



University of Maribor

Faculty of Mechanical Engineering



Report on EURAMET Supplementary Comparison Calibration of a transducer 60 mm

**EURAMET project 1513
EURAMET.L–S2.1.n01
(renamed from EURAMET.L–S31)**

Final report

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1 Document control

Version Draft A.1	Issued on 6 January 2023
Version Draft A.2	Issued on 17 February 2023
Version Draft A.3	Issued on 4 September 2023
Version Draft B	Issued on 14 September 2023

2 Introduction

The metrological equivalence of national measurement standards and of calibration certificates issued by national metrology institutes is established by a set of key and supplementary comparisons chosen and organized by the Consultative Committees of the CIPM or by the regional metrology organizations in collaboration with the Consultative Committees.

At its meeting October 2020 (organised and hosted online by DFM Denmark), the EURAMET Technical Committee for Length (TC-L) decided to perform a supplementary comparison on precise linear transducers, named EURAMET.L-S31., with MIRS/UM-FS/LTM (Slovenian DI for length) as the pilot laboratory. The comparison was registered in the KCDB in December 2020, artefact circulation started in March 2021 and was completed in July 2022.

3 Organization

3.1 Participants

Table 1. List of participant laboratories and their contacts.

Laboratory Code	Contact person, Laboratory	Phone, Fax, email
MIRS/UM-FS/LTM (pilot)	University of Maribor Faculty of Mechanical Engineering Smetanova 17 SI-2000 Maribor Slovenia	Bojan Acko +386 2 220 7581 bojan.acko@um.si
BEV	Bundesamt für Eich – und Vermessungswesen Arltgassee 35 AT-1160 Wien Austria	Michael Matus +43 1 21 110 6540 michael.matus@bev.gv.at
CEM	Centro Espanol de Metrologia C/del Alfar 2 ES-28760 Tres Cantos (Madrid) Spain	Mar Pérez +34 91 807 48 01 mmperezh@cem.es
DFM	Danish Fundamental Metrology (DFM) Kogle Allé 5 DK-2970 Hørsholm Denmark	Jan Hald +45 2545 9019 jha@dfm.dk
DTI	Teknologisk Institut (DTI) Gregersensvej 8H 2630 Taastrup Denmark	Jens Bo Toftegaard +45 7220 3034 jbt@teknologisk.dk

Laboratory Code	Contact person, Laboratory	Phone, Fax, email
JV	Justervesenet - Norwegian Metrology Service Fetveien 99 NO-2007 Kjeller Norway	Helge Karlsson +47 64 84 84 84 hka@justervesenet.no
METAS	Federal Institute of Metrology METAS Lindenweg 50 CH-3003 Bern-Wabern Switzerland	Felix Meli, Christian Kottler +41 58 387 03 46 felix.meli@metas.ch christian.kottler@metas.ch
VSL	VSL Thijssseweg 11 P.O.Box 654 NL-2600 AR Delft The Netherlands	Richard Koops +31 15 2691 642 rkoops@vsl.nl
VTT MIKES	VTT Technical Research Centre of Finland Ltd, Centre for Metrology MIKES Tekniikantie 1 FI-02150 Espoo Finland	Antti Lassila +358 40 7678584 antti.lassila@vtt.fi

* According to documents provided by Danish authorities and the rules in CIPM MRA-P-11 and CIPM MRA-G-11; relevant data for Danish CIPM MRA purposes are exclusively based on DFM results.

3.2 Schedule

The original schedule of measurements was set in version 4 of the technical protocol, issued in March 2021. It is represented in the first 9 rows of Table 2 (until January 2022). However, two participants (BEV and VSL; marked with *) expressed a wish to repeat their measurements due to technical issues and one more participant (VTT MIKES) expressed a wish to join the comparison at the EURAMET TC-L meeting in October 2021 (organised and hosted online by BMM Montenegro). After all participants have confirmed the proposed changes, version 5 of the technical protocol was issued (November 2021). New schedule has included three more measurements (marked with **) and the final measurement in the pilot laboratory.

Table 2. Schedule of the comparison.

RMO	Laboratory	Original schedule	Date of measurement	Results received
EURAMET	MIRS/UM-FS/LTM	March 2021	March 2021	
	BEV*	April 2021	April 2021	May 2021
	DTI	June 2021	June 2021	October 2021
	VSL*	July 2021	August 2021	-
	DFM	September 2021	September 2021	October 2021
	CEM	October 2021	October 2021	July 2022
	JV	November 2021	November 2021	December 2021

RMO	Laboratory	Original schedule	Date of measurement	Results received
	METAS	December 2021	December 2021	February 2022
Pilot lab	MIRS/UM-FS/LTM	January 2022	February 2022	
	BEV**	March 2022	March 2022	March 2022
	VSL**	April 2022	April 2022	September 2022
	VTT MIKES**	May 2022	June 2022	September 2022
Pilot lab	MIRS/UM-FS/LTM	June 2022	July 2022	

Actual measurements have been performed in accordance with the original schedule with slight delays due to technical problems and minor problems with ATA carnet when the package has travelled from and back to EU. The transducer had to travel back to the pilot laboratory after the measurements in VSL in April 2022 in order to be repaired. However, no significant impact on the measurement results was established after the repair. Some more significant time delays were detected in reporting, but these had no significant influence on the overall duration of the comparison.

4 Artefact

4.1 Description of artefact

The artefact was a transducer with the measurement range (0 to 60) mm and the resolution of 10 nm. The artefact is shown in Fig. 1. The artefact was equipped with a spherical tip $\varnothing 5$ mm made of Tungsten carbide.

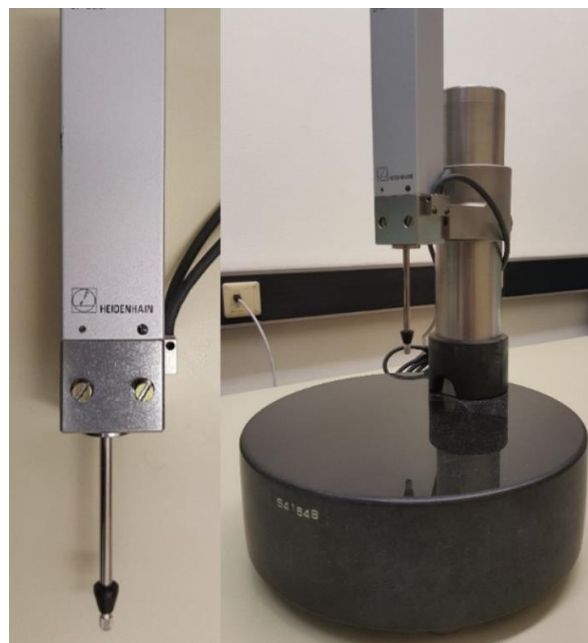


Figure 1: Heidenhain transducer Certo CP 60M

Table 3. Producer’s specifications of the artefact

Measuring range	(0 to 60) mm
Resolution	10 nm (by using the supplied display unit)
Repeatability	< 0,03 μm
System accuracy	$\pm 0,1 \mu\text{m}$ (with linear error compensation up to $\pm 0,03 \mu\text{m}$)
Scale material	Zerodur (thermal expansion α not specified; assumed to be negligible in the specified measuring range at temperatures 19 °C to 21 °C)
Material of the components in the measuring loop	Invar; $\alpha = 1 \times 10^{-6} \text{ K}^{-1}$
Guides	Ball-bearing guide with low friction
Plunger (probe) actuation	By gravity (chosen for the transducer in comparison)

Display unit

The transducer was supplied with an easy-to-use display unit shown in Fig. 2. Precise instructions for using the display unit were provided.



Figure 2: Display unit

4.2 Stability of artefact

The artefact was measured at the beginning (March 2021) and at the end of the circulation loop (July 2022) by the pilot laboratory. In addition, two intermediate control measurements were performed, when the artefact was returned to the pilot due to customs arrangements (February 2022) and service (May 2022). Originally, only one intermediate check was planned to be performed in January 2022, but since the mechanics of the artefact was fixed in May 2022, an additional control measurement was performed.

Deviations of two intermediate and the final measurements from the initial measurement at MIRS/UM-FS/LTM are presented in Figure 3. The results in all measurement points remained within the uncertainty limits ($k = 2$), specified by the pilot.

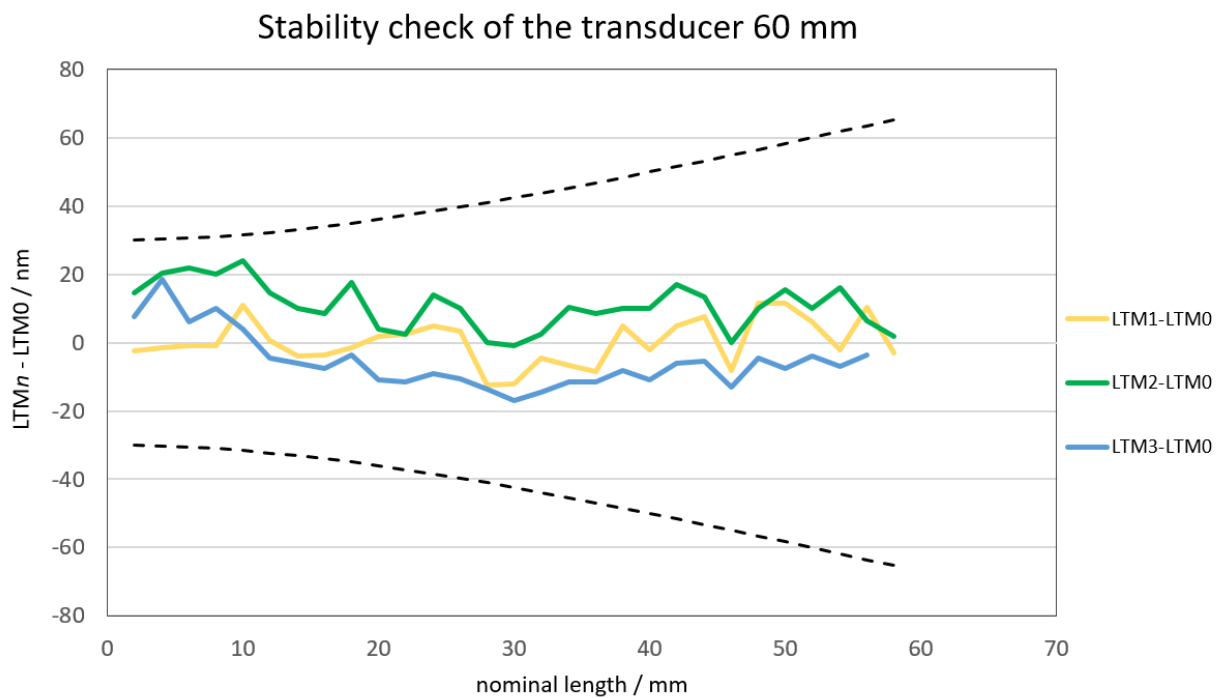


Figure 3. Stability of the transducer – deviations of two intermediate results (LTM1 and LTM2) and the final results (LTM3) from the initial measurements (LTM0). Dashed lines indicate the uncertainty of the initial measurements for a coverage factor $k = 2$.

4.3 Condition of artefact at start/end of comparison

The initial condition of the transducer was checked before the measurement loop and was documented in the technical protocol. Based on the history of the transducer and the display unit, no significant changes in functionality and in metrological characteristics were expected during the period of measurements.

After receiving the transducer back in January 2022, we have noticed that the tip of the transducer was changed by one of the participants, but we couldn't find out where. We have performed extensive investigation of the impact of this changes on the metrological characteristics and the impact was not detectable. Changes of biases in respect to our initial measurement were within our measurement uncertainty in all measurement points.



Figure 4. Original tip (left) and the “new” tip of unknown source (right)

As explained in Chapter 3.2, additional measurements were performed in the period from March to June 2022. The transducer had to travel back to the pilot laboratory after the measurements in April 2022 in order to be repaired (mechanical malfunction was detected by a participant after performing the measurements). However, no significant impact on the measurement results was established after fixing it.

5 Measuring instructions

5.1 Measurands

The measurand was a displacement error (bias) of the transducer in defined measurement points (Table 1) in the measurement direction perpendicular to the lower housing surface.

The origin (reference point, "0") had to be defined in respect to the internal reference mark.

Table 4. Measurement points

Nominal lengths from defined origin in mm									
2	4	6	8	10	12	14	16	18	20
22	24	26	28	30	32	34	36	38	40
42	44	46	48	50	52	54	56	58	

It was recommended to measure in both directions: inwards (+) and outwards (-). However, the **measurand of the comparison was only the average detected bias (\bar{B})**.

$$\bar{B} = (B_+ + B_-)/2 \quad (1)$$

where:

\bar{B} – measurand for the comparison

B_+ – bias in positive direction (probe moving inwards)

B_- – bias in negative direction (probe moving outwards)

5.2 Traceability

Length measurements had to be traceable to the latest realisation of the metre as set out in the current "*Mise en Pratique*".

Temperature measurements had to be made using the International Temperature Scale of 1990 (ITS-90).

5.3 Measurement conditions

Since the measurement force of the transducer is produced by gravity, it was strongly recommended to orient the transducer vertically. Such orientation is also proposed by the producer.

However, it was also allowed to perform the measurements in horizontal orientation if a proper connection of the transducer with the standard instrument could have been assured. In this case, the results had to be properly corrected (measurement force, shaft bending, ...).

The measured values had to be referred to the following reference conditions:

- temperature of 20 °C (ITS-90).

If necessary, corrections had to be applied based upon the thermal expansion, stated in Table 3.

5.4 Measurement instructions

The calibration had to be carried out as for a normal customer. The participants were free to choose their own method of measurement. It was recommended to measure in both directions: inwards (+) and outwards (-). However, the quantity for the comparison was only the average detected bias (\bar{B}), because some laboratories might have chosen to measure only in one direction (e. g. only outwards if gauge blocks were used).

Before calibration, the transducer had to be inspected for damages. Any scratches or other damages had to be documented.

The measurements had to be reported for the measuring conditions, given in 5.3. A table for reporting the results was prepared by the pilot laboratory and distributed to the participants.

No other measurements were to be attempted by the participants and the artefact was not allowed to be used for any purpose other than described in the technical protocol. The artefact was not allowed to be given to any party other than the participants in the comparison.

5.5 Measurement uncertainty

The uncertainty of measurement had to be estimated according to the *ISO Guide to the Expression of Uncertainty in Measurement*. The participants were encouraged to use their usual model for the uncertainty calculation.

The participants were requested to report a detailed measurement uncertainty budget using a prepared table.

The comparison results were assessed by considering expanded uncertainties with $k = 2$.

6 Results

6.1 Results and standard uncertainties as reported by participants

Reported results (biases B_+ , B_- , and \bar{B} ; see Equation 1) are presented in Tables 5, 6 and 7, while the reported expanded uncertainties are presented in Table 8. Most laboratories have calculated expanded uncertainties by using the coverage factor $k = 2$, only DFM has used $k = 2,1$.

Since laboratories were allowed to measure only in one direction (see 5.4), these single measurement values were considered as “average values” in the comparison and are presented in Table 7. There were 2 such laboratories in the comparison – BEV and VSL (BEV has reported their results only as “average” values, while VSL has indicated that their measurements have been performed in “outwards” direction).

Table 5. Reported biases B_+ for 29 measurement positions (for probe movement “inwards”)

Positions	Participants								
	MIRS/UM- FS/LTM	DTI	DFM	CEM	JV	METAS	BEV*	VSL*	VTT MIKES
	B_+ / nm	B_+ / nm	B_+ / nm	B_+ / nm	B_+ / nm	B_+ / nm	B_+ / nm	B_+ / nm	B_+ / nm
2 mm	-10	0	7	-14	-110	-34	/	/	14
4 mm	-30	0	21	-28	-170	-29	/	/	10
6 mm	5	-100	35	-39	-260	2	/	/	29
8 mm	8	-100	26	-47	-280	-1	/	/	28
10 mm	10	-200	20	-79	-320	-5	/	/	39
12 mm	10	-300	32	-89	-340	-4	/	/	34
14 mm	20	-300	40	-106	-380	20	/	/	32
16 mm	15	-500	26	-105	-410	20	/	/	42
18 mm	10	-400	45	-119	-450	27	/	/	43
20 mm	21	-700	33	-137	-480	17	/	/	38
22 mm	32	-500	39	-143	-520	25	/	/	42
24 mm	22	-600	34	-145	-540	31	/	/	40
26 mm	20	-700	32	-148	-570	25	/	/	41
28 mm	30	-800	39	-149	-600	28	/	/	42
30 mm	52	-900	58	-152	-650	45	/	/	66
32 mm	53	-1000	62	-149	-660	58	/	/	64
34 mm	57	-1000	64	-137	-700	54	/	/	69
36 mm	68	-1000	71	-136	-710	47	/	/	77
38 mm	72	-1000	84	-139	-730	61	/	/	87
40 mm	81	-1200	74	-140	-710	73	/	/	94
42 mm	98	-1200	93	-119	-730	89	/	/	107
44 mm	122	-1300	111	-110	-690	104	/	/	114
46 mm	136	-1300	121	-89	-670	108	/	/	131
48 mm	142	-1500	128	-73	-630	123	/	/	143
50 mm	150	-1400	146	-57	-590	149	/	/	153
52 mm	160	-1500	154	-50	-570	170	/	/	188
54 mm	180	-1700	175	-15	-610	171	/	/	172
56 mm	193	-1700	179	1	-630	184	/	/	189
58 mm	210	-1700	196	2	-700	209	/	/	196

* Not reported

Table 6. Reported biases B_{-} for 29 measurement positions (for probe movement “outwards”)

Positions	Participants								
	MIRS/UM- FS/LTM	DTI	DFM	CEM	JV	METAS	BEV*	VSL	VTT MIKES
	B_{-}/nm	B_{-}/nm	B_{-}/nm	B_{-}/nm	B_{-}/nm	B_{-}/nm	B_{-}/nm	B_{-}/nm	B_{-}/nm
2 mm	5	100	6	-8	-90	-7	/	22	9
4 mm	9	100	26	-24	-130	3	/	-1	8
6 mm	15	-100	35	-18	-180	33	/	27	17
8 mm	10	-200	26	-28	-240	24	/	16	7
10 mm	12	-200	24	-43	-310	18	/	4	13
12 mm	25	-300	34	-62	-360	19	/	16	17
14 mm	30	-400	37	-83	-390	42	/	0	17
16 mm	32	-400	30	-84	-440	46	/	-13	27
18 mm	21	-600	30	-93	-490	56	/	6	17
20 mm	23	-500	26	-105	-510	48	/	-29	22
22 mm	19	-600	20	-126	-550	49	/	-9	23
24 mm	28	-700	26	-115	-560	57	/	-24	21
26 mm	29	-800	39	-116	-590	53	/	-37	32
28 mm	41	-900	52	-117	-610	55	/	-33	37
30 mm	52	-900	61	-117	-650	75	/	10	59
32 mm	66	-1000	67	-110	-680	85	/	-5	64
34 mm	72	-1100	72	-102	-710	88	/	-13	75
36 mm	79	-1100	74	-101	-740	82	/	-13	80
38 mm	88	-1100	81	-99	-780	96	/	-11	90
40 mm	99	-1200	93	-97	-780	106	/	-20	96
42 mm	112	-1200	102	-84	-820	118	/	-7	115
44 mm	127	-1300	120	-69	-840	129	/	-5	108
46 mm	148	-1400	117	-62	-880	142	/	-3	147
48 mm	151	-1500	146	-62	-890	159	/	/	148
50 mm	171	-1400	160	-44	-880	184	/	-6	175
52 mm	178	-1600	171	-34	-830	194	/	-24	180
54 mm	192	-1500	187	-29	-810	210	/	-18	190
56 mm	210	-1600	212	-1	-760	223	/	-29	206
58 mm	216	-1700	187	19	-720	217	/	-17	229

* Not reported

Table 7. Reported average biases \bar{B} for 29 measurement positions

Positions	Participants								
	MIRS/UM-FS/LTM	DTI	DFM	CEM	JV	METAS	BEV	VSL	VTT MIKES
	\bar{B} / nm	\bar{B} / nm	\bar{B} / nm	\bar{B} / nm	\bar{B} / nm	\bar{B} / nm	\bar{B} / nm	\bar{B} / nm	\bar{B} / nm
2 mm	-2,5	100	6	-11	-100	-21	16	22	12
4 mm	-10,5	100	24	-26	-150	-13	4	-1	9
6 mm	10	-100	35	-28	-220	18	-4	27	23
8 mm	9	-100	26	-37	-260	11	-14	16	17
10 mm	11	-200	22	-61	-315	6	23	4	26
12 mm	17,5	-300	33	-76	-350	7	40	16	25
14 mm	25	-400	39	-94	-385	31	35	0	25
16 mm	23,5	-500	28	-95	-425	33	27	-13	34
18 mm	15,5	-500	37	-106	-470	42	64	6	30
20 mm	22	-600	29	-121	-495	32	96	-29	30
22 mm	25,5	-600	29	-135	-535	37	53	-9	33
24 mm	25	-600	30	-130	-550	44	90	-24	30
26 mm	24,5	-800	35	-132	-580	39	89	-37	37
28 mm	35,5	-800	46	-133	-605	41	102	-33	40
30 mm	52	-900	59	-134	-650	60	72	10	62
32 mm	59,5	-1000	65	-129	-670	72	93	-5	64
34 mm	64,5	-1000	68	-120	-705	71	91	-13	72
36 mm	73,5	-1100	72	-119	-725	64	118	-13	79
38 mm	80	-1000	83	-119	-755	78	110	-11	88
40 mm	90	-1200	83	-119	-745	90	138	-20	95
42 mm	105	-1200	98	-101	-775	104	126	-7	111
44 mm	124,5	-1300	116	-89	-765	116	124	-5	111
46 mm	142	-1400	119	-76	-775	125	132	-3	139
48 mm	146,5	-1500	137	-68	-760	141	134	/	146
50 mm	160,5	-1400	153	-50	-735	166	129	-6	164
52 mm	169	-1600	163	-42	-700	182	139	-24	184
54 mm	186	-1600	181	-22	-710	191	136	-18	181
56 mm	201,5	-1600	196	0	-695	204	141	-29	197
58 mm	213	-1700	191	10	-710	213	168	-17	212

Table 8. Expanded uncertainties U , as reported by the participants.

Positions	Participants								
	MIRS/UM- FS/LTM	DTI	DFM	CEM	JV	METAS	BEV	VSL	VTT MIKES
	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm
2 mm	30	640	23	100	252	18	124	60	13
4 mm	30	640	24	100	253	19	124	60	13
6 mm	31	640	24	100	253	19	124	60	13
8 mm	31	640	25	100	254	20	124	61	13
10 mm	32	640	26	100	256	21	124	61	14
12 mm	32	640	27	100	257	22	124	61	14
14 mm	33	640	29	100	259	24	124	62	14
16 mm	34	640	30	100	262	25	124	62	14
18 mm	35	640	32	100	264	27	124	63	15
20 mm	36	640	34	100	267	28	124	63	15
22 mm	37	640	35	100	270	30	124	64	16
24 mm	38	640	37	100	273	32	124	65	16
26 mm	40	640	39	100	277	34	124	65	17
28 mm	41	640	41	100	281	36	124	66	17
30 mm	42	640	43	100	284	38	124	67	18
32 mm	44	640	45	100	289	40	124	68	18
34 mm	45	640	47	100	293	42	124	69	19
36 mm	47	640	50	100	298	43	124	70	19
38 mm	48	640	52	100	302	46	124	71	20
40 mm	50	640	54	100	307	48	124	72	21
42 mm	52	640	56	100	312	50	124	73	21
44 mm	53	640	58	100	318	52	124	74	22
46 mm	55	640	61	100	323	54	124	76	23
48 mm	57	640	63	100	329	56	124	77	23
50 mm	58	640	65	100	335	58	124	78	24
52 mm	60	640	67	100	340	60	124	79	25
54 mm	62	640	70	100	346	62	124	81	25
56 mm	64	640	72	100	352	64	124	82	26
58 mm	65	640	74	100	359	66	124	83	27

6.2 Measurement uncertainties

No unique instruction on how to report the measurement uncertainties was provided by the pilot laboratory. The participants were encouraged to use their usual model for the uncertainty calculation. The participants were requested to report a detailed measurement uncertainty budget using the table, which was attached as Appendix A.2 to the technical protocol.

Uncertainty budgets of all participants are presented in Appendix A.1 in a form in which they have been reported by the participants.

Short summary of the reported standard and expanded uncertainties is presented in Table 9.

Table 9. Standard and expanded uncertainties reported by the participants
 (expression form was unified by the pilot)

Participant	Reported standard uncertainty	Reported expanded uncertainty
MIRS/UM-FS/LTM	Q[15 nm , $0,58 \cdot 10^{-6} L$]	Q[30 nm , $1,1 \cdot 10^{-6} L$]
DTI	320 nm	640 nm
DFM	Q[11 nm , $0,58 \cdot 10^{-6} L$]	Q[23 nm , $1,22 \cdot 10^{-6} L$]
CEM	50 nm	100 nm
JV	Q[126 nm , $2,2 \cdot 10^{-6} L$]	Q[252 nm , $4,4 \cdot 10^{-6} L$]
METAS	Q[8.9 nm , $0,57 \cdot 10^{-6} L$]	Q[18 nm , $1,1 \cdot 10^{-6} L$]
BEV	62 nm	124 nm
VSL	Q[30 nm , $0,5 \cdot 10^{-6} L$]	Q[60 nm , $1,0 \cdot 10^{-6} L$]
<i>VSL also reported their CMC in KCDB:</i>		Q[0,1 μm , $1 \cdot 10^{-6} L$]; $k = 2$
VTT MIKES	Q[6.6 nm , $0,2 \cdot 10^{-6} L$]	Q[13 nm , $0,4 \cdot 10^{-6} L$]

6.3 Changes to results after Draft A.1 (optional)

No changes were made to the results after Draft A.1. However, tables with results and measurement uncertainties as well as graphs were rearranged in such manner, that the participants are listed according to the term of performing measurements.

7 Analysis

7.1 Calculation of the CRV

The comparison reference value was derived as weighted mean for each marking position.

Few datasets had to be excluded as outliers in order to derive consistent data sets, following the procedure given in [2]. The procedure is also briefly discussed in [3] and [5].

In the applied procedure, the total number of participants submitting a result is denoted by I , while the reported measured value of each participant is denoted by x_i and its associated standard uncertainty by $u(x_i)$. The normalised weight, w_i , for the result x_i is given by:

$$w_i = C \cdot \frac{1}{[u(x_i)]^2} \quad (2)$$

where the normalising factor, C , is given by:

$$C = \frac{1}{\sum_{i=1}^I \left(\frac{1}{u(x_i)} \right)^2} \quad (3)$$

The weighted mean, \bar{x}_w , is given by:

$$\bar{x}_w = \sum_{i=1}^I w_i \cdot x_i \quad (4)$$

The uncertainty of the weighted mean is calculated by:

$$u(\bar{x}_w) = \frac{1}{\sqrt{\sum_{i=1}^I \left(\frac{1}{u(x_i)} \right)^2}} = \sqrt{C} \quad (5)$$

7.2 Deviations from the CRV and degrees of equivalence

After deriving the weighted mean and its associated standard uncertainty, the deviation of each laboratory's result from the weighted mean is determined simply as $d_i = x_i - \bar{x}_w$. The uncertainty of this deviation is calculated as a combination of the uncertainties of the result, $u(x_i)$, and the uncertainty of the weighted mean $u(\bar{x}_w)$. The uncertainty of the deviation from the weighted mean is given by equation (6), which includes a minus sign to take into account the correlation between the two uncertainties.

$$u(x_i - \bar{x}_w) = \sqrt{[u(x_i)]^2 - [u_{int}(\bar{x}_w)]^2} \quad (6)$$

The degree of equivalence (DoE) was determined by calculating the E_n value for each laboratory's result, where E_n is defined as the ratio of the deviation from the weighted mean, divided by the expanded uncertainty of this deviation:

$$E_n = \frac{|x_i - \bar{x}_w|}{2 \cdot u(x_i - \bar{x}_w)} \quad (7)$$

The uncertainty in the DoE was calculated using either

$$u(x_i - \bar{x}_w) = \sqrt{[u(x_i)]^2 - [u_{int}(\bar{x}_w)]^2} \quad \text{for results which contributed to the weighted mean}$$

or

$$u(x_i - \bar{x}_w) = \sqrt{[u(x_i)]^2 + [u_{int}(\bar{x}_w)]^2} \quad \text{for results which made no contribution.}$$

7.3 Consistency check and outlier identification

For the determination of the comparison reference value CRV, statistical consistency of the results contributing to the CRV was performed by means of analysing the Birge ratio R_B which compares the observed spread of the results with the spread expected from the individual reported uncertainties.

The application of least squares algorithms and the χ^2 -test leads to the Birge ratio

$$R_B = \frac{u_{\text{ext}}(\bar{x}_w)}{u(\bar{x}_w)}, \quad (8)$$

where $u_{\text{ext}}(\bar{x}_w)$ is the external standard deviation

$$u_{\text{ext}}(\bar{x}_w) = \sqrt{\frac{1}{(I-1)} \cdot \frac{\sum_{i=1}^I w_i (x_i - \bar{x}_w)^2}{\sum_{i=1}^I w_i}}. \quad (9)$$

The Birge ratio has an expectation value of $R_B = 1$, when considering standard uncertainties. For a coverage factor of $k = 2$, the expectation value is increased and the data in a comparison are consistent provided that

$$R_B < \sqrt{1 + \sqrt{8/(I-1)}} \quad (10)$$

where I is the number of participants [2]. The iterative process of excluding results from contributing to the CRV is stopped as soon as Eq. (10) is fulfilled, even if values for $|E_n| > 1$ are remaining [5].

As indicated in Ch. 5.1, the **measurand of the comparison was only the average detected bias (\bar{B})**. Therefore, Tables 10 to 13 refer only to \bar{B} .

Table 10. Comparison reference values \bar{x}_w and associated standard uncertainties $u(\bar{x}_w)$ - the following outliers were removed from CRV calculation: one (JV) in positions 14 mm, 16 mm and 18 mm, two (JV and CEM) in positions 20 mm, 22 mm, 24 mm, 26 mm, 28 mm and 30 mm, three (JV, DTI and CEM) in positions 32 mm, 34 mm, 36 mm, 38 mm, 40 mm, 42 mm, and 44 mm, and four (JV, DTI, CEM and VSL) in positions 46 mm, 48 mm, 50 mm, 52 mm, 54 mm, 56 mm and 58 mm

	Measurement position / mm									
	2	4	6	8	10	12	14	16	18	20
\bar{x}_w / nm	1,6	3,4	21,6	15,1	17,9	20,6	26,1	29,1	29,3	28,0
$u(\bar{x}_w)$ / nm	4,5	4,5	4,6	4,7	4,9	5,0	5,2	5,4	5,5	5,8
	Measurement position / mm									
	22	24	26	28	30	32	34	36	38	40
\bar{x}_w / nm	31,1	30,2	33,5	38,1	58,3	62,0	67,1	72,2	81,0	87,7
$u(\bar{x}_w)$ / nm	6,0	6,2	6,4	6,6	6,9	7,1	7,4	7,6	7,9	8,1
	Measurement position / mm									
	42	44	46	48	50	52	54	56	58	
\bar{x}_w / nm	102,5	107,4	135,7	144,4	162,0	179,1	181,4	196,5	209,1	
$u(\bar{x}_w)$ / nm	8,4	8,6	9,2	9,4	9,7	10,0	10,3	10,6	10,9	

Table 11. Deviations from the comparison reference values

Positions	Participants								
	MIRS/UM- FS/LTM	DTI	DFM	CEM	JV	METAS	BEV	VSL	VTT MIKES
	d_i / nm	d_i / nm	d_i / nm	d_i / nm	d_i / nm	d_i / nm	d_i / nm	d_i / nm	d_i / nm
2 mm	-4,1	98,4	4,4	-12,6	-101,6	-22,6	14,4	20,4	10,4
4 mm	-13,9	96,6	20,6	-29,4	-153,4	-16,4	0,6	-4,4	5,6
6 mm	-11,6	-121,6	13,4	-49,6	-241,6	-3,6	-25,6	5,4	1,4
8 mm	-6,1	-115,1	10,9	-52,1	-275,1	-4,1	-29,1	0,9	1,9
10 mm	-6,9	-217,9	4,1	-78,9	-332,9	-11,9	5,1	-13,9	8,1
12 mm	-3,1	-320,6	12,4	-96,6	-370,6	-13,6	19,4	-4,6	4,4
14 mm	-1,1	-426,1	12,9	-120,1	-411,1	4,9	8,9	-26,1	-1,1
16 mm	-5,6	-529,1	-1,1	-124,1	-454,1	3,9	-2,1	-42,1	4,9
18 mm	-13,8	-529,3	7,7	-135,3	-499,3	12,7	34,7	-23,3	0,7
20 mm	-6,0	-628,0	1,0	-149,0	-523,0	4,0	68,0	-57,0	2,0
22 mm	-5,6	-631,1	-2,1	-166,1	-566,1	5,9	21,9	-40,1	1,9
24 mm	-5,2	-630,2	-0,2	-160,2	-580,2	13,8	59,8	-54,2	-0,2
26 mm	-9,0	-833,5	1,5	-165,5	-613,5	5,5	55,5	-70,5	3,5
28 mm	-2,6	-838,1	7,9	-171,1	-643,1	2,9	63,9	-71,1	1,9
30 mm	-6,3	-958,3	0,7	-192,3	-708,3	1,7	13,7	-48,3	3,7
32 mm	-2,5	-1062,0	3,0	-191,0	-732,0	10,0	31,0	-67,0	2,0
34 mm	-2,6	-1067,1	0,9	-187,1	-772,1	3,9	23,9	-80,1	4,9
36 mm	1,3	-1172,2	-0,2	-191,2	-797,2	-8,2	45,8	-85,2	6,8
38 mm	-1,0	-1081,0	2,0	-200,0	-836,0	-3,0	29,0	-92,0	7,0
40 mm	2,3	-1287,7	-4,7	-206,7	-832,7	2,3	50,3	-107,7	7,3
42 mm	2,5	-1302,5	-4,5	-203,5	-877,5	1,5	23,5	-109,5	8,5
44 mm	17,1	-1407,4	8,6	-196,4	-872,4	8,6	16,6	-112,4	3,6
46 mm	6,3	-1535,7	-16,7	-211,7	-910,7	-10,7	-3,7	-138,7	3,3
48 mm	2,1	-1644,4	-7,4	-212,4	-904,4	-3,4	-10,4	/	1,6
50 mm	-1,5	-1562,0	-9,0	-212,0	-897,0	4,0	-33,0	-168,0	2,0
52 mm	-10,1	-1779,1	-16,1	-221,1	-879,1	2,9	-40,1	-203,1	4,9
54 mm	4,6	-1781,4	-0,4	-203,4	-891,4	9,6	-45,4	-199,4	-0,4
56 mm	5,0	-1796,5	-0,5	-196,5	-891,5	7,5	-55,5	-225,5	0,5
58 mm	3,9	-1909,1	-18,1	-199,1	-919,1	3,9	-41,1	-226,1	2,9

Table 12. Expanded uncertainties for the deviations from the comparison reference values.

Positions	Participants								
	MIRS/UM- FS/LTM	DTI	DFM	CEM	JV	METAS	BEV	VSL	VTT MIKES
	<i>U</i> / nm	<i>U</i> / nm	<i>U</i> / nm	<i>U</i> / nm	<i>U</i> / nm	<i>U</i> / nm	<i>U</i> / nm	<i>U</i> / nm	<i>U</i> / nm
2 mm	28,7	640,1	21,4	99,6	252,0	15,7	123,7	59,4	9,4
4 mm	28,9	640,1	21,8	99,6	252,5	16,1	123,7	59,4	9,4
6 mm	29,2	640,1	22,4	99,6	253,2	16,8	123,7	59,6	9,4
8 mm	29,6	640,1	23,2	99,6	254,3	17,7	123,6	59,8	9,5
10 mm	30,1	640,1	24,2	99,5	255,6	18,7	123,6	60,0	9,5
12 mm	30,7	640,1	25,4	99,5	257,3	19,9	123,6	60,4	9,6
14 mm	31,4	640,1	26,8	99,5	259,6	21,3	123,6	60,7	9,7
16 mm	32,3	640,1	28,3	99,4	261,9	22,8	123,5	61,2	9,8
18 mm	33,2	640,1	29,9	99,4	264,4	24,4	123,5	61,7	9,9
20 mm	34,2	640,1	31,5	100,7	267,2	26,0	123,5	62,2	10,0
22 mm	35,2	640,1	33,3	100,7	270,2	27,7	123,4	62,8	10,2
24 mm	36,4	640,1	35,1	100,8	273,5	29,5	123,4	63,4	10,4
26 mm	37,6	640,1	37,0	100,8	277,0	31,3	123,3	64,1	10,6
28 mm	38,8	640,1	39,0	100,9	280,8	33,1	123,3	64,9	10,8
30 mm	40,1	640,1	41,0	100,9	284,8	35,0	123,2	65,7	11,1
32 mm	41,5	640,2	43,0	101,0	289,0	36,9	123,2	66,5	11,4
34 mm	42,9	640,2	45,1	101,1	293,4	38,8	123,1	67,4	11,7
36 mm	44,3	640,2	47,2	101,2	298,0	40,7	123,1	68,3	12,0
38 mm	45,8	640,2	49,3	101,2	302,8	42,7	123,0	69,3	12,4
40 mm	47,3	640,2	51,4	101,3	307,8	44,7	122,9	70,3	12,7
42 mm	48,8	640,2	53,6	101,4	312,9	46,7	122,9	71,3	13,1
44 mm	50,4	640,2	55,7	101,5	318,2	48,7	122,8	72,4	13,4
46 mm	51,8	640,3	57,8	101,7	323,7	50,5	122,6	77,8	13,1
48 mm	53,4	640,3	60,0	101,8	329,3	52,5	122,6	/	13,5
50 mm	55,0	640,3	62,2	101,9	335,1	54,5	122,5	80,5	13,8
52 mm	56,6	640,3	64,4	102,0	341,0	56,5	122,4	81,9	14,2
54 mm	58,3	640,3	66,6	102,1	347,0	58,6	122,3	83,3	14,6
56 mm	59,9	640,3	68,8	102,2	353,1	60,6	122,2	84,8	15,0
58 mm	61,6	640,4	71,1	102,3	359,3	62,6	122,1	86,2	15,4

Table 13. E_n values (values exceeding an absolute value of 1,0 are marked in red).

Positions	Participants								
	MIRS/UM- FS/LTM	DTI	DFM	CEM	JV	METAS	BEV	VSL	VTT MIKES
	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm	U/nm
2 mm	0,1	0,2	0,2	0,1	0,4	1,4	0,1	0,3	1,1
4 mm	0,5	0,2	0,9	0,3	0,6	1,0	0,0	0,1	0,6
6 mm	0,4	0,2	0,6	0,5	1,0	0,2	0,2	0,1	0,2
8 mm	0,2	0,2	0,5	0,5	1,1	0,2	0,2	0,0	0,2
10 mm	0,2	0,3	0,2	0,8	1,3	0,6	0,0	0,2	0,8
12 mm	0,1	0,5	0,5	1,0	1,4	0,7	0,2	0,1	0,5
14 mm	0,0	0,7	0,5	1,2	1,6	0,2	0,1	0,4	0,1
16 mm	0,2	0,8	0,0	1,2	1,7	0,2	0,0	0,7	0,5
18 mm	0,4	0,8	0,3	1,4	1,9	0,5	0,3	0,4	0,1
20 mm	0,2	1,0	0,0	1,5	2,0	0,2	0,6	0,9	0,2
22 mm	0,2	1,0	0,1	1,6	2,1	0,2	0,2	0,6	0,2
24 mm	0,1	1,0	0,0	1,6	2,1	0,5	0,5	0,9	0,0
26 mm	0,2	1,3	0,1	1,6	2,2	0,2	0,5	1,1	0,4
28 mm	0,1	1,3	0,2	1,7	2,3	0,1	0,5	1,1	0,2
30 mm	0,2	1,5	0,0	1,9	2,5	0,1	0,1	0,7	0,3
32 mm	0,1	1,7	0,1	1,9	2,5	0,3	0,3	1,0	0,2
34 mm	0,1	1,7	0,0	1,9	2,6	0,1	0,2	1,2	0,4
36 mm	0,0	1,8	0,0	1,9	2,7	0,2	0,4	1,2	0,6
38 mm	0,0	1,7	0,0	2,0	2,8	0,1	0,2	1,3	0,6
40 mm	0,0	2,0	0,1	2,0	2,7	0,1	0,4	1,5	0,6
42 mm	0,1	2,0	0,1	2,0	2,8	0,0	0,2	1,5	0,6
44 mm	0,3	2,2	0,2	1,9	2,7	0,2	0,1	1,6	0,3
46 mm	0,1	2,4	0,3	2,1	2,8	0,2	0,0	1,8	0,2
48 mm	0,0	2,6	0,1	2,1	2,7	0,1	0,1	/	0,1
50 mm	0,0	2,4	0,1	2,1	2,7	0,1	0,3	2,1	0,1
52 mm	0,2	2,8	0,3	2,2	2,6	0,1	0,3	2,5	0,3
54 mm	0,1	2,8	0,0	2,0	2,6	0,2	0,4	2,4	0,0
56 mm	0,1	2,8	0,0	1,9	2,5	0,1	0,5	2,7	0,0
58 mm	0,1	3,0	0,3	1,9	2,6	0,1	0,3	2,6	0,2

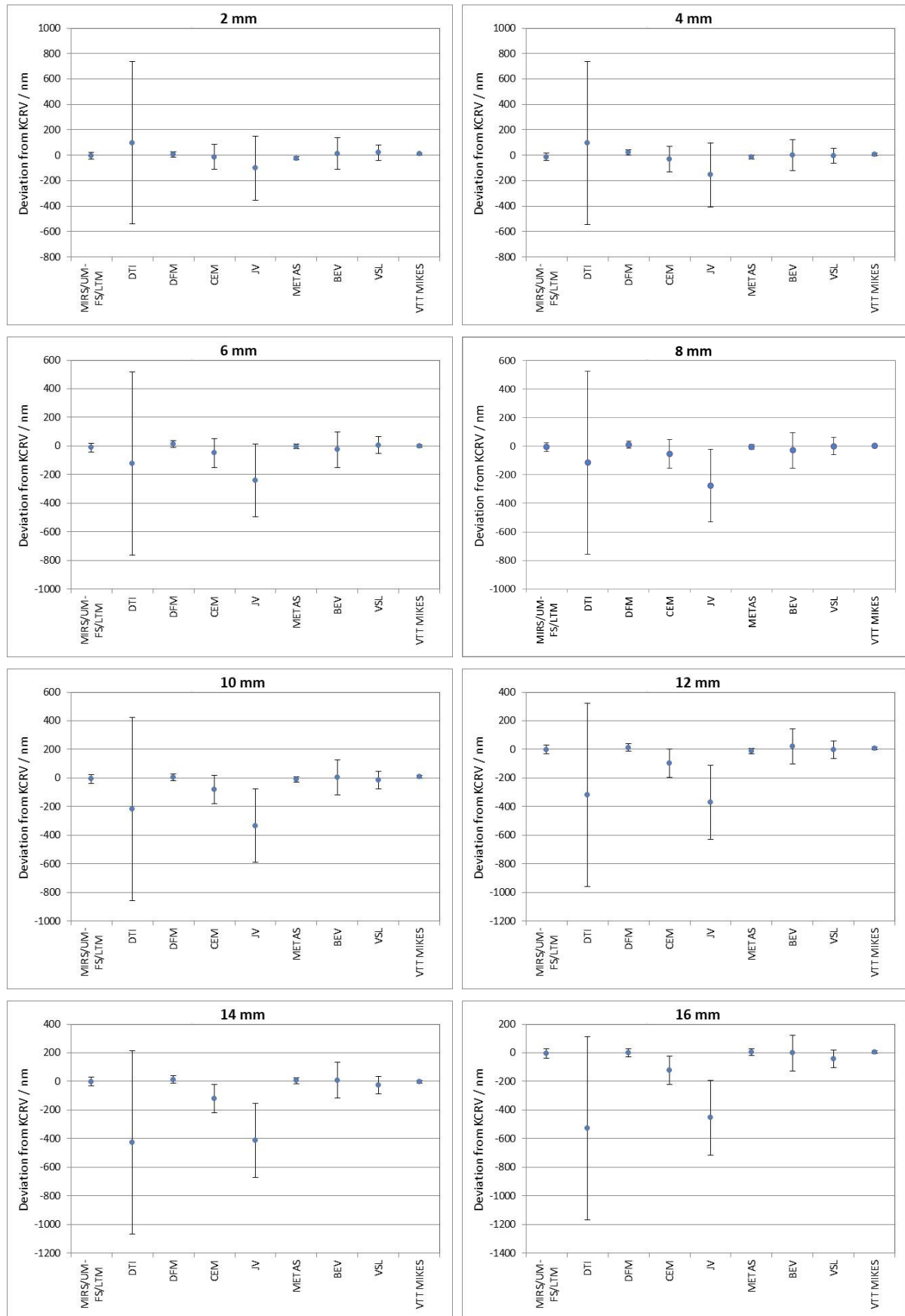


Figure 5. Deviations of the reported values from the CRV in points 2 mm to 16 mm. The error bars indicate the expanded measurement uncertainties ($k = 2$) of the deviation from the CRV.

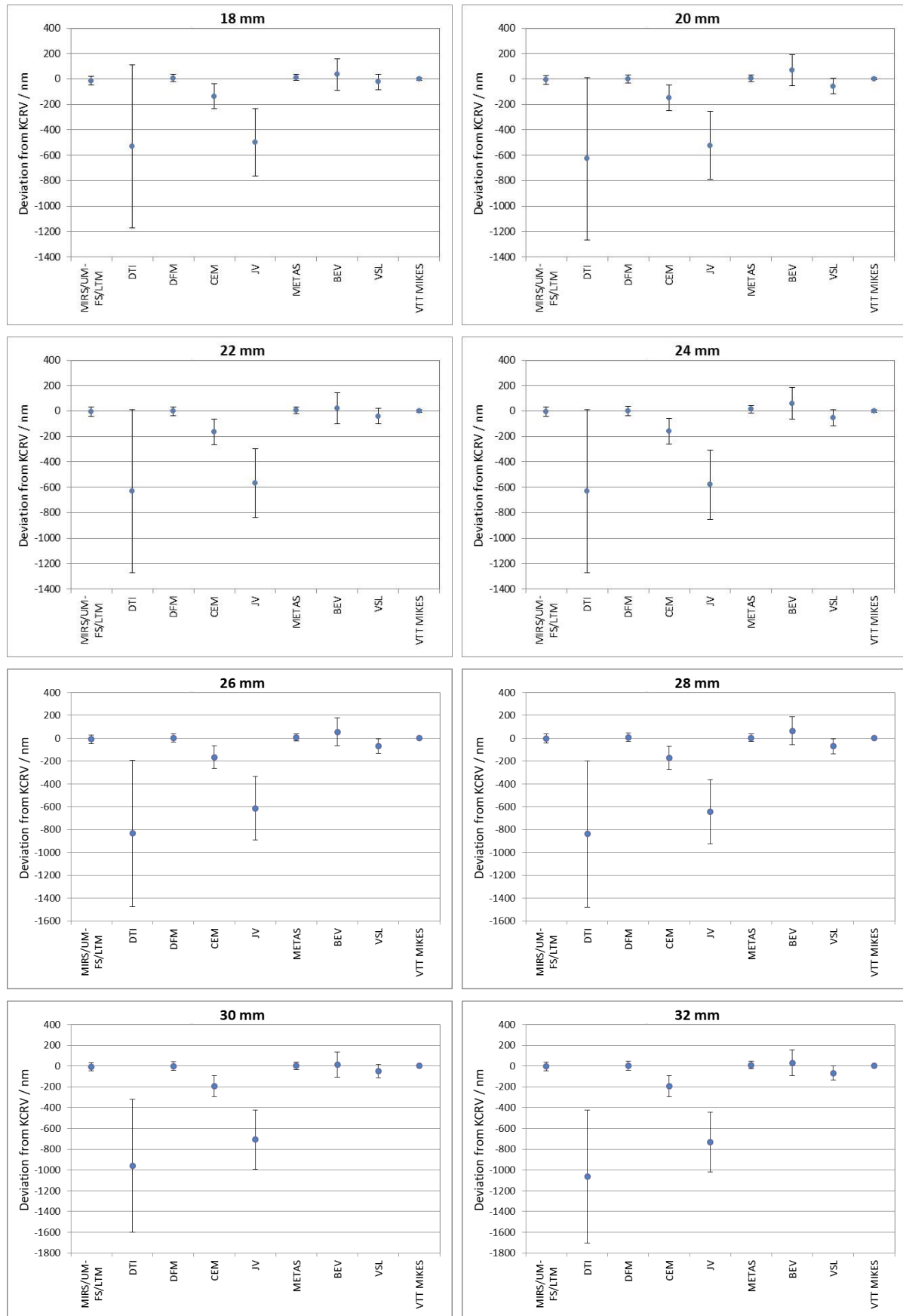


Figure 6. Deviations of the reported values from the CRV in points 18 mm to 32 mm. The error bars indicate the expanded measurement uncertainties ($k = 2$) of the deviation from the CRV.

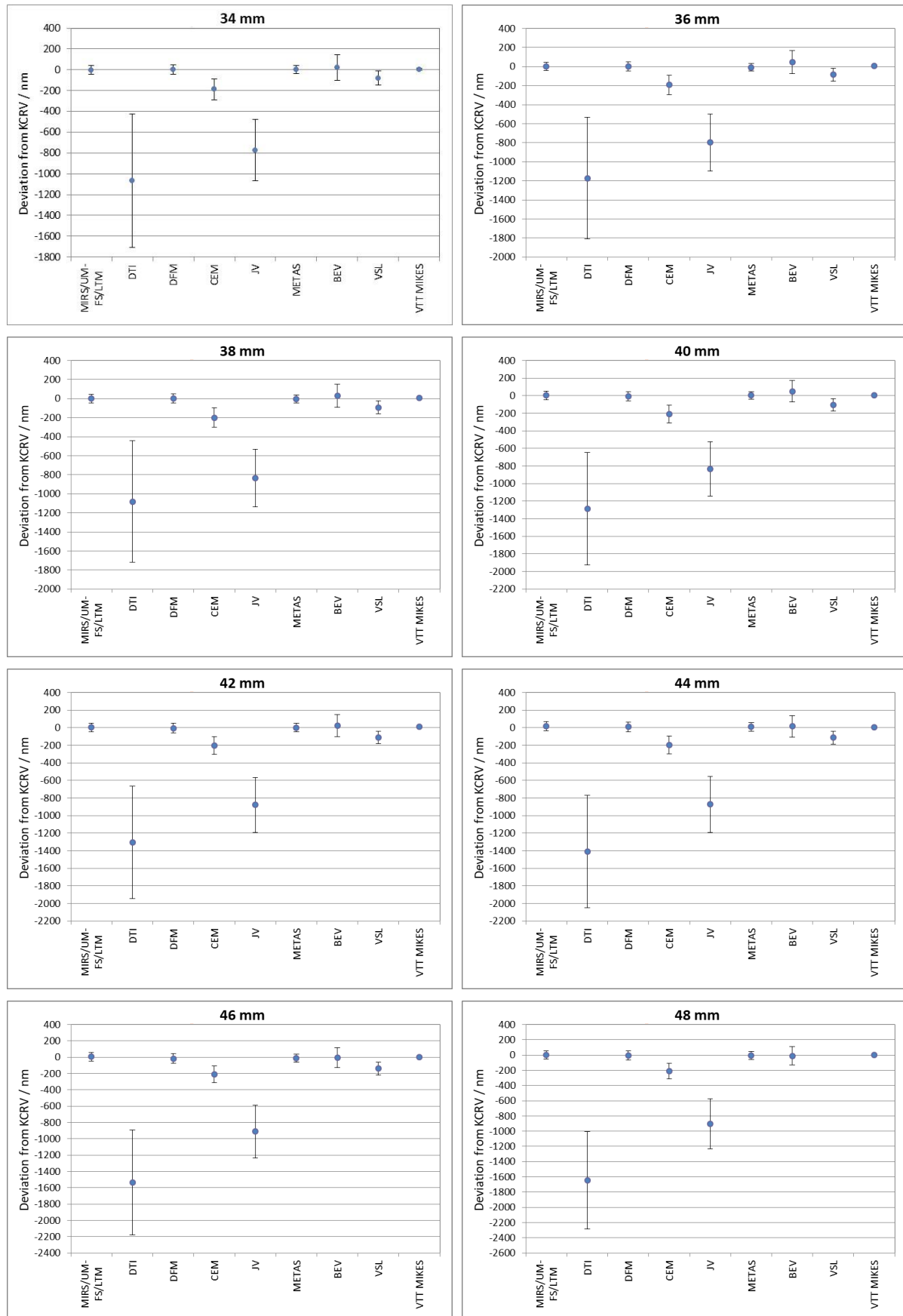


Figure 7. Deviations of the reported values from the CRV in points 34 mm to 48 mm. The error bars indicate the expanded measurement uncertainties ($k = 2$) of the deviation from the CRV.

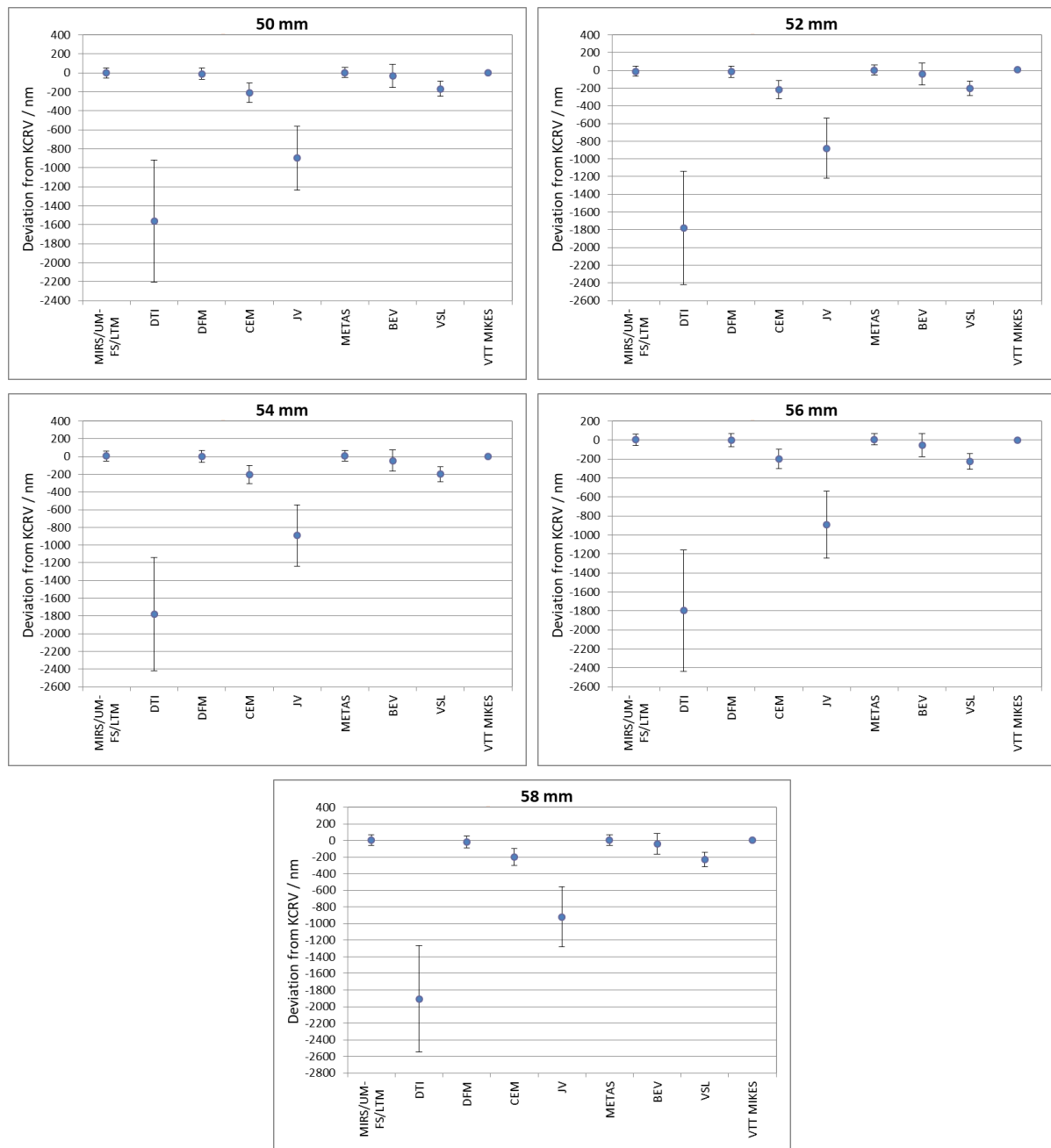


Figure 8. Deviations of the reported values from the CRV in points 50 mm to 58 mm. The error bars indicate the expanded measurement uncertainties ($k = 2$) of the deviation from the CRV.

7.4 Discussion of results

The degrees of equivalence (DoE) represented by E_n values for each participant's results, are summarised in Table 13. Five out of nine participants have more than 95 % of E_n values within the acceptance limits ($E_n \leq 1,0$). Four participants don't meet the equivalence criterion with no significant indication on the reason for this. These participants have used different types of equipment and different orientations of the transducer (2 labs in horizontal and 3 labs in vertical position). They have performed the measurements in very different time intervals (DTI quite at the beginning of the loop in June 2021, while VSL has repeated their measurements in April 2022 – near to the end of the loop). Therefore, no time related influence can be indicated. Furthermore, these 4 participants didn't meet the equivalence criterion in quite many measurement points (from 50 % to 79 %).

Only two participants (VSL and METAS) have their CMCs already published in the BIPM KCDB. Both laboratories have reported slightly better measurement uncertainties in this comparison from the published CMCs in the KCDB.

7.5 Linking of result to other comparisons

The results of this comparison are not linked to any other comparison.

8 References

- [1] CIPM MRA-G-11: Measurement comparisons in the CIPM MRA, 2021
- [2] M G Cox: 2002 Metrologia 39 589 The evaluation of key comparison data, 2002
- [3] F. Pollinger: EURAMET 1433 (EURAMET.L-S27/EURAMET.L-S2.3.n01) Draft A.2 Report, 2022
- [4] A. Lewis, T. Coveney: CCL/WG-MRA/GD-3 Guide to preparation of Key Comparison Reports in Dimensional Metrology
- [5] A Lewis et al: CCL-GD-3.2 KC Report Template, 1999

Appendix A1 Reported measurement uncertainties

Uncertainty budgets presented in this appendix are copied directly from the participant's reports. No special instructions for reporting the uncertainty budgets were given in the technical protocol, only an empty template table was created and handed over by the pilot.

MIRS/UM-FS/LTM Slovenia

<i>Quantity</i> X_i	<i>Quantity estimate</i> x_i	<i>Standard uncertainty</i> $u(x_i)$	<i>Sensitivity coefficient</i> $c_i = \partial f / \partial x_i$	<i>Uncertainty contribution</i> $u_i(B)$
<i>Error in reading the result on the transducer</i> L_T	0 nm	3 nm	1	3 nm
<i>Thermal expansion coefficient of the transducer</i> α_T	$8 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$	$0,58 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$	$0,1 \text{ } ^\circ\text{C} \cdot L$	$6 \cdot 10^{-8} \cdot L$
<i>Temperature deviation of the transducer</i> θ_T	0 $^\circ\text{C}$	0,02 $^\circ\text{C}$	$8 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1} \cdot L$	$2 \cdot 10^{-7} \cdot L$
<i>Laser interferometer indication</i> L_{LI}	L	$10 \text{ nm} + 3 \cdot 10^{-7} \cdot L$	1	$10 \text{ nm} + 3 \cdot 10^{-7} \cdot L$
<i>LI cosine error influence</i> e_{\cos}	0 nm	$3 \cdot 10^{-7} \cdot L$	1	$3 \cdot 10^{-7} \cdot L$
<i>Dead path contribution (LI)</i> e_{mp}	0 nm	10 nm	1	10 nm
<i>Transducer cosine error influence</i> $e_{\cos T}$	0 nm	$6 \cdot 10^{-8} \cdot L$	1	$6 \cdot 10^{-8} \cdot L$

Combined standard uncertainty: $u_c(B) = \sqrt{(15 \text{ nm})^2 + (5 \cdot 10^{-7} \cdot L)^2}; k = 2$

Expanded uncertainty (95 %): $U(B) = \sqrt{(30 \text{ nm})^2 + (10^{-6} \cdot L)^2}; k = 2$

BEV Austria

<i>Quantity</i> X_i	<i>Quantity estimate</i> x_i	<i>Standard uncertainty</i> $u(x_i)$	<i>Sensitivity coefficient</i> $c_i = \partial f / \partial x_i$	<i>Uncertainty contribution</i> $u_i(B)$
LI	0	$L \cdot 0,5 \cdot 10^{-6}$	1	(0 – 30) nm
alignement	0	20 nm	1	20 nm
repeatability	0	(20 – 40) nm	1	40 nm
Vertical calibration	0	(0 – 30) nm	1	(0 – 30) nm

Combined standard uncertainty: $u_c(B) = 62 \text{ nm}$

Expanded uncertainty (95 %): $U(B) = 124 \text{ nm}$

DTI Denmark

Uncertainty budget was not reported in the requested form.

Combined standard uncertainty: $u_c(B) = \pm 0,00032$ mm

Expanded uncertainty (95 %): $U(B) = \pm 0,00064$ mm

VSL The Netherlands

Parameter	Value	Unit	Standard unc.	Comb. Stand. Unc. Std onz	Sensitivity	Unit	Length independent /um	Length dependent
1 Transducer reading								
Resolution (zero points and en measurement point)	0.01	um	0.004					
Zero point drift	0.02	um	0.012					
				0.012		1 --		0.012
2 Laser reading								
Resolution (zero points and en measurement point)	0.01	um	0.004					
Standard uncertainty counting system	0.008	um	0.008					
Repeatability	0.01	um	0.010					
				0.013		1 --		0.013
3 Laser wavelength								
Air temperature								
Standard uncertainty thermistors	0.05	K	0.050					
Max drift during measurement	0.2	K	0.115					
				0.126	9.56E-07	K ¹ x L		1.20E-07 x L
Air pressure								
Standard uncertainty barometer	0.1	mbar	0.100					
Max drift during measurement	1	mbar	0.577					
				0.586	2.68E-07	mbar ¹ x L		1.57E-07 x L
Humidity								
Standard uncertainty measurement system	2.5	%RV	2.500					
Max drift during measurement	5	%RV	2.887					
				3.819	8.46E-09	%RV ¹ x L		3.23E-08 x L
Edlen equation								
Uncertainty in the Edlen correction	1.00E-08	--	1.00E-08					
				1.00E-08		1 x L		1.00E-08 x L
Laser wavelength								
Relative standard uncertainty	2.05E-09	--	2.05E-09					
Max drift after recalibration	4.10E-09	--	2.37E-09					
				3.13E-09		1 x L		3.13E-09 x L
4 Dead path								
Dead path length	75	mm						
Air temperature								
Max drift during measurement	0.2	K	0.115					
				0.115	9.56E-07	K ¹ x Ldp		
Air pressure								
Max drift during measurement	0.5	mbar	0.289					
				0.289	2.68E-07	mbar ¹ x Ldp		
Humidity								
Max drift during measurement	5	%RV	2.887					
				2.887	8.46E-09	%RV ¹ x Ldp		
Total effect for given dead path			um					0.010
5a Laser alignment								
Lateral translation laser spot over alignment distance	0.1	mm						
Alignment distance	100	mm	7.22E-08					
				7.22E-08		1 x L		7.22E-08 x L
5b Alignment transducer								
Radiale displacement of probe tip over pinole surface	0.1	mm						
Alignment distance	100	mm	2.89E-07					
squareness error pinole over 8 mm	0.0040	mm	2.89E-07					
				4.08E-07		1 x L		4.08E-07 x L
6 Transducer temperature								
Material								
Standard uncertainty thermistors	0.05	K	0.050					
Max drift during measurement	0.1	K	0.058					
				0.076	0.00E+00	K ¹ x L		0.00E+00 x L
Thermal expansion								
Uncertainty in the TEC	1.00E-06	K ¹ x L	5.77E-07					
Deviation from standard temperature of 20 °C				5.77E-07		0.3 K		1.73E-07 x L
7 Abbe error correction								
ABBE-offset horizontal	0.5	mm						
Yaw	0.003	o	0.015					
ABBE-offset vertical	0.5	mm						
Pitch	0.003	o	0.015					
				0.021		1 --		0.021
							0.030	4.92E-07

Combined standard uncertainty: $u_c(B) = Q[0.03, 0.5E-06 L]$

Expanded uncertainty (95 %): $U(B) = Q[0.06, 1.0E-06 L]$

CMC in KCDB is $U = Q[0.1, 1E-06 L]$ for $k = 2$.

DFM Denmark

Quantity X_i	Quantity estimate x_i	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c_i = \partial f / \partial x_i$	Uncertainty contribution $u_i(B)$
Pressure (hPa)	1015.70	0.30	-15.5	-4.64
Temperature (°C)	21.79	0.15	61.0	9.21
Relative humidity (%)	46.3	5.0	0.636	3.18
Laser wavelength vacuum (nm)	632.991 452	0.000 020	91617.0	1.83
Cosine error (factor)	0.999 999 78	0.000 000 31	-57 992 765.2	-17.86
Beam displacement (Abbe's error) (mm)	0.50	0.25	65.0	16.24
Stage rotation (µrad/mm)	1.12	0.56	29.0	16.24
Polarization mixing and electronic errors (nm)	0	1.0	1.00	1.00
Interferometer counts*	187 682 068	55.7	0.309	17.21
Instrument (DUT) resolution (nm)	0	2.9	1.00	2.89
Thermal expansion coefficient DUT scale /K	1.00E-07	5.0E-08	103 749 015.8	5.19
Thermal expansion coefficient DUT plunger /K	1.00E-06	3.0E-07	10 798 166.5	3.24
DUT plunger length (mm)	70	10	0.154	1.54
Temperature drift during calibration (°C)	0.154	0.077	70.0	5.39
			$u_c(B)$	36.6

* includes repeatability

Uncertainty budget above is representative at $L = 58$ mm.

Combined standard uncertainty: $u_c(B) = \sqrt{(11 \text{ nm})^2 + \left(0.58 \text{ nm} \frac{L}{1 \text{ mm}}\right)^2}$

Expanded uncertainty (95 %): $U(B) = 2.1 \sqrt{(11 \text{ nm})^2 + \left(0.58 \text{ nm} \frac{L}{1 \text{ mm}}\right)^2}$

CEM Spain

Values expressed in nm.

<i>Quantity</i> X_i	<i>Quantity estimate</i> x_i	<i>Standard uncertainty</i> $u(x_i)$	<i>Sensitivity coefficient</i> $c_i = \partial f / \partial x_i$	<i>Uncertainty contribution</i> $u_i(B)$
Repeatability	46	16	1	16
Resolution	10	3	1	3
Interferometer Uncert.	29	15	1	15
Stage effects	70	40	1	40
Thermal expansion	29	8	1	8

Combined standard uncertainty: $u_c(B) = 50$ nm

Expanded uncertainty (95 %): $U(B) = 100$ nm

JV Norway

$$Bias = (L_{T,Pos} - L_{T,0}) - (L_{Ref} + \delta L_{interf,nonlin}) + \delta L_{Alignx} + \delta L_{Aligny} + \delta L_{T,Temp} + \delta L_{Pos,Target}$$

Quantity X_i	Quantity estimate x_i	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c_i = \partial f / \partial x_i$	Uncertainty contribution $u_i(B)$
Reference laser, double pass interferometer, wavelength	L	$1 \cdot 10^{-10}$ Relative to L	-L	$-10^{-10} L$
Refractice index of air	n(T,p,RH%)	$0.87 \cdot 10^{-6}$ Relative to L	-L	$-0.87 \cdot 10^{-6} L$
Interferometer nonlinearity *)	0 nm	100 nm	-1	-100 nm
Alignment, x: 0,1 mm/100 mm	0 nm	$1.4 \cdot 10^{-6} L$	1	$1.4 \cdot 10^{-6} L$
Alignment, y: 0,1 mm/100 mm	0 nm	$1.4 \cdot 10^{-6} L$	1	$1.4 \cdot 10^{-6} L$
Transducer position at target, Res	0 nm	3 nm	1	3 nm
Transducer position at 0, Res	0 nm	3 nm	-1	- 3 nm
Transducer repeatability	0 nm	42 nm	1	42 nm
Nonexact positioning of probe at targets	0 nm	10 nm	1	10 nm
Temperature variation during measurement	0 nm	64 nm	1	64 nm

* Interferometer nonlinearity is not measured, this is a conservative estimate.

Combined standard uncertainty: $u_c(B) = Q[126 \text{ nm} , 2.2 \cdot 10^{-6} L]$

Expanded uncertainty (95 %): $U(B) = Q[252 \text{ nm} , 4.4 \cdot 10^{-6} L]$

METAS Switzerland

The uncertainty estimation considers 15 known contributions from various sources such as the interferometer (wavelength, refractive index with p, H, t, Edlen and resolution), the stage (short and long range angular motions), the alignment (cosine errors and Abbbe offsets), temperature and the DUT (resolution and repeatability).

The given results are averages of several measurement cycles.

Quantity X_i	Quantity estimate x_i	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c_i = \partial f / \partial x_i$	Uncertainty contribution const /nm $u_i(B)$	Uncertainty contribution rel. $u_i(B)$
Laser frequency	λ	2E-09	1		2E-09
Resol. Interferometer	5 nm	2.9 nm	1	2.9	
Air pressure	p	0.87 mbar	0.27E-6L /mbar		2.34E-07
Air temperature	t	0.23 K	0.92E-6L/K		2.12E-07
Air humidity	rH	5.80 %	0.01E-6L /%		5.8E-08
Edlen	n	1.15E-08	1		1.2E-08
Cos. Error Laser lin	$\cos \alpha$	2.9E-04	1.5E-04		4.2E-08
Cos. Error transducer lin	$\cos \alpha$	2.9E-04	1.5E-04		4.2E-08
Abbe offset sort range	dA	0.46 mm	7.7E-06	3.5	
Abbe offset lin	dA	0.46 mm	1.0 E-6 /mm		4.6E-07
Repeatability	s	7.0 nm	1	7.0	
Resol. Tansducer	5 nm	2.9 nm	1	2.9	
Temp cal.	Tc	0.015 K	2E-7 /K		3.0E-09
Transducer Temp	Tt	0.12 K	2E-7 /K		5.8E-09
CTE Transducer	$d\alpha$	5.8E-7/K	0.12 K		6.96E-08

Combined standard uncertainty: $u_c(B) = \sqrt{(8.9 \text{ nm})^2 + (5.68 \cdot 10^{-7} * L)^2}$

Expanded uncertainty (95 %): $U(B) = \sqrt{(18 \text{ nm})^2 + (1.1 \cdot 10^{-6} * L)^2} \quad (k = 2)$

VTT MIKES Finland

Quantity X_i	Quantity estimate x_i	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c_i = \partial f / \partial x_i$	Uncertainty contribution $u_i(B)$
B , repeatability	0	0.011 μm	0.41	0.004 μm
δL_{abbe}	0	0.014 μm	0.3	0.004 μm
δL_{cos}	0	5.2E-04 rad	2.6E-04 rad L	1.3E-07 L
L_R	0.058 m			
d , periodic nonlin	0	0.005 μm	0.41	0.002 μm
λ/n , equation & wavelength	1.00027	1.40E-08	1 L	1.4E-08 L
t , air temperature	20 °C	0.10 °C	9.6E-07 L/°C	9.6E-08 L
p , air pressure	1013 hPa	35 Pa	2.7E-09 L/Pa	9.5E-08 L
h , air humidity	47% rh	3%	8.5E-09 L/%	2.6E-08 L
δd_{cos}	0	3.0E-04 rad	1.5E-04 rad L	4.5E-08 L
α , TEC of transducer	0	1.0E-07 1/°C	0.20 °C L	2.0E-08 L
$\Delta t_{20\text{C}}$	20 °C	0.10 °C	1.0E-07 L/°C	1.0E-08 L

Combined standard uncertainty: $u_c(B) = Q[0.0066 \mu\text{m}; 2.0\text{E-}7\text{L}]$

Expanded uncertainty (95 %): $U(B) = Q[0.013 \mu\text{m}; 4.0\text{E-}7\text{L}]$

Appendix A2 Equipment and measuring procedures of the participants

Uncertainty budgets presented in this appendix are copied directly from the participant's reports.

MIRS/UM-FS/LTM Slovenia

The measurement set-up is constructed on a custom made Newport 3D stage (Fig. 1), where the motorised vertical axis (Z) is used for generating movement of the LI optics.

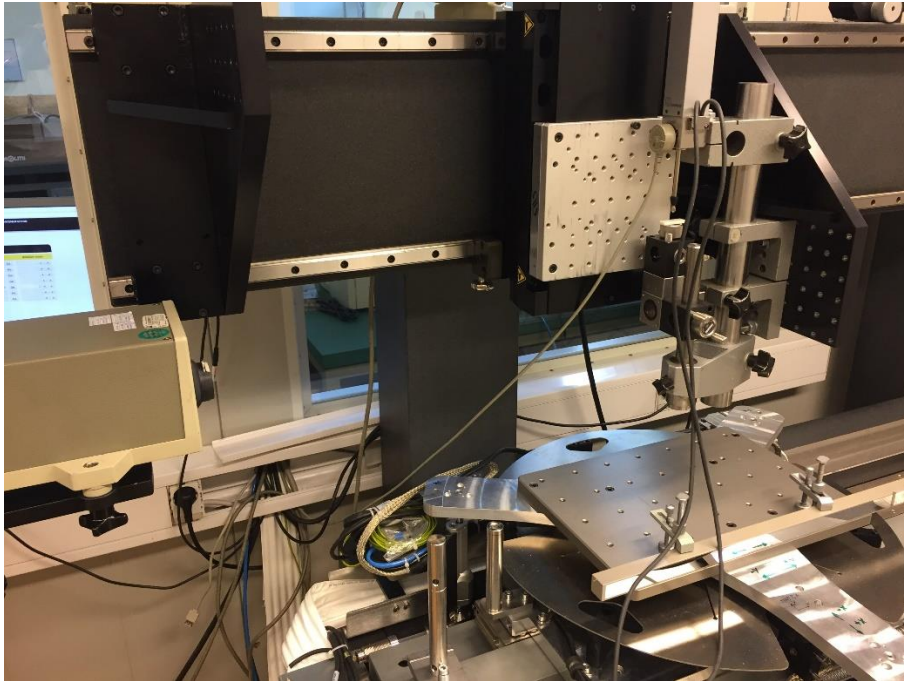


Figure 1: Measurement set-up for calibrating transducers

The transducer is mounted on a fixed holder and adjusted with the Z-stage movement by using a dial gauge with 1 μm resolution (Fig. 2). The adjustment is performed by following instructions in the technical protocol. The transducer was in fact oriented by using two vertical surfaces (coloured in Fig. 2) for practical reasons. Perpendicularity of these surfaces to the lower horizontal surface (defining the measurand) was checked on a CMM and was within few micrometres.

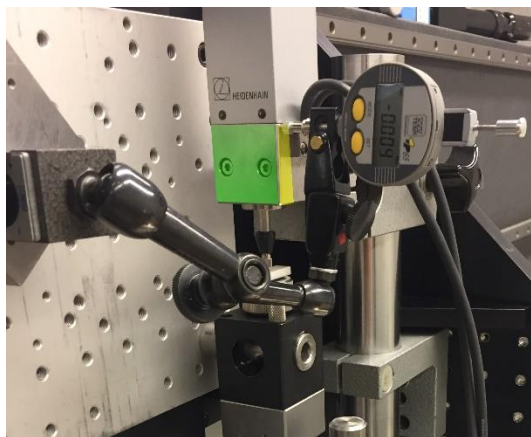


Figure 2: Adjustment of the transducer to the measurement direction (Z-stage movement)

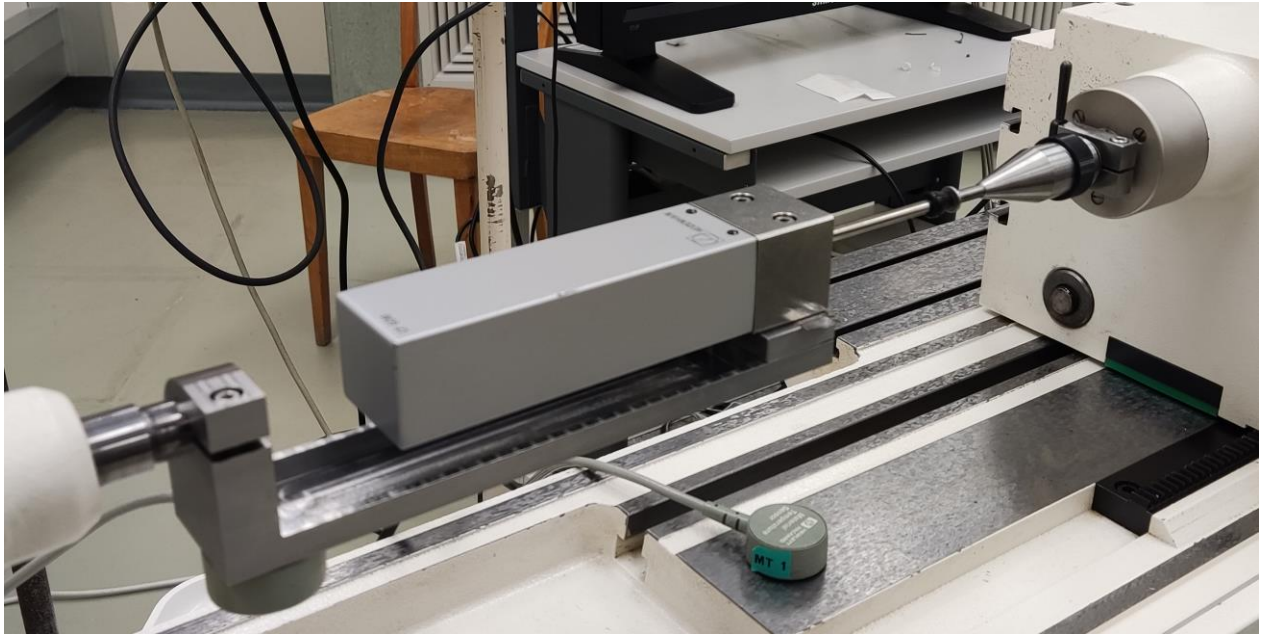
The laser beam is adjusted with the Z-stage movement by using a position sensing detector (Fig. 3). The direction can be adjusted within few micrometres on a 100 mm path, which produces a cosine error loer than $3 \cdot 10^{-7} \cdot L$.



Figure 3: Position sensing detector for adjusting the laser beam to the measurement direction

BEV Austria

The calibration procedure was a two step process. The prescribed nominal positions were calibrated using a linear length measuring machine equipped with a laser interferometer (SIP 3002). The transducer was mounted on holder firmly attached to the fixed probe of the length measuring machine. The transducer had to be calibrated in horizontal position this way. Since the motor was deactivated by the pilot, a proper contact between the machine's probe and the device under test had to be assured. For this reason the machine was carefully moved only in one direction. Only the inwards direction was realized for this comparison.



For the alignment of the transducer the reference faces were aligned using a dial gauge parallel to the machine axis. The alignment was better than $2 \mu\text{m} / 10 \text{mm}$.

During the measurement rather large linear deviations were found which could not be correlated to dimensional parameters.

For the correction of this effect we calibrated the transducer in vertical position (two-point calibration) using a stand as in Fig. 2 of the technical protocol and sets of calibrated gauge blocks with a range of 56.5 mm. The result of this calibration was combined with the values obtained by horizontal measurements.

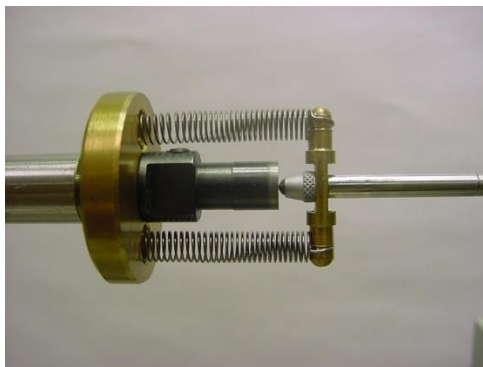
DTI Denmark

Calibration was performed with horizontal spindle and with the Laser interferometer mounted in a horizontal guide unit.

The reading is obtained at the moment the digit changes.



Contact with surface is made by a spring system.



We did not have an adapter, so we 3D printed one.



VSL The Netherlands

VSL normally calibrates displacement transducers using a 1D measurement machine (ULM 01-600D) where the position of the machine is measured with a laser interferometer. The measurements are performed in a cleanroom environment with controlled temperature and humidity conditions of respectively $(20.0 \pm 0.1) ^\circ\text{C}$ and $(45 \pm 5) \%RH$. With the standard procedure transducers are calibrated in a horizontal position. For this comparison, we have adapted the measurement procedure since the transducer in this comparison was designed to operate in a vertical direction. The procedure was augmented with a measurement cycle using end gauges with the transducer in the vertical direction, see fig. 1. This could however, only be done for a limited data set.

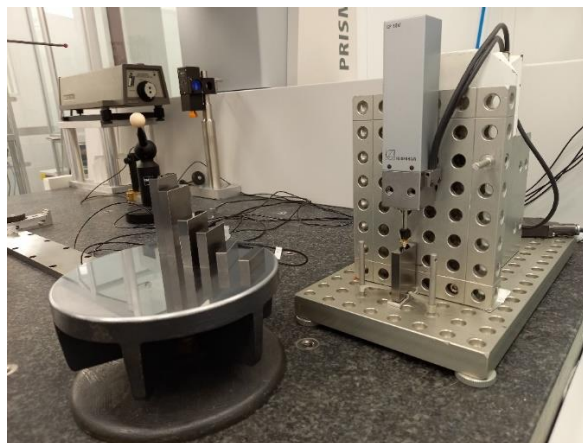


Figure 1. A set of end gauges was used to provide reference data for the vertical measurement position that were subsequently used to correct measurements performed in the horizontal direction.

In order to measure over the range and interval according to the protocol, that transducer was subsequently measured in the horizontal direction using our standard procedure with the ULM and laser interferometer, see fig. 2.



Figure 2. Calibrating the transducer in the horizontal position on the ULM with a laser interferometer.

Since we consider the procedure with the end gauges for this transducer as more appropriate, we used the calibration result with the end gauges as reference data to correct the data set obtained with the ULM and laser interferometer.

Transducers with an optical encoder are only calibrated for one measurement direction so the table contains only data for the outgoing measurement.

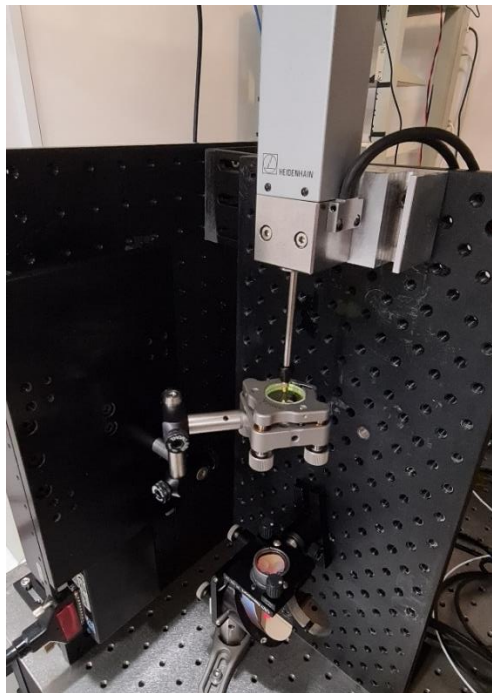
DFM Denmark

The transducer is calibrated using a laser interferometer (Zygo ZMI 2000). The transducer is mounted vertically. A motorised translational stage (Newport XPS) is used to move the transducer. The tip of the transducer rests on the back surface of a mirror. The front surface of the mirror is used as the measurement mirror in the displacement interferometer. The mirror is fixed to the translational stage.

The laser beams in the interferometer are aligned to be parallel to the transducer plunger axis. The residual angle (cosine error) is measured by attaching a position sensitive photodiode to the plunger and then moving the plunger up/down.

Residual rotations of the translational stage (pitch, yaw) are measured with the interferometer. The displacement between the plunger axis and the interferometer laser beams are measured visually. These two measurements are used to calculate the Abbe error.

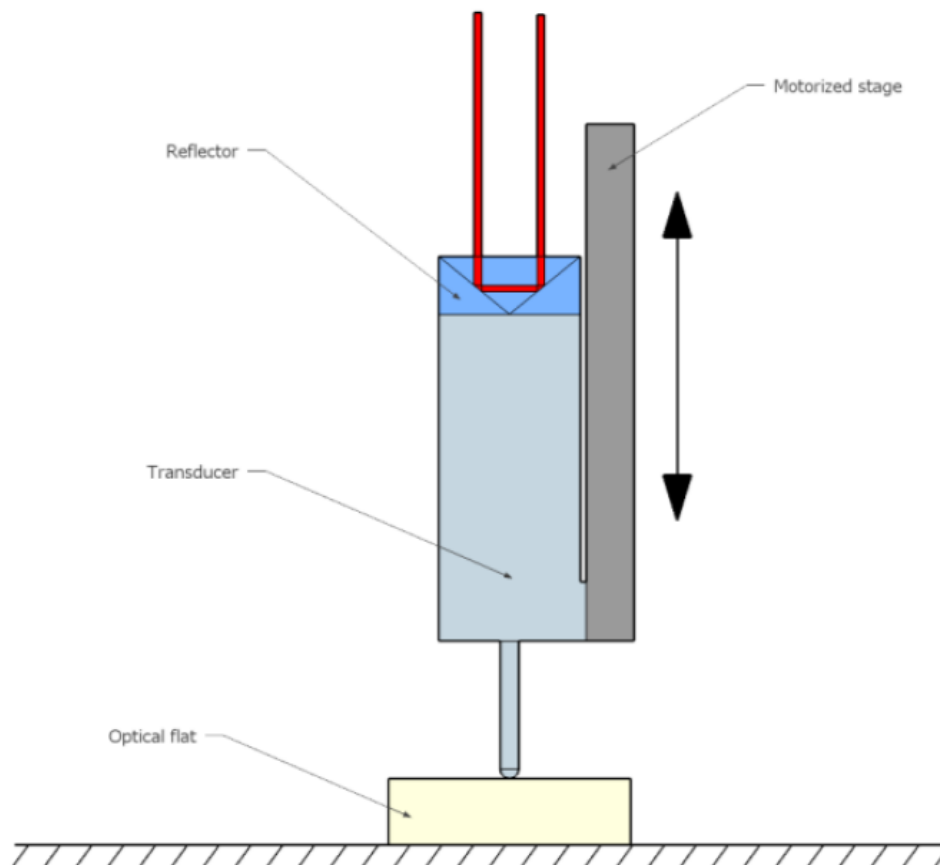
Positions are measured up and down in one run. Each run is repeated 4 times. Data from each run are corrected for linear drift.



CEM Spain

The measurements were carried out by mounting the transducer on motorized linear stage, which movements were simultaneously measured by an interferometer. Measurements were compensated calculating the refractive index by means of temperature, pressure and humidity sensors.

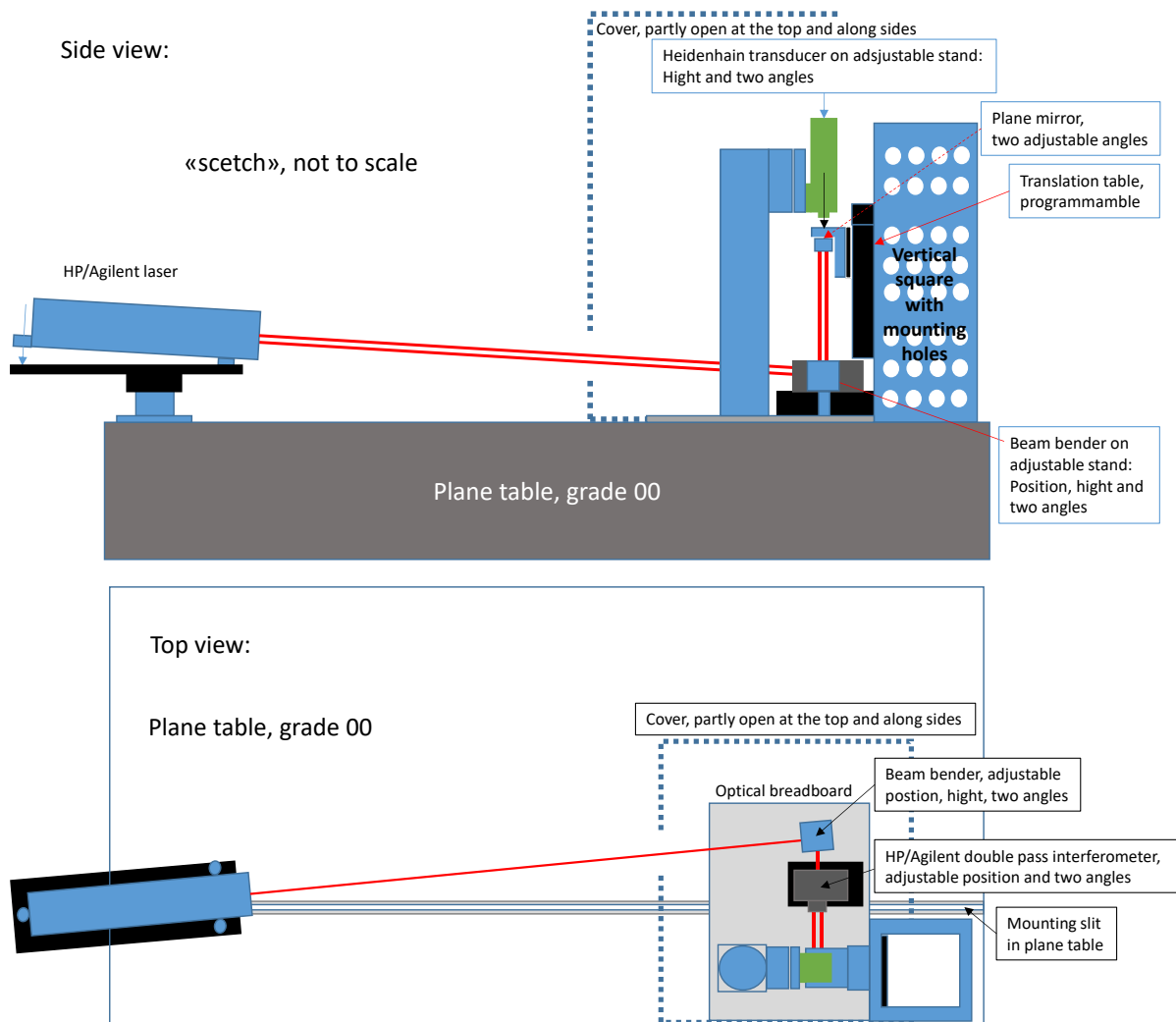
The transducer was mounted vertically and a reflector was fixed to the top of the housing, which was the moving element, while the probe was in contact with a static optical flat.



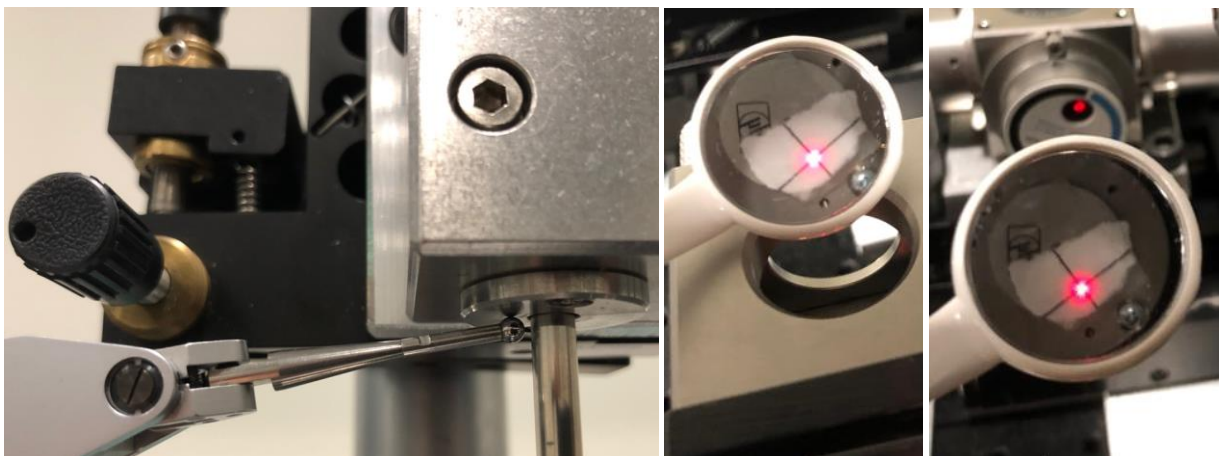
A rough alignment was performed by minimizing the difference between the readings of the interferometer and the transducer. A linear component in the measurements could not be completely eliminated, so the reported results are corrected eliminating the mentioned linear error.

Measurements were carried out in both directions (inwards and outwards).

JV Norway



Alignment of transducer in two angles with respect to the plane table using digital “*vippeindikator*”, eng: “tilt indicator”(?), with $0.1 \mu\text{m}$ resolution. Alignment of the transducer to the plane table is within $\text{ca } \pm 1 \mu\text{m} / 10 \text{mm}$. Alignment of the laser to the transducer is within $\text{ca } \pm 0.1 \text{mm} / 100 \text{mm}$ in x- and y-direction.



METAS Switzerland

The METAS built measurement setup calibrates electronic transducers in vertical orientation. The position of the motorized vertical stage is measured by an interferometer respecting the Abbe principle. The transducer is aligned with respect to its reference mounting surfaces and its probe is placed on a short gauge block on the moving part of the stage. The gauge block is serving as anvil with high surface quality and flatness. The plunger follows the anvil by its gravitational weight. The measurements are PC controlled and fully automated.

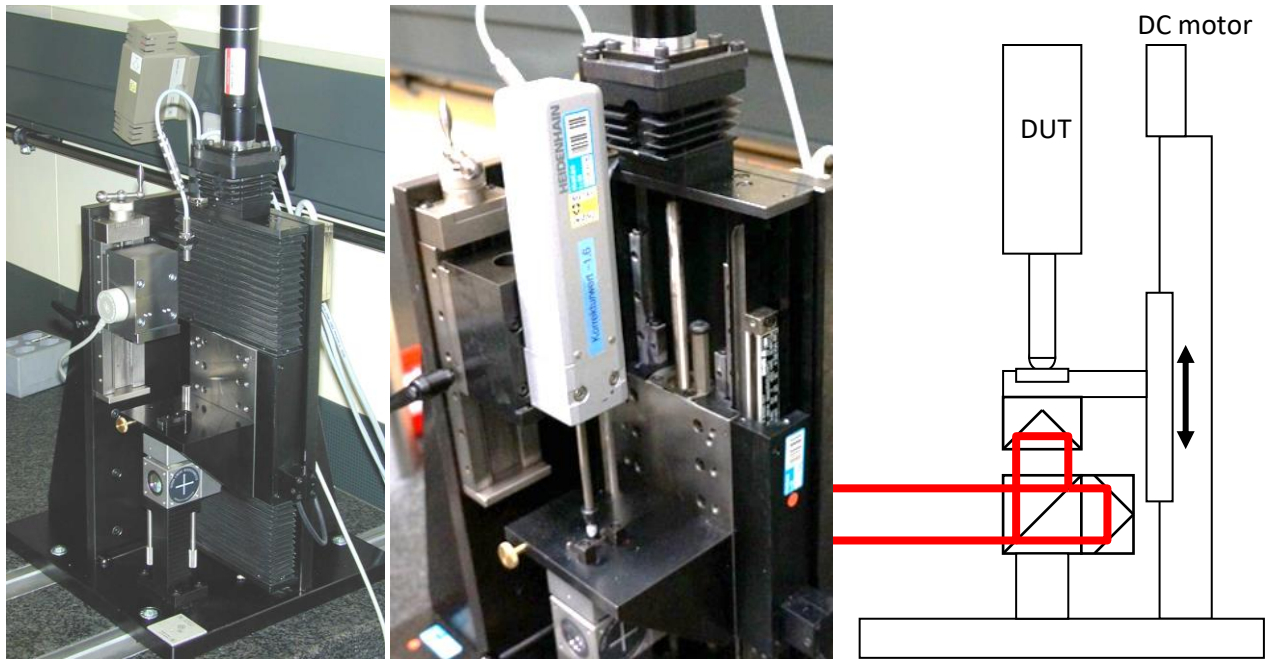


Figure 1: Measurement rig with and without DUT as well as a simple sketch with interferometer beams in red.

Measurement uncertainty estimation:

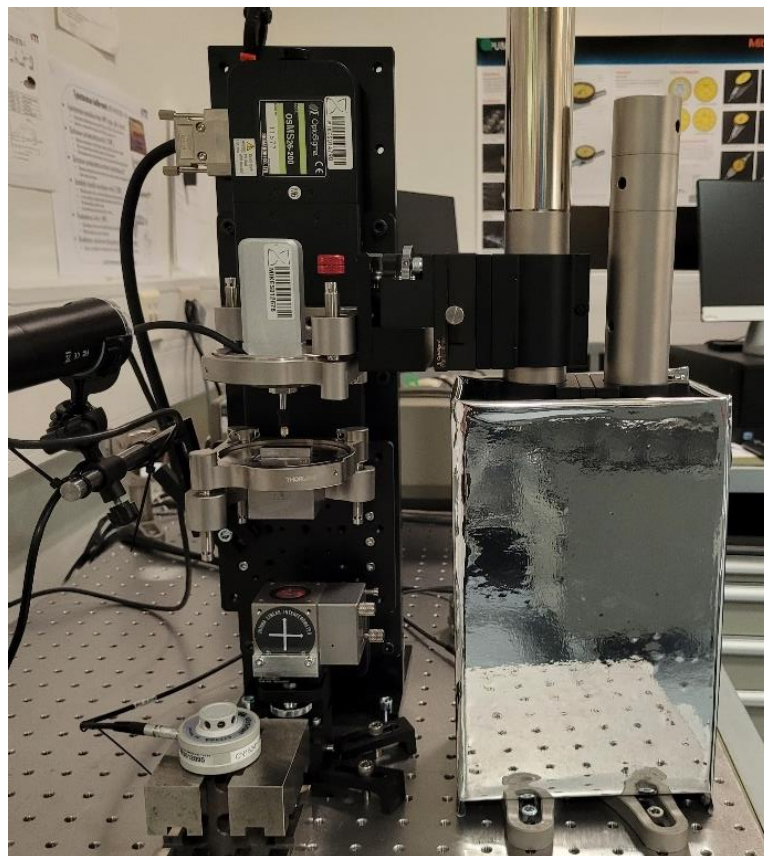
The uncertainty estimation considers 15 known contributions from various sources such as the interferometer (wavelength, refractive index with p , H , t , Edlen and resolution), the stage (short and long range angular motions), the alignment (cosine errors and Abbe offsets), temperature and the DUT (resolution and repeatability).

The final measurement uncertainty is length dependent with a constant value of 18 nm and a linear term of $1.1E-6$.

The given results in section 2 are averages of several measurement cycles.

VTT MIKES Finland

Purpose built interferometric measurement instrument is used for calibration of length transducers at VTT MIKES, see figure below. The instrument is consisted of 200 mm high quality vertical linear stage with adjustable mounting plate for interferometer corner cube pointing downwards and a short gauge block pointing upwards. A heterodyne laser interferometer beam is adjusted to the corner cube and parallel with the movement of the vertical stage. The surface of the gauge block is adjusted normal to movement axis of the linear stage using auxiliary laser pointer and position sensitive detector. Environmental parameters air pressure, temperature and humidity are measured, and the refractive index of air is adjusted accordingly during the measurements.



The encoder under test is fixed to a high-quality mirror holder with suitable mounting plate. It is adjusted, utilising a small microscope fixed to the carriage, so that its measurement axis is parallel with linear stage movement and concentric with laser interferometer measurement axis. The linear stage, laser interferometer and encoder are connected to a PC with a measurement software. The needed measurement parameters can be given to the software, which then carries out the measurement automatically. When doing the measurements overscan is used in beginning and turning points. During single inward or outward run the stage is moved only to single direction until needed measurement point is reached.

The results presented are averages of three separate measurement sequences with inward and outwards runs made at slightly different position of the linear stage. For each measurement sequence separate adjustment of the encoder position was made. The data offsets were adjusted so that encoder and reference reading in nominal 0 position are equal to 0.

The instrument is situated in accurately air-conditioned laboratory room.