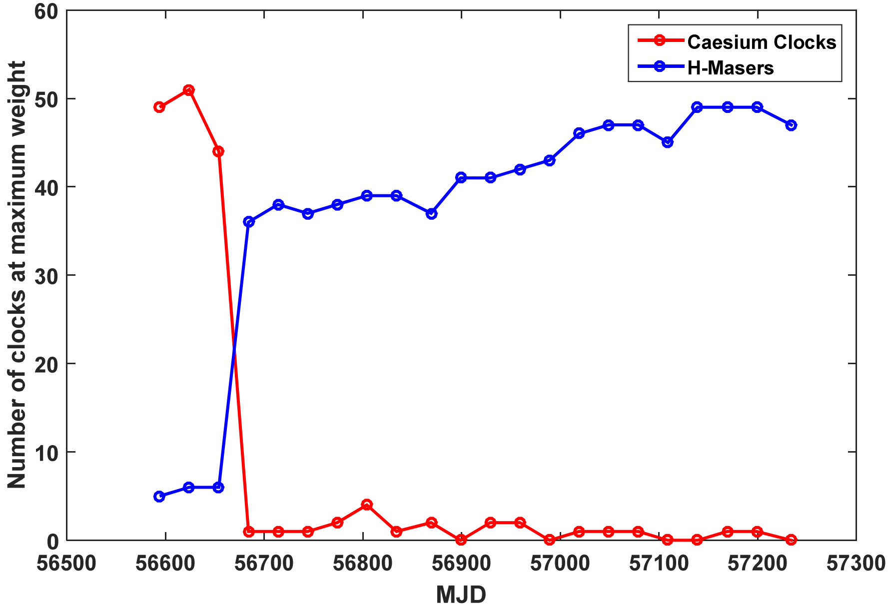
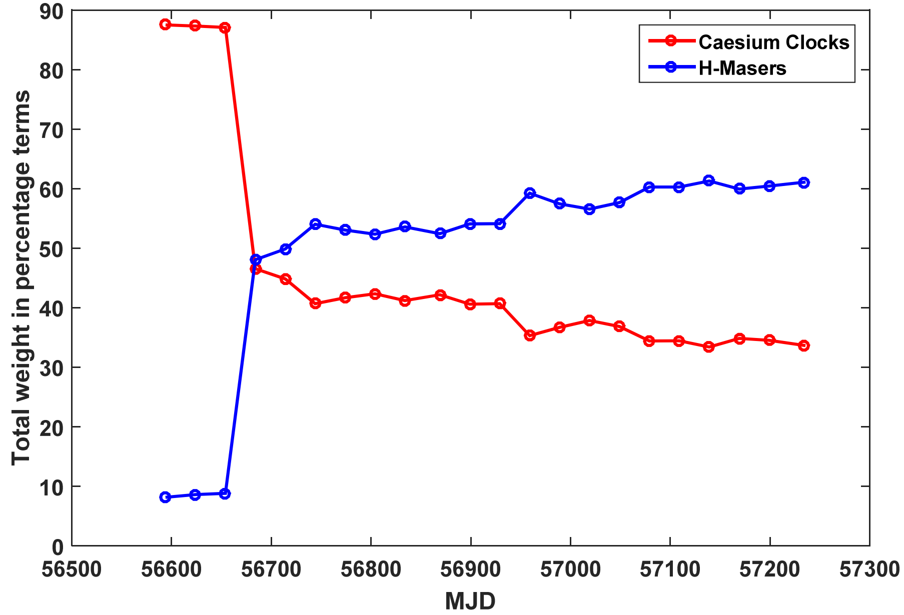
Weight Analysis - 1

By changing the weighting algorithm the role of the caesium clocks and Hydrogen Masers changed completely.





	H-Masers	Caesium
Total weight	~50-60%	~30-40%
Num at w_{max}	~45	~1

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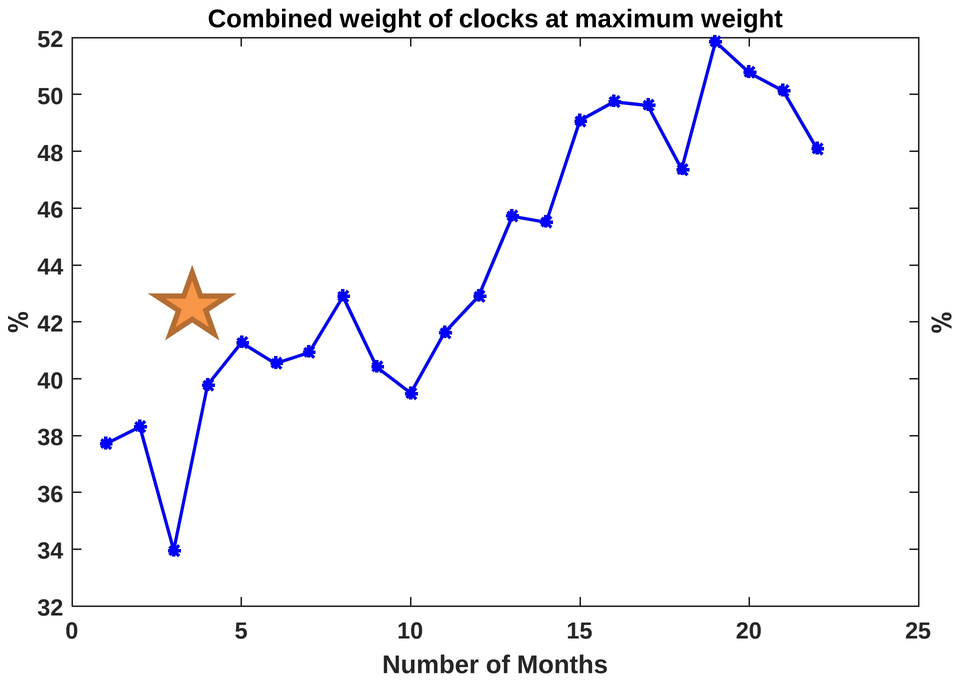
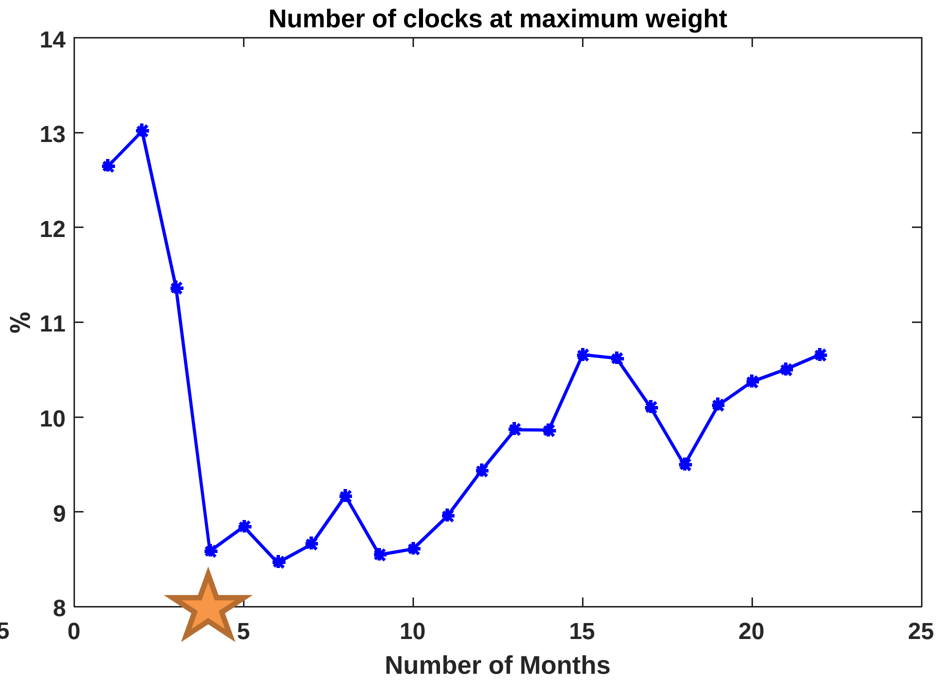
Weight Analysis - 2

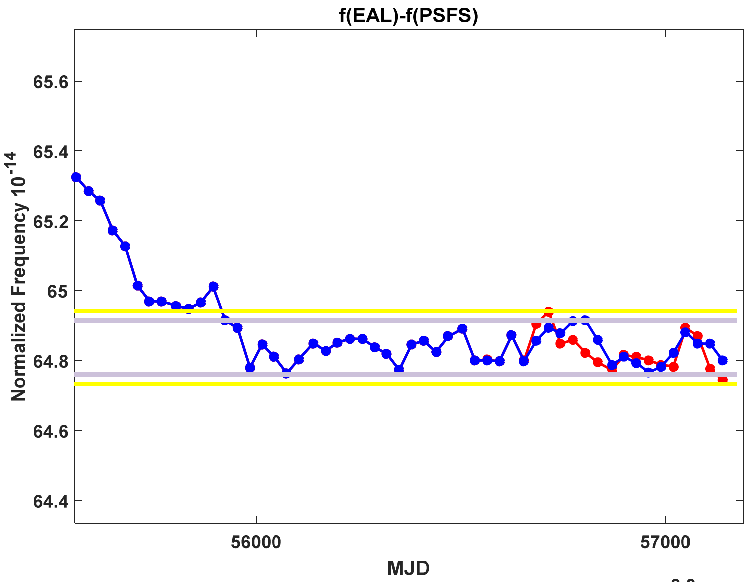
By changing the maximum weight from w_{max} = 2.5/N to w_{max} = 4/N the number of clocks at maximum weight decreases but the combined weight of clocks at maximum weight increases.

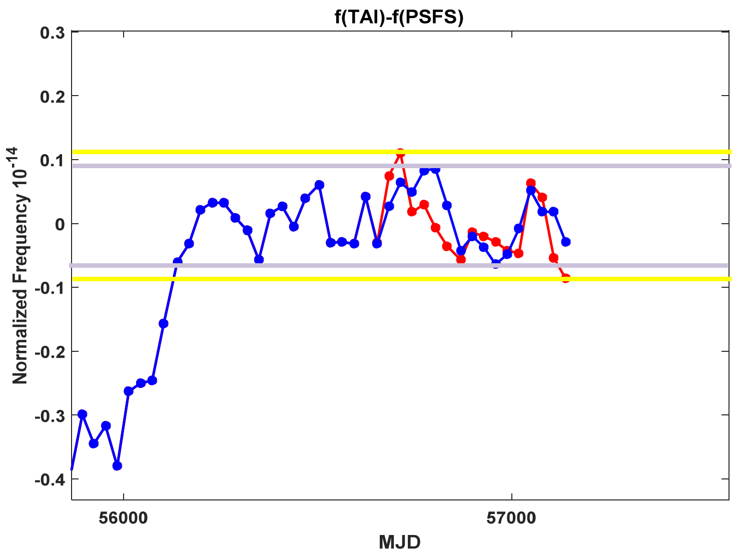




Weighting algorithm changed

The performance of EAL and TAI and with respect to PSFS





Max and Min for
the new algorithm

New algorithm
Old algorithm

Max and Min for
the old algorithm

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The Uncertainty of [*UTC*-*UTC*(*k*)]
(with G. Petit and A. Harmegnies)

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Present: the uncertainty of [*UTC-UTC*(*k*)]

The calculation of the uncertainties is obtained by applying the law of the propagation uncertainty on:

$$x_j(t) = EAL - h_j(t) = \sum_{i=1}^{N} w_i \left[h_i'(t) - x_{i,j}(t) \right]$$

The obtained solution is depending on the time link uncertainties and on the atomic clock weights:

$$u_{x_j}^2 = \sum_{i=1}^{N} w_i^2 u_{x_{i,j}}^2 + 2\sum_{i=1}^{N-1} \sum_{k=i+1}^{N} w_i w_k u_{(x_{i,j}, x_{k,j})}$$

Until now the data are expressed as time links to a pivot laboratory (PTB) and are considered uncorrelated.

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Present: Odd feature 1 (pivot uncertainty)

Date 2015 0h UTC	JAN 27	FEB 1	FEB 6	FEB 11	FEB 16	FEB 21	FEB 26	Uncertainty/ns			Notes
MJD	57049	57054	57059	57064	57069	57074	57079	uA	uB	u	
Laboratory k				[UTC-UTC(k)]/ns							
AOS (Borowiec)	-0.8	0.2	0.7	1.4	0.6	-1.0	-3.2	0.3	5.1	5.1	
BEV (Wien)	12.1	8.2	-3.7	-9.0	-19.2	-27.7	-38.5	0.3	3.1	3.1	
BIM (Sofiya)	2019.8	2017.3	2043.3	2041.9	2065.6	2057.3	2071.9	1.5	7.0	7.2	
BIRM (Beijing)	28.0	28.4	24.1	13.8	5.9	-4.6	-4.4	1.5	20.0	20.1	
CAO (Cagliari)	-2937.7	-3053.5	-3177.4	-3298.8	-3419.3	-3544.6	-3668.8	8.0	7.0	10.7	
CH (Bern-Wabern)	12.8	10.5	6.7	7.4	7.9	7.9	6.2	0.3	1.3	1.3	
DFNT (Tunis)	-	4876.6	5077.1	5276.7	5481.0	5662.4	5843.2	1.5	20.0	20.1	
IT (Torino)	-1.8	-0.6	-0.4	-0.1	-0.1	-0.3	-0.6	0.3	1.4	1.4	
NICT (Tokyo)	-4.7	-6.1	-7.1	-9.3	-13.8	-17.3	-16.0	0.3	4.7	4.7	
NIST (Boulder)	3.5	2.6	2.5	3.0	2.5	2.6	3.5	0.3	4.8	4.8	
OP (Paris)	1.1	1.4	0.9	1.4	1.1	1.2	1.1	0.3	1.3	1.3	
PTB (Braunschweig)	-0.6	-0.1	0.5	1.5	1.6	2.1	1.9	0.1	0.8	0.8	
USNO (Washington DC)	0.6	0.6	0.8	1.3	1.3	1.2	1.8	0.2	1.0	1.0	

6 - Time links used for the computation of TAI and their uncertainties.

Link	Type	uA/ns	uB/ns	Calibration Type	Calibration Dates
AOS /PTB	GPSPPP	0.3	5.0	LC(GPS P3)	2011 Jun
BEV /PTB	GPSPPP	0.3	3.0	BC(GPS MC)	2012 Mar
BIM /PTB	GPS MC	1.5	7.0	GPS EC/GPS EC	2007 Nov/2006 Sep
BIRM/PTB	GPS MC	1.5	20.0	NA /GPS EC	NA /2006 Sep
CAO /PTB	GPS MC	8.0	7.0	GPS EC/GPS EC	2004 Nov/2006 Sep
CH /PTB	TWGPPP	0.3	1.0	LC(TWSTFT)/BC(GPS PPP)	2008 Sep/2009 Aug
DFNT/PTB	GPS MC	1.5	20.0	NA /GPS EC	NA /2006 Sep
IT /PTB	TWGPPP	0.3	1.2	LC(TWSTFT)/BC(GPS PPP)	2008 Sep/2009 Aug
NICT/PTB	GPSPPP	0.3	5.0	GPS EC/GPS EC	2005 Jun/2004 Aug
NIST/PTB	TWSTFT	0.3	5.0	LC(TWSTFT)/BC(GPS PPP)	2005 May/2009 Aug
OP /PTB	TWGPPP	0.3	1.1	LC(TWSTFT)/BC(GPS PPP)	2008 Sep/2009 Aug
USNO/PTB	TWSTFT	0.3	1.0	TW EC	2014 Jun

Due to the absence of correlations, the uncertainty of the pivot PTB is underestimated and unrealistically small.

Present: Odd feature 2 (dependence on one link)

Laboratory k		Uncertainty/ns uA	uB	u
CH	(Bern-Wabern)	0.3	2.1	2.1
CNM	(Queretaro)	2.0	5.3	5.7
DTAG	(Frankfurt/M)	0.3	10.2	10.2
EIM	(Thessaloniki)	7.5	5.3	9.2
ESTC	(Noordwijk)	0.3	5.3	5.3
IT	(Torino)	0.3	2.1	2.2
NICT	(Tokyo)	0.3	4.9	4.9
NIST	(Boulder)	0.3	5.1	5.1
OP	(Paris)	0.3	2.1	2.1
PTB	(Braunschweig)	0.1	1.8	1.8
ROA	(San Fernando)	0.3	5.3	5.3
USNO	(Washington DC)	0.2	3.4	3.4

6 - Time links and their uncertainties.

Link	Type	uA/ns	uB/ns
CH /PTB	TWGPPP	0.3	1.0
CNM /PTB	GPS MC	2.0	5.0
DTAG/PTB	GPSPPP	0.3	10.0
EIM /PTB	GPS MC	7.5	5.0
ESTC/PTB	GPSPPP	0.3	5.0
IT /PTB	TWGPPP	0.3	1.2
NICT/PTB	TWGPPP	0.3	5.0
NIST/PTB	TWGPPP	0.3	5.0
OP /PTB	TWGPPP	0.3	1.1
ROA /PTB	TWGPPP	0.3	5.0
USNO/PTB	GPSPPP	0.3	5.0

Impact on OP, PTB uncertainties

USNO time link uncertainties

Laboratory k		Uncertainty/ns uA	uB	u
CH	(Bern-Wabern)	0.3	1.2	1.3
CNM	(Queretaro)	2.0	5.0	5.4
DTAG	(Frankfurt/M)	0.3	10.0	10.0
EIM	(Thessaloniki)	7.5	5.0	9.0
ESTC	(Noordwijk)	0.3	5.0	5.0
IT	(Torino)	0.3	1.4	1.4
NICT	(Tokyo)	0.3	4.6	4.6
NIST	(Boulder)	0.3	4.8	4.8
OP	(Paris)	0.3	1.2	1.3
PTB	(Braunschweig)	0.1	0.7	0.7
ROA	(San Fernando)	0.3	5.0	5.0
USNO	(Washington DC)	0.2	0.9	0.9

6 - Time links and their uncertainties.

Link	Type	uA/ns	uB/ns
CH /PTB	TWGPPP	0.3	1.0
CNM /PTB	GPS MC	2.0	5.0
DTAG/PTB	GPSPPP	0.3	10.0
EIM /PTB	GPS MC	7.5	5.0
ESTC/PTB	GPSPPP	0.3	5.0
IT /PTB	TWGPPP	0.3	1.2
NICT/PTB	TWGPPP	0.3	5.0
NIST/PTB	TWGPPP	0.3	5.0
OP /PTB	TWGPPP	0.3	1.1
ROA /PTB	TWGPPP	0.3	5.0
USNO/PTB	TWGPPP	0.3	1.0

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Proposed new formalism

Two problems: absence of correlations and not optimal use of all available data.

Proposed solution: Consider a classical estimation problem

$$C\,Y = L$$

Where the solution $Y = (C^T S_L^{-1} C)^{-1} (C^T S_L^{-1}) L$ is the optimal solution if redundancy (and the unique solution if no redundancy)

S_L is the input variance matrix of the measurement vector L

$S_Y = (C^T S_L^{-1} C)^{-1}$ is the variance matrix of the vector Y

The variance matrix S_Y will directly provide the total uncertainty u of [UTC-UTC(k)] when S_L has been filled with both u_A and u_B.

- Approach valid for a redundant system: all available info (TW) may be used.
- GNSS measurements introduced in their original form: UTC(k) – GNSStime

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TEST: Proposed solution by using GNSS data wrt GNSS time scale

Example of current data used

Lab1	Ref/LAB	Type	u_A /ns	u_B / ns
CH	PTB	TWGPPP	0.3	1
CNM	PTB	GPS MC	1	5
DTAG	PTB	GPSPPP	0.3	10
IT	PTB	GPSPPP	0.3	5
EIM	PTB	GPS MC	7.5	5
OP	PTB	TWGPPP	0.3	1
ROA	PTB	TWGPPP	0.3	5
USNO	PTB	GPSPPP	0.3	5

Example of new data proposed

Lab1	Ref/LAB	Type	u_A /ns	u_B / ns
CH	PTB	TWGPPP	0.3	1
CNM	GNSS	GPS MC	1	3.5
DTAG	GNSS	GPSPPP	0.3	6
IT	GNSS	GPSPPP	0.3	3.5
EIM	GNSS	GPS MC	7.5	3.5
OP	PTB	TWGPPP	0.3	1
ROA	PTB	TWGPPP	0.3	5
USNO	GNSS	GPSPPP	0.3	3.5
PTB	GNSS	GPSPPP	0.3	3.5

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Proposed solution exemple

- Least square method resolution of non-redondant system to obtain EAL wrt UTC(k)

Design Matrix					
-1	0	0	0	0	1
0	-1	0	0	0	1
0	0	-1	0	1	0
0	0	0	-1	1	0
0	0	0	0	-1	1
W_{10057}	W_{10048}	W_{10011}	W_{10020}	W_{10005}	W_{IGRT}

X

EAL - lab
EAL – 10057
EAL – 10048
EAL – 10011
EAL – 10020
EAL – 10005
EAL – IGRT

=

Lab - REF
10057 - IGRT
10048 - IGRT
10011 - 10005
10020 - 10005
10005 - IGRT
Prediction

Y

L

$$C\,Y = L$$

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Proposed solution exemple

Resolution of previous system : $C\,Y = L$

$$Y = (C^T S_L^{-1} C)^{-1} C^T S_L^{-1} L$$

where $(C^T S_L^{-1} C)^{-1}$ = S_Y

$S_L =$

Time links Variance-covariance Matrix					
U_{GNSS}	cov	0	0	cov	0
cov	U_{GNSS}	0	0	cov	0
0	0	U_{TW}	0	0	0
0	0	0	U_{TW}	0	0
cov	cov	0	0	U_{GNSS}	0
0	0	0	0	0	U_{pred}

with cov = 0.03ns

Test with correlations (1): u_A using original GNSS data

Introducing correlations: use of original GNSS data, expressed with respect to a reference timescale to which an uncertainty can be attributed

Laboratory	u - Correlated / [ns]	u - Uncorrelated / [ns]
AOS	0.27	0.33
IT	0.34	0.32
NIST	0.34	0.32
OP	0.33	0.32
PTB	**0.18**	**0.14**
USNO	0.25	0.23
SCL	5.99	6.0
ZA	1.49	1.5

The low value of the pivot in the uncorrelated case is less marked in the correlated case. Differences vary with the amount of correlation introduced.

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Test with correlations (2): Dependence on USNO time link

In the case of correlated data, the total uncertainty u depends less on the USNO link quality than in the case of uncorrelated measures (all values in ns).

LAB	USNO / uB=1 ns	USNO / uB=5 ns
AOS	4.1	4.4
IT	2.0	2.6
NIST	3.8	4.1
OP	2.0	2.5
PTB	**1.7**	**2.3**
USNO	1.8	3.8
SCL	11.3	11.5
VMI	19.8	19.9

A First application of the Kalman Filter routine to UTC
(with Federica Parisi)

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The Kalman Filter Algorithm

- The Kalman Filter (KF) is a very powerful tool used in many fields. If the parameters are well estimated the KF is able to «clean» the noise affecting the data.

- We introduce the Kalman Filter as basic filter to deal with the white phase noise in time transfer systems,

- We use it to build time scale.

- Many work is still necessary .....

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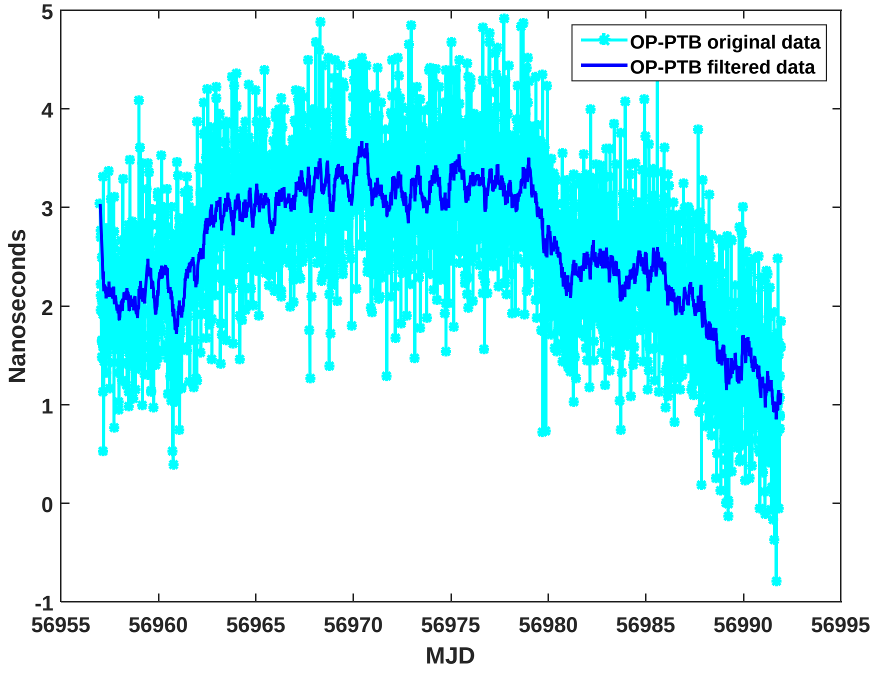
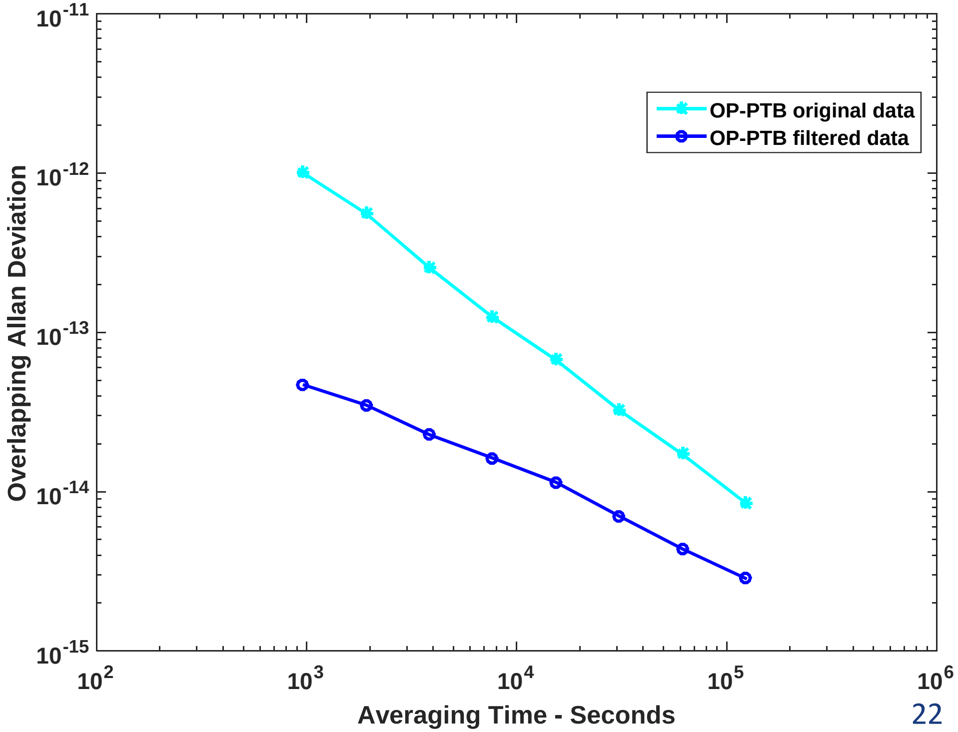
EXAMPLE: OP – PTB GPSP3 data



The Kalman Filter routine is used principally as a filter to noisy measurements to clean white noise (caracteristic of time transfer noise).



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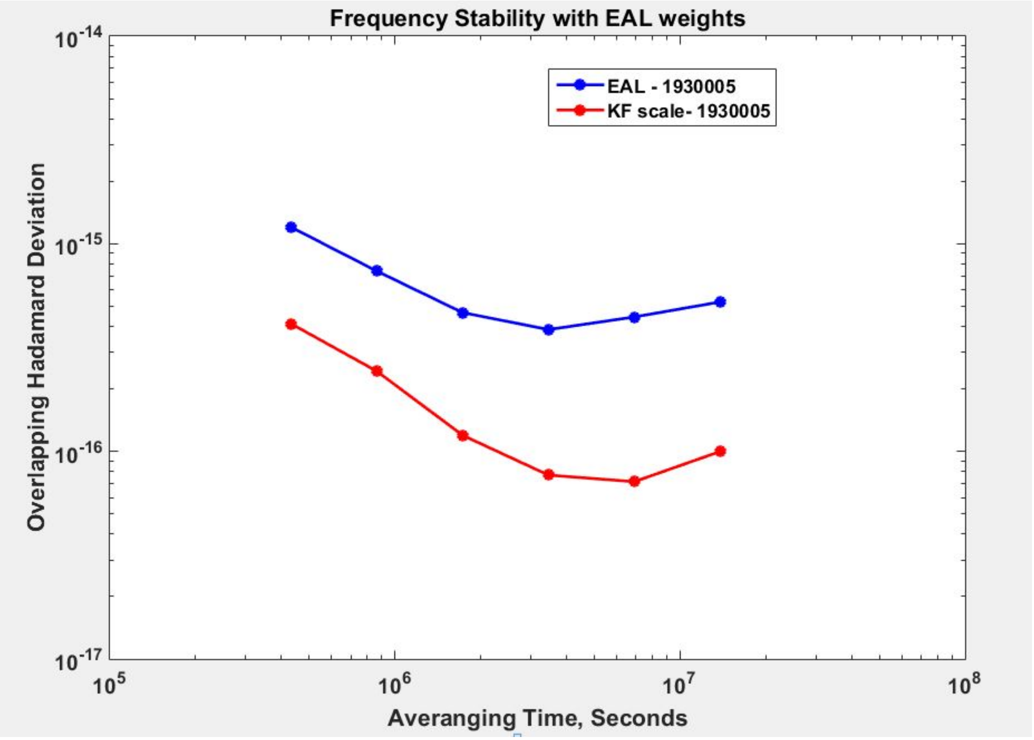
Test of Kalman Filter on 139 atomic clocks

- 139 atomic clocks always present without time and frequency steps are used
- EAL was calculated only with these clocks
- KF time scale is calculated with these clocks and with the weights used in EAL
- The only difference between EAL and KF time scale is the «prediction component»



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Conclusions

The effects of the new weighting algorithm are already evident in the short term stability of EAL (limited by the time transfer noise).

A new solution has been proposed to solve the odd features of the uncertainties of [*UTC*-*UTC*(*k*)].

We started a study on the use of redudant time link and on the effect on the uncertainties.

A new work on the use of the Kalman Filter for UTC has been started.

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