

# Linking the results of the regional key comparison EURAMET.AUV.V-K3 to those of the CIPM key comparison CCAUV.V-K3

C. Hof Federal institute of metrology (METAS), Switzerland

Claire Bartoli Laboratoire National de métrologie et d'Essais (LNE), France

## Abstract

During 2014 and 2015, fourteen national metrology institutes (NMI) took part in the CIPM key comparison CCAUV.V-K3 on the primary calibration of the complex voltage sensitivity of an accelerometer in the low frequency range. The five Euramet participants in this comparison had also participated in the RMO comparison EURAMET.AUV.V-K3 involving a total of ten Euramet NMIs. The results of the five CIPM participants have been used to link the results of the remaining five NMIs to the results in the CIPM key comparison. The degrees of equivalence between the result for each NMI and the key comparison reference value (KCRV) has been calculated and the results are given in the form of a matrix and graph for the ten Euramet NMIs.

## 1. Introduction

As part of a major comparison programme [1], Euramet conducted the RMO key comparison EURAMET.AUV.V-K3 [2] on the calibration of an accelerometer QA 700. Ten laboratories participated in this comparison that took place between December 2011 and August 2013. The specific task of this comparison was the measurement of the magnitude and phase of the complex voltage sensitivity of the servo accelerometer QA 700, SN 39477 at specified frequencies and amplitudes in the low frequency domain (from 0.1 Hz to 200 Hz) as no comparison had ever been performed in this range. The voltage sensitivity was calculated as the ratio of the amplitude of the output of the accelerometer to the amplitude of the acceleration at its reference surface with primary means in accordance with ISO 16063-11 : 1999 “Methods for the calibration of vibration and shock transducers - Part 11: Primary vibration calibration by laser interferometry” [3].

Five participants of this comparison subsequently participated in the CIPM key comparison CCAUV.V-K3 [4] that took place between August 2014 and August 2015. This CIPM key comparison was designed to determine the exact same quantity of a different vibration measuring chain consisting of an accelerometer SA704, S/N 1040 in combination with a signal conditioner type MSA-I, S/N 02011001, both manufactured by NIM China. The frequency range in the CIPM key comparison reached from 0.1 Hz to 40 Hz specifying identical frequencies within the common subset of the range.

The fact that the same quantity was measured on a common subset of identical frequencies in both comparisons allows a detailed linking of the RMO measurement results to the CIPM key comparison to be readily calculated.

In the current report, the CIPM key comparison CCAUV.V-K3 is referred to as the CIPM comparison and the RMO key comparison EURAMET.AUV.V-K3 is referred to as

the RMO comparison.

**Table 1. Details of the participants in the EURAMET.AUV.V-K3**

<b>NMI</b>	<b>Full name</b>	<b>Country</b>	<b>Regional metrology organization</b>	<b>Participation in the CIPM comparison</b>
LNE (pilot)	Laboratoire National de métrologie et d'Essais	France	Euramet	yes
CMI	Czech Metrology Institute	Czech Republic	Euramet	
SP*	SP Technical Research Institute of Sweden	Sweden	Euramet	
METAS	Swiss federal office of metrology	Switzerland	Euramet	yes
INRIM	Istituto Nazionale di Ricerca Metrologica	Italy	Euramet	
GUM	Central Office of Measures	Poland	Euramet	yes
CEM	Centro Español de Metrología	Spain	Euramet	
PTB	Physikalisch-Technische Bundesanstalt	Germany	Euramet	yes
DPLA	Danish Primary Laboratory of Acoustics	Denmark	Euramet	yes
MIKES	Centre for Metrology and Accreditation	Finland	Euramet	

\* now "National Metrology Center RISE", (Sweden)

## 2. Model for the linkage

In order to link the results of the RMO comparison to those of CIPM comparison a well-established model [5, 6] was used.

The measurand in the CIPM comparison is denoted by  $X$ . The values  $x_1, u(x_1), \dots, x_N, u(x_N)$  denote the best estimates and associated standard uncertainties of the laboratories and  $x$  denotes the KCRV.

The measurand in the RMO comparison is denoted by  $Y$ . The values  $y_1, u(y_1), \dots, y_M, u(y_M)$  denote the best estimates and associated standard uncertainties of the laboratories and  $y$  denotes the weighted mean of the linking laboratories in the RMO comparison.

Furthermore,  $G = \{1, \dots, p\}$  ( $p \leq \min(N, M)$ ) is the index set of the linking laboratories.

The laboratories are labelled such that any number within  $G$  denotes the same laboratory in both comparisons.

The value  $R = X/Y$  denotes the linking coefficient between the two measurands to make the link between the two comparisons. The linking coefficient is estimated using the KCRV of the CIPM comparison and the combined results in the RMO comparison of the linking laboratories. The estimated linking coefficient is then applied to the results of the RMO comparison.

Any correlations of the results of the linking laboratories in both comparisons are considered as being negligible and as a consequence, the various estimators  $x_1, \dots, x_N, y_1, \dots, y_N$  are treated as being uncorrelated."

These quantities can then be expressed as

$$x = \frac{\sum_{l=1}^N \frac{x_l}{u^2(x_l)}}{\sum_{l=1}^N \frac{1}{u^2(x_l)}} \quad u^2(x) = \frac{1}{\sum_{l=1}^N \frac{1}{u^2(x_l)}} \quad (1)$$

$$y = \frac{\sum_{l \in G} \frac{y_l}{u^2(x_l)}}{\sum_{l \in G} \frac{1}{u^2(x_l)}} \quad u^2(y) = \frac{1}{\sum_{l \in G} \frac{1}{u^2(x_l)}} \quad (2)$$

Then  $R$  is estimated according to

$$r = \frac{x}{y} \quad u^2(r) = \frac{u^2(x)}{y^2} + \frac{x^2}{y^4} u^2(y) \quad (3)$$

$Z = RY$  denotes the linked measurand of the RMO comparison and

$$z_l = ry_l \quad u^2(z_l) = y_l^2 u^2(r) + r^2 u^2(y_l) + 2ry_l u(r, y_l) \quad l = 1, \dots, M \quad (4)$$

$$u(r, y_l) = \begin{cases} -\frac{x}{y^2} u^2(yr), & l \in G \\ 0, & \text{otherwise} \end{cases}$$

are the corresponding estimates including the associated uncertainties.

The degrees of equivalence are defined as the differences between the linked results of the RMO comparison and the KCRV of the CIPM comparison

$$d_i = z_i - x \quad i = 1, \dots, M \quad (5)$$

and the uncertainties associated with these differences where

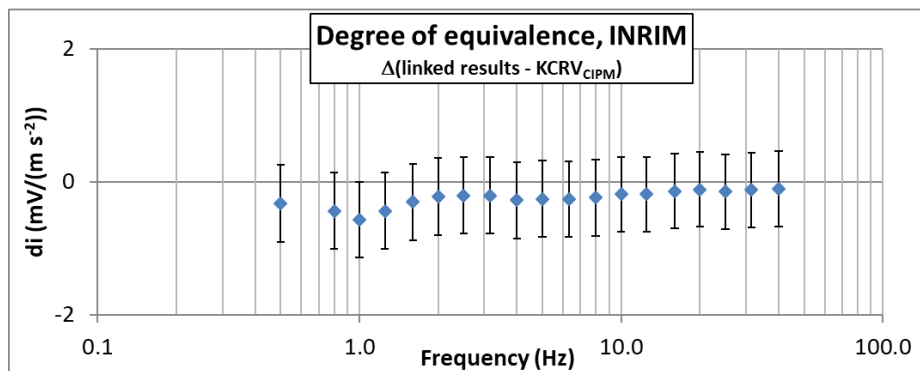
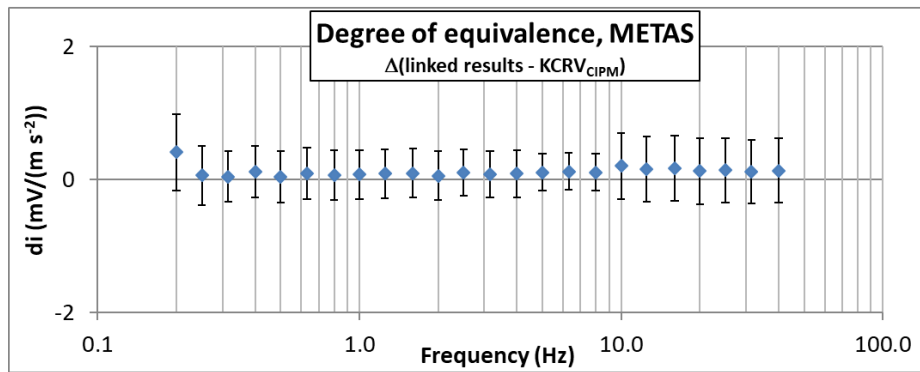
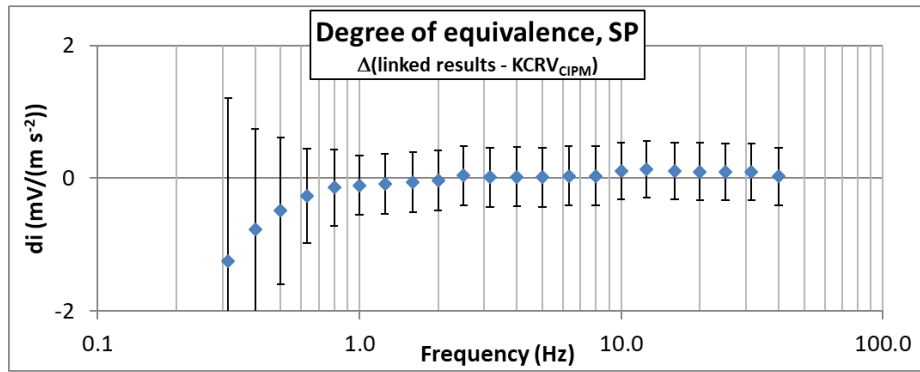
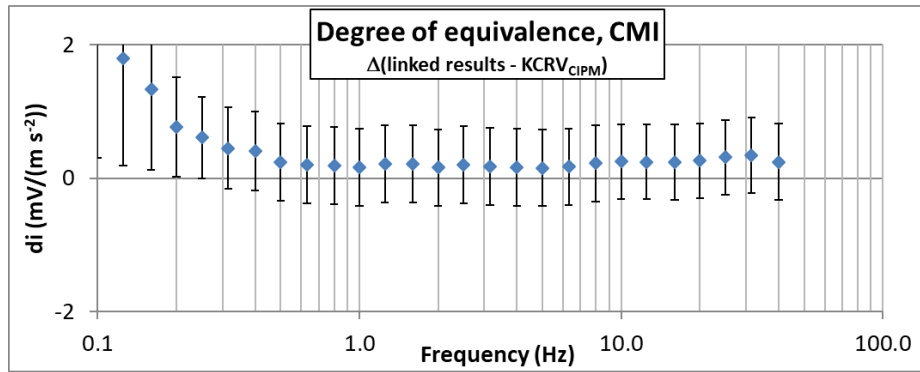
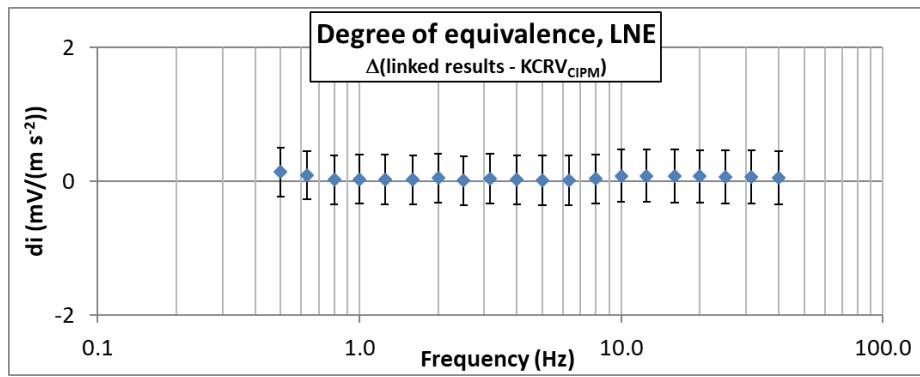
$$u^2(d_i) = u^2(z_i) + \left[1 - 2\frac{z_i}{x}\right] u^2(x) \quad i = 1, \dots, M$$

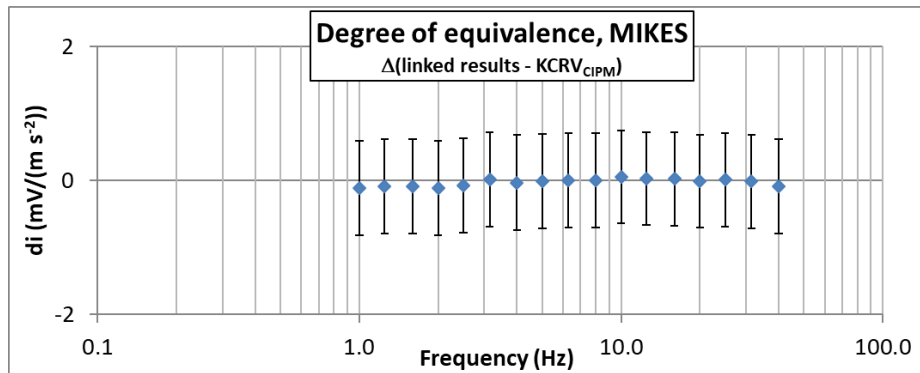
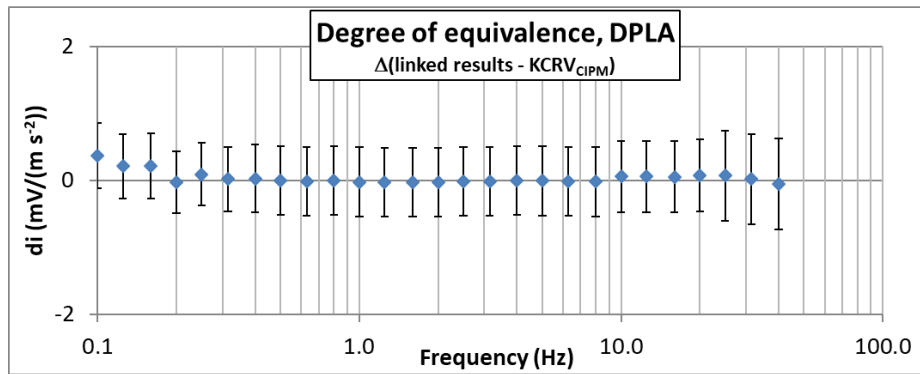
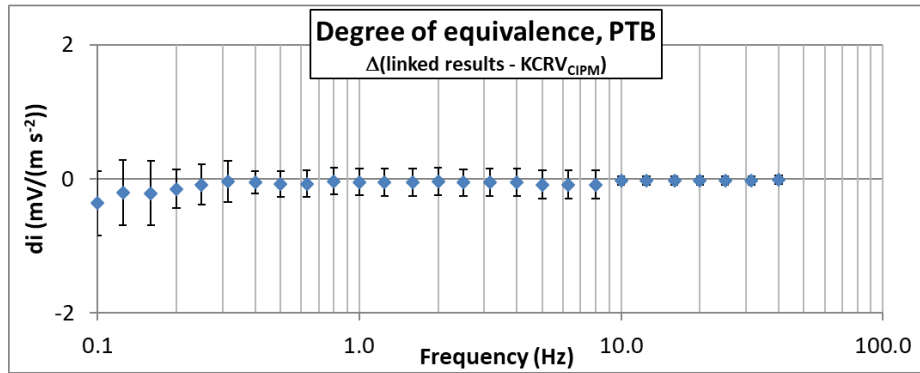
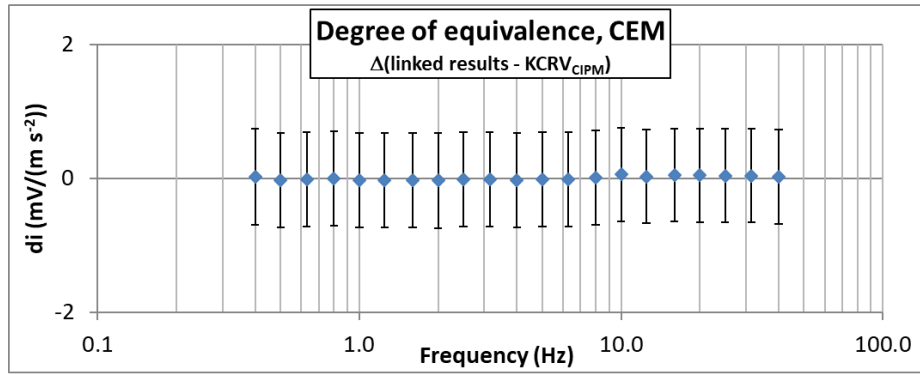
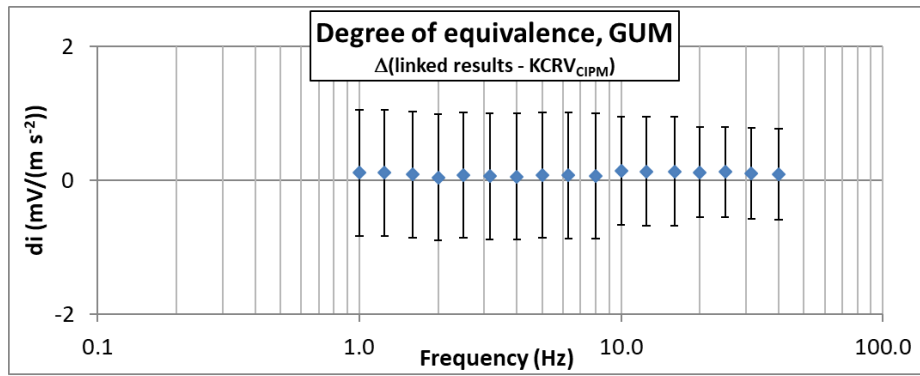
### 3. Results

The following table shows the so obtained degrees of equivalence and the associated uncertainties for the amplitude of the complex sensitivity .

**Table 1. Degrees of equivalence and the associated expanded uncertainties for the amplitude of the complex sensitivity**

$f$ (Hz)	LNE		CMI		SP		METAS		INRIM		GUM		CEM		PTB		DPLA		MIKES	
	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$	$d_i$	$2u_i$
0.100			2.73	2.42											-0.37	0.48	0.37	0.49		
0.125			1.79	1.60											-0.21	0.48	0.21	0.48		
0.160			1.34	1.21											-0.21	0.48	0.22	0.48		
0.200			0.77	0.75			0.41	0.57							-0.15	0.29	-0.03	0.46		
0.250			0.61	0.62			0.06	0.44							-0.09	0.30	0.09	0.47		
0.315			0.45	0.61	-1.24	2.45	0.04	0.38							-0.04	0.31	0.02	0.48		
0.400			0.41	0.59	-0.77	1.51	0.12	0.39					0.02	0.72	-0.05	0.17	0.02	0.50		
0.500	0.14	0.36	0.25	0.58	-0.49	1.10	0.04	0.39	-0.33	0.58			-0.03	0.71	-0.08	0.20	0.00	0.51		
0.630	0.09	0.36	0.20	0.58	-0.26	0.71	0.09	0.38					-0.02	0.71	-0.07	0.20	-0.02	0.51		
0.800	0.02	0.36	0.19	0.58	-0.14	0.58	0.06	0.38	-0.44	0.58			0.00	0.71	-0.04	0.20	0.00	0.51		
1.000	0.03	0.37	0.16	0.58	-0.11	0.45	0.07	0.37	-0.57	0.57	0.11	0.94	-0.03	0.71	-0.05	0.20	-0.03	0.51	-0.11	0.71
1.250	0.03	0.37	0.22	0.58	-0.08	0.45	0.09	0.37	-0.44	0.57	0.11	0.94	-0.03	0.71	-0.05	0.20	-0.03	0.51	-0.08	0.71
1.600	0.02	0.37	0.22	0.58	-0.06	0.45	0.09	0.37	-0.30	0.57	0.08	0.94	-0.03	0.71	-0.05	0.20	-0.03	0.51	-0.08	0.71
2.000	0.05	0.37	0.16	0.58	-0.03	0.45	0.05	0.37	-0.22	0.58	0.04	0.94	-0.03	0.71	-0.04	0.20	-0.03	0.51	-0.11	0.71
2.500	0.01	0.37	0.20	0.58	0.04	0.45	0.11	0.35	-0.21	0.57	0.08	0.94	-0.02	0.71	-0.06	0.20	-0.02	0.51	-0.07	0.71
3.150	0.04	0.37	0.18	0.58	0.01	0.45	0.08	0.35	-0.21	0.57	0.06	0.94	-0.02	0.71	-0.06	0.20	-0.02	0.51	0.01	0.71
4.000	0.02	0.37	0.16	0.58	0.02	0.45	0.08	0.35	-0.28	0.57	0.06	0.94	-0.03	0.71	-0.05	0.20	0.00	0.52	-0.03	0.71
5.000	0.02	0.37	0.15	0.57	0.02	0.45	0.11	0.28	-0.26	0.57	0.08	0.94	-0.01	0.70	-0.09	0.21	-0.01	0.52	-0.01	0.70
6.300	0.01	0.37	0.17	0.57	0.04	0.45	0.12	0.28	-0.26	0.57	0.07	0.94	-0.02	0.70	-0.09	0.21	-0.02	0.52	0.01	0.70
8.000	0.03	0.37	0.22	0.57	0.03	0.45	0.10	0.28	-0.24	0.57	0.07	0.94	0.01	0.70	-0.09	0.21	-0.02	0.52	0.01	0.70
10.000	0.08	0.39	0.25	0.56	0.11	0.43	0.20	0.49	-0.19	0.56	0.14	0.81	0.06	0.69	-0.03	0.06	0.06	0.53	0.06	0.69
12.500	0.08	0.39	0.25	0.56	0.14	0.43	0.16	0.49	-0.19	0.56	0.13	0.81	0.03	0.69	-0.03	0.06	0.05	0.53	0.03	0.69
16.000	0.08	0.39	0.24	0.56	0.10	0.43	0.16	0.49	-0.14	0.56	0.13	0.81	0.05	0.69	-0.03	0.06	0.05	0.53	0.02	0.69
20.000	0.07	0.39	0.26	0.56	0.10	0.43	0.12	0.49	-0.12	0.56	0.12	0.68	0.04	0.70	-0.03	0.06	0.07	0.54	-0.01	0.70
25.000	0.07	0.39	0.31	0.56	0.09	0.43	0.14	0.48	-0.15	0.56	0.13	0.68	0.04	0.70	-0.03	0.06	0.07	0.68	0.01	0.70
31.500	0.07	0.40	0.34	0.57	0.10	0.43	0.12	0.48	-0.12	0.56	0.11	0.68	0.04	0.70	-0.02	0.06	0.01	0.68	-0.01	0.70
40.000	0.05	0.40	0.25	0.57	0.03	0.43	0.13	0.48	-0.11	0.57	0.09	0.68	0.03	0.70	-0.02	0.06	-0.06	0.68	-0.08	0.70





The situation is somewhat different regarding the phase data evaluated during these two comparisons.

- First of all, the phase data of the two measurement artefacts are not related by a proportionality factor. The linking approach used to analyze the amplitude data can therefore not be applied to these phase data.
- Furthermore, the value of the phase response obtained by the weighted mean of the linking laboratories of the RMO comparison equals zero for practically all frequencies when taking into account the corresponding uncertainty.

Therefore, we decided renounce to any further processing of the phase response data with the purpose of establishing a link between the two key comparisons.

## 5. Conclusions

In the here presented analysis the RMO results were converted into the corresponding data equivalent to the CIPM comparison.

It was found that the measurement results obtained by all RMO comparison participants are also consistent with the KCRV of the CIPM comparison. The only exception to this observation are the low frequency results up to 0.2 Hz provided by CMI. This finding is, however, not surprising when considering, that these results were also inconsistent with the KCRV of the RMO comparison [2].

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